or

$$\frac{\hbar^2 pk}{2m} + \frac{\hbar^2 k^2}{2m} = \hbar\omega_p.$$

Since p is about k_0 ($\hbar^2 k_0^2/2m = \epsilon_F$, the Fermi energy), and $k < \omega_p/v_0$ is of the order of 10^8 cm^{-1}, the condition holds, so that plasmons may decay and transfer their energy to electrons. Therefore groups of electrons originating from plasmon decay may occur in secondary electron emission spectra, since the plasmon energy $\hbar\omega_p$ is very large compared with the single-particle energy $\hbar k v_0$.

The details of the rapid analyzer, experimental procedure, and results concerning the abovementioned sample materials will be published as soon as possible.

I am indebted to Professor Dr. O. Glemser for many useful discussions.

[1] D. Bohm and D. Pines, Phys. Rev. 82, 625 (1951), and 85, 338 (1952).

[2] R. H. Ritchie, Phys. Rev. 106, 874 (1957).

[3] E. A. Stern and R. A. Ferrell, Phys. Rev. 120, 130 (1960).

[4] F. Fujimoto and K. Komaki, J. Phys. Soc. Jap. 25, 1679 (1968); C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1963), p. 48.

[5] C. J. Powell and J. B. Swan, Phys. Rev. 118, 640 (1960).

[6] D. M. Roessler and W. C. Walker, Phys. Rev. 166, 599 (1968).

[7] E. T. Arakawa and M. W. Williams, J. Phys. Chem. Solids 29, 735 (1968).

[8] Fujimoto and Komaki, Ref. 4.

[9] N. B. Gornyi, Fiz. Tverd. Tela 8, 1939 (1966) [Sov. Phys. Solid State 8, 1535 (1966)].

[10] R. Kollath, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1956), Vol. 21, pp. 232-291.

[11] D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1964).

CURRENT-PHASE RELATIONS IN SUPERCONDUCTING BRIDGES

T. A. Fulton and R. C. Dynes
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 8 June 1970)

The current-phase relation $I_s(\varphi)$ for an Anderson-Dayem bridge is investigated making experimental use of the quantum interference between two such bridges. Near T_c and for critical currents $\lesssim 10\ \mu$A, $I_s(\varphi)$ is found to be nearly sinusoidal.

In a recent experiment Simmonds and Parker[1] measured the current-voltage (I-V) characteristic of a superconducting weak link comprising a Sn thin-film constriction, or Anderson-Dayem bridge.[2] Their measurements were conducted near the critical temperature T_c of Sn. In this region the critical currents of the bridges are small enough so that thermal noise is expected to influence markedly the shape of the I-V curve. Their results were compared with Ambegaokar and Halperin's[3] theoretical prediction of the shape of the I-V curve for a resistively shunted, zero-capacitance Josephson tunnel junction[4] in the presence of thermal noise.

A fundamental but unknown parameter in this comparison is the shape of the current-phase relation $I_s(\varphi)$, which relates the magnitude of the supercurrent flow in the bridge to the phase difference between the order parameters of the superconductors connected by the bridge. Simmonds and Parker used as model $I_s(\varphi)$ both the sinusoid appropriate to Josephson junctions and a sawtooth, the latter providing a somewhat better fit to their data. It is not obvious, however, that such bridges necessarily possess a continuous, single-valued $I_s(\varphi)$ after the manner of Josephson junctions.[5] For example the phase difference required to increase the supercurrent to its maximum, I_c, could exceed π, corresponding to an effectively multivalued current-phase relation. Such a situation is compatible with the observed ac Josephson behavior and with the vortex-flow picture of phase slippage of Anderson and Dayem.[2] In the absence of a definite knowledge of the shape of $I_s(\varphi)$ the interpretation of results of experiments on the weak-link properties of bridges, e.g. that of Simmonds and Parker,[1] or, say, on electromagnetic radiation response[2,6] must be somewhat tentative.

To resolve this question we have investigated experimentally the shape of $I_s(\varphi)$ for Anderson-Dayem bridges. The experiment involves measuring the dependence on applied magnetic field B of the critical current I_c of a quantum interferometer[7] comprising two bridges in parallel. We sketch here how one may gain information concerning $I_s(\varphi)$ from such a measurement. For a double weak-link interferometer, having current-phase relations $I_{s1}(\varphi_1)$ and $I_{s2}(\varphi_2)$ and critical currents I_{c1} and $I_{c2} \gg I_{c1}$ for the individual weak links, the usual analysis[8] shows that I_c is an oscillatory function of B, repeating whenever the magnetic flux Φ linking the area enclosed by the interferometer loop increases by a flux quantum $\Phi_0 = 2.07\times 10^{-15}$ Wb. One of us has given elsewhere[9] a discussion of the shapes of the $I_c(B)$ "fringe patterns" which occur in the general case of an interferometer having an arbitrary loop self-inductance L and geometrical asymmetry, assuming only that the $I_{s1}(\varphi_1)$ and $I_{s2}(\varphi_2)$ are periodic, odd, and reasonably smooth. It can be shown that if the inequality

$$2\pi L/\Phi_0 < \left| \left(\frac{dI_{s1}}{d\varphi_1}\right)^{-1}_{\varphi_1=\pi} + \left(\frac{dI_{s2}}{d\varphi_2}\right)^{-1}_{\varphi_2=0} \right| \tag{1}$$

is obeyed, the $I_c(B)$ fringes will almost always be smoothly continuous, while if (1) does not hold, the minima of fringes will be cusplike. Further, if $I_{s1}(\varphi_1)$ is not a smoothly continuous, single-valued function then the $I_c(B)$ fringes will not be smooth for any L, so that observation of smooth fringes shows that $I_{s1}(\varphi_1)$ is such a well-behaved function. For sinusoidal $I_s(\varphi)$, (1) becomes $2\pi LI_{c1}/\Phi_0 + I_{c1}/I_{c2} < 1$, and similar values of L, I_{c1}, and I_{c2} will be required to meet (1) for other shapes of $I_s(\varphi)$.

If $I_c(B)$ is a smooth, continuous function it turns out one can construct uniquely from it an "effective" current-phase relation $f(\theta)$, related to $I_{s1}(\varphi_1)$ by

$$f(\theta) = I_{s1}(\varphi(\theta)),$$

where

$$\theta = \varphi_1 + 2\pi L_1 I_{s1}(\varphi_1)/\Phi_0.$$

Here L_1 is a constant inductance parameter, $L_1 I_{s1}$ being the magnetic flux linking the interferometer loop due solely to I_{s1}, so that $L_1 \leqslant L$ for most geometries. The $f(\theta)$ is essentially a plot of I_{s1} against the effective phase variable θ, φ and θ being related by the single parameter L_1. If $L_1 = 0$ then $f(\theta)$ is identically $I_{s1}(\varphi_1)$, while for nonzero values of L_1, $f(\theta)$ is a skewed version of $I_{s1}(\varphi_1)$, rising more slowly and descending more steeply with a delay in phase of the maximum value by $2\pi L_1 I_{c1}/\Phi_0$. If (1) holds, $f(\theta)$ is single-valued. Although $f(\theta)$ has the same shape as the oscillatory part of $I_c(B)$ only if (1) is strongly obeyed, i.e., $I_{c1} \ll I_{c2}$ and $2\pi LI_{c1} \ll \Phi_0$, the amplitude $(2I_{c1})$ of $f(\theta)$ and the relative separations of its maxima and minima are identical to those of the $I_c(B)$ oscillations in all cases where (1) is obeyed, so that these important features can be taken directly from the raw data. To determine $I_{s1}(\varphi_1)$ finally from $f(\theta)$ one must either know L_1 or be sure it is small enough that $\theta \approx \varphi_1$. Both to satisfy (1) and to meet this criterion it is desirable to keep L as small as possible.

In our experiments the interferometer was formed by evaporating a thin (500-1000 Å) Sn film on a glass substrate and scribing it to form a double bridge configuration (inset, Fig. 1). The bridges were typically constrictions tapering at ≈45° half-angle to a narrowest dimension of 1-5 μm. The interferometer loops were roughly rectangular, 3-10 μm wide by 20-100 μm long, the bridges being separated by the larger dimension. For loops of this size we estimate $L \sim 10^{-10}$-10^{-11} H. In some cases L was further reduced by evaporating a superconducting over-

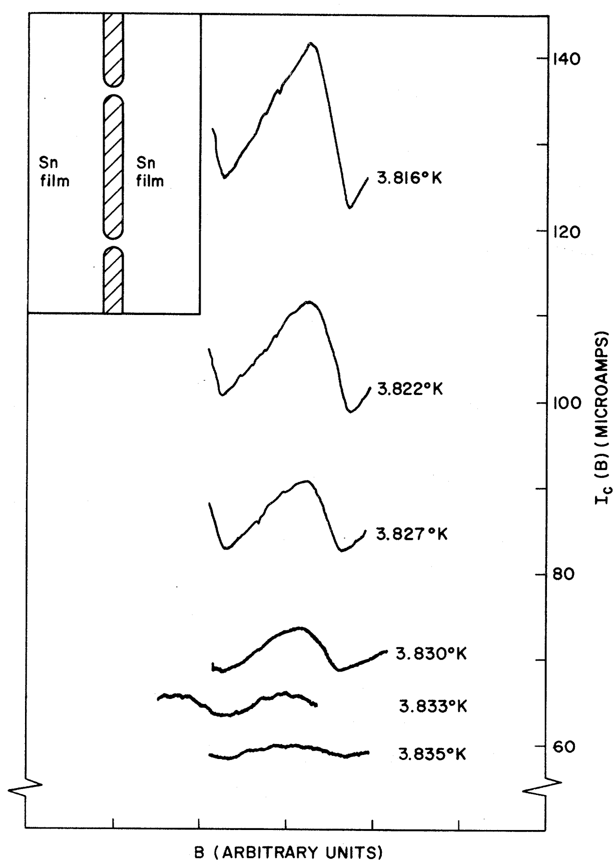

FIG. 1. I_c vs B over one period of the interference pattern for various temperatures for one interferometer. Inset: schematic diagram of the double bridge configuration used. Typical dimensions are given in the text.

lay of Pb or Sn, electrical insulation from the device being provided by heavy oxidation of the surface of the device in an oxygen plasma discharge. The interferometers were immersed in liquid He and shielded from stray magnetic fields by Mumetal. I_c was recorded continuously versus applied field B, using a feedback arrangement which locked the current bias to the voltage onset at I_c. For devices with an overlay B was applied to the loop by passing current (a few mA) through one of the large Sn films forming one side of the interferometer, parallel to the long dimension of the loop.

The $I_c(B)$ recorded in this fashion showed well-defined quantum interference oscillations whose periods were in rough accord with the measured areas of the loops. The concentration of the perpendicular applied field at the film edges by the demagnetizing effects of the thin film precluded an exact comparison. The major problem in measuring the shapes of the oscillations was the trapping of flux in the thin films due to the small critical perpendicular fields, resulting in hysteresis of the $I_c(B)$ fringe patterns with respect to the direction of field sweep. The procedure adopted was to increase the field to some value and then measure $I_c(B)$ while decreasing the field. In this way about one to one and one-half fringes could be plotted reversibly, with flux jumping and hysteresis setting in for larger sweeps.

Although the quantum interference patterns are observed at low temperatures as well as near T_c, for the $L \approx 10^{-10}$-10^{-11} H encountered in these devices the inequality (1) is satisfied only for $I_c \lesssim 10$ μA, a condition occurring only near T_c. Representative $I_c(B)$ curves for one particular interferometer in this temperature range are shown in Fig. 1. For this device the bridges were 1.4 and 2.0 μm wide, the film thickness was 700 Å, and the loop area was ≈460 μm^2. A Pb-Sn sandwich overlay was applied to reduce L. The T_c's of the two bridges of this device differed slightly.[10] For T lying between the two T_c's the bridge supercurrent was due to just one of the bridges, a condition existing for $0 \geq I_c(T) \lesssim 50$ μA. The T_c of the second bridge was signaled by the appearence of a quantum interference pattern at a critical current $I_c \approx 60$ μA and a corresponding temperature of $T = 3.84°$. As T was decreased further the maximum I_c and the modulation depth both increased rapidly.

The oscillations of $I_c(B)$ were initially smoothly continuous for modulation depths of $\lesssim 10$ μA, while at lower T the minima of $I_c(B)$ became cusplike, indicating that (1) was satisfied in the smaller domulation regions but was violated eventually due to the increasing critical current I_{c1} of the weaker bridge. The fact that the fringes are smoothly continuous at low amplitude shows that the Anderson-Dayem bridges do indeed possess a smoothly varying, single-valued current-phase relation in this regime, and since $I_{c1} \ll I_{c2}$ the observed $I_c(B)$ at the lower modulation depths have the form of $f(\theta)$ for this bridge, particularly the two lower curves of Fig. 1. The shapes of the curves suggest a rather smooth, symmetric $f(\theta)$ not too different from a sinusoid.

In Fig. 2 we show similar data for another interferometer which give a better indication of the shape of $f(\theta)$. The bridge widths for this device were 3.7 and 4.6 μm, the film thickness was 700 Å, and the loop area was ≈50 μm^2. In this case the critical currents of the two bridges were more nearly the same at these temperatures than in the previous case, so that the $I_c(B)$ curves show a larger percentage modulation with cusps at the minima until the critical currents are small, but a transition to smooth minima does occur in the lower two curves of Fig. 2. Here, since (1) is only just satisfied, the $I_c(B)$ is not

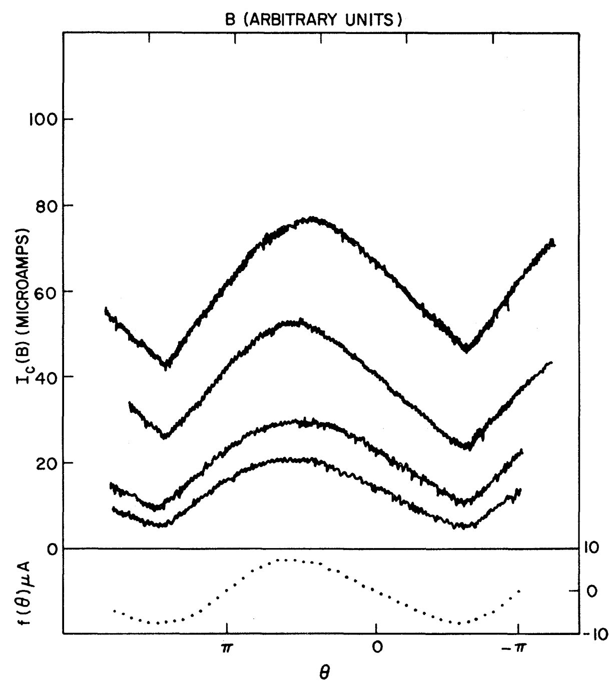

FIG. 2. I_c vs B over one period of the interference pattern for a second interferometer in the region of 3.80°K. Higher critical currents correspond to lower temperatures. Inset: the function $f(\theta)$ extracted from the highest temperature $I_c(B)$ pattern from this figure.

a good representation of $f(\theta)$, and the $f(\theta)$ derived[9] from the lowest curve is shown by the dotted line. Note that the maximum and minimum of $f(\theta)$ are located at 0.6π and -0.6π. Although the value of L_1 is not known, we recall that the effect of a nonzero value of L_1 is to cause the separation of the extrema of $f(\theta)$ to exceed that of $I_s(\varphi)$ by just $4\pi L_1 I_{c1}/\Phi_0$. The true $I_s(\varphi)$ is then less skewed than the $f(\theta)$. In view of the fairly small skewness of $f(\theta)$ we believe that the true $I_s(\varphi)$ for the bridge is probably essentially sinusoidal, with the skewness of $f(\theta)$ attributable entirely to the nonzero inductance. The value of L_1 required to skew a sinusoid to the observed extent is 2.7×10^{-11} H, a reasonable value considering the size of the interferometer loop.

A periodic $I_c(B)$ necessarily results from the periodicity of the free energy in applied flux of a multiply connected structure,[11] whatever the critical current mechanism. One might question, then, our use of a weak-link picture for the analysis of $I_c(B)$. A description of supercurrent flow in these terms, however, is actually rather general. Consider specifically a bridge between bulk superconductors, where "bulk" means that the Landau-Ginzburg equations with an order parameter of constant magnitude describe the supercurrent flow. Here φ, the gauge-invariant phase difference[12] taken along a fixed path joining a point in each bulk material, obeys generally the Gor'kov-Josephson frequency relation,[4,12] $\hbar d\varphi/dt = 2\Delta\mu$, where $\Delta\mu$ is the chemical potential difference between the points. By the thermodynamics arguments of Josephson[4] the supercurrent I_s must be a function of φ in the absence of dissipation. This function $I_s(\varphi)$, which could be measured in principle by incorporating the structure into a superconducting ring and applying a magnetic field,[13] we take as the current-phase relation for the bridge. For double bridges between bulk materials an $I_{s1}(\varphi_1)$ and an $I_{s2}(\varphi_2)$ can be similarly defined for each bridge and the constraint $\varphi_2-\varphi_1 = 2\pi\Phi/\Phi_0$ follows from the bulk nature of the materials. Under these conditions the weak-link approach applies. The $I_s(\varphi)$ for any particular bridge could at one extreme be a continuous, single-valued function periodic for φ changing by 2π, as for a Josephson junction. Alternately, I_s might increase continuously with φ over a range much larger than 2π, with $I_s(\varphi)$ ending abruptly at the critical current,[5] as for a long, narrow, thin-film strip. In both cases the arguments of Ref. 9 apply. In the present experiments, however, discontinuous $I_s(\varphi)$ are ruled out by the observation of a smoothly continuous $I_c(B)$.

In summary, we have presented here evidence that Anderson-Dayem bridges at temperatures near T_c and for critical currents of the order of 10 μA possess continuous, single-valued supercurrent-phase relations and that the shape of these relations in this regime is nearly sinusoidal.

---

[1]M. Simmonds and W. H. Parker, Phys. Rev. Lett. 24, 876 (1970).

[2]P. W. Anderson and A. H. Dayem, Phys. Rev. Lett. 13, 195 (1964).

[3]V. Ambegaokar and B. I. Halperin, Phys. Rev. Lett. 22, 1364 (1969).

[4]B. D. Josephson, Phys. Lett. 1, 251 (1962), and Advan. Phys. 14, 419 (1965).

[5]D. E. McCumber, J. Appl. Phys. 39, 2503 (1968).

[6]A. H. Dayem and J. J. Wiegan, Phys. Rev. 155, 419 (1967).

[7]R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, Phys. Rev. Lett. 12, 159 (1964).

[8]E.g., J. E. Mercereau, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969).

[9]T. A. Fulton, to be published.

[10]A. F. G. Wyatt, V. M. Dmitriev, and W. S. Moore, in *Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow U. S. S. R., 31 August–6 September 1966* (VINITI, Moscow, 1967).

[11]W. A. Little and R. D. Parks, Phys. Rev. Lett. 9, 9 (1962), and Phys. Rev. 133, A97 (1964).

[12]E.g., B. D. Josephson, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969).

[13]F. Bloch, Phys. Rev. Lett. 21, 1241 (1968).