THEORY OF RESISTIVITY IN COLLISIONLESS PLASMA*

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(Received 8 June 1970)

An electron drift driven by an applied constant electric field causes an ion acoustic instability. A steady state is proposed in which ballistic clumps of plasma behave like dressed test particles and collisionally scatter each other. The applied field is balanced by the dynamical friction force on the clumps. The resulting conductivity is $\sigma \approx 10\omega_{pe}/k\lambda_D$.

In a collisionless plasma the resistivity is generally believed to be due to the scattering of particles by waves driven unstable by the applied field. However, because of the formation of plasma clumps[1,2] it appears likely that a rather different state develops in which the electron and ion clumps behave like macroparticles and collisionally scatter each other.

We assume that the plasma contains a uniform electric field $\vec{E}_0$ (parallel to a strong magnetic field) which causes the electrons to acquire an average velocity u which in turn drives an ion-acoustic instability. We further assume that a quasisteady state develops in which macroscopic properties such as the average distribution function $f(\vec{v})$ change slowly compared with the ion-acoustic frequency $\omega_s = kv_s \approx k(T_e/m_i)^{1/2}$. ($e$ = electron, i = ion; q_e, m_e, n_e, and T_e are electron charge, mass, average number density, and temperature; $\omega_{pe}^2 = 4\pi n_e q_e^2/m_e$, $T_e = \frac{1}{2}m_e v_e^2$, $\lambda_D^2 = T_e/4\pi n_e q_e^2$.) This problem has been considered by a number of authors.[3-5]

There are two basic questions to be answered. First, what ultimately limits the wave growth? Second, what, if anything, inhibits the electrons from freely accelerating? Several processes can limit wave growth; however, we shall assume that for a sufficiently large field E_0 the required energy and momentum transfer to the ions can be achieved only by trapping a portion of the ions in the waves.[6]

To answer the second question one must consider the types of wave-particle interactions that are possible. The conventional collisionless theory for $f(\vec{v})$ provides only for diffusion. However, diffusion can retard only the positive-slope portion of $f_e(\vec{v})$ and then only for $v \approx v_s$. For large wave amplitudes the resonance can be broadened or one can include higher-order diffusion and thereby involve a greater portion of velocity space in the resonance. But the negative-slope portion of f_e remains a problem if diffusion is the only operative process. The negative-slope portion must be retained if a quasisteady state is to be achieved. What is needed is a drag or dynamical friction force equal to $-\vec{E}_0$ which will retard electrons independent of $\partial f_e/\partial\vec{v}$. Such a force, which has previously been omitted from the theory, can occur in collisionless plasma.

To understand this force consider the development of the instability. As the electrons feed energy to the wave at the linear growth rate $\gamma_e \approx \frac{1}{2}\pi^{1/2}(u/v_e)\omega_s$, the wave grows and the ion wave-particle resonance width increases. Eventually some ions become resonant (trapped) and absorb wave energy at a rate γ_i sufficient to make the total nonlinear growth rate γ_{NL} vanish, i.e., $\gamma_{NL} = \gamma_i + \gamma_e = 0$. However, this is not an acceptable steady state because the resonant ion and electron scattering by the waves will cause ballistic clumps to form. These clumps will then act like discrete test particles and continuously emit undamped waves by the Cherenkov process, leading to an increasing wave energy density. Therefore, for a steady state we must require that γ_{NL} be sufficiently negative to just balance the Cherenkov emission. The state is analogous to the dressed test-particle model for linearly stable plasma.[7-10]

One can understand the formation of clumps as follows: Since plasma particles are randomly scattered by the electric field of the turbulent plasma, the distribution function will develop chaotic fine-grain fluctuations in phase space. Such fine-grain structures are ignored in the "smoothed" distribution function used in the quasilinear theory. These fluctuations can be thought of as a superposition of clumps of plasma. Consider fluctuations of size $\Delta^3 r\Delta^3 v$ in phase space. If $\Delta^3 r\Delta^3 v$ is small, then all particles of the same species lying inside $\Delta^3 r\Delta^3 v$ will move together as a clump since they will have approximately the same velocity and will experience approximately the same forces. The smaller the clump size $\Delta^3 r\Delta^3 v$, the longer the clump lifetime. For sufficiently small clumps, the lifetime can be longer than the clump collision time or other relaxation times of interest. Such clumps will then behave like a single large particle or macroparticle whose charge and mass is the aggregate of the charges and masses of the individual particles which comprise the clump. In the steady-state case, the clump population is constant in time. This is possible, even though the clumps have a finite lifetime, because the turbulent plasma is constantly creating new clumps.

The equation describing the formation of the clumps will be contained in a subsequent publication. Related problems of clump formation, dressing, and Cherenkov emission have been studied in the coherent-wave case[11-13] and the bump-on-tail instability.[2] Clumps which are basically ballistic modes have been considered recently in other connections.[14-16]

When $\gamma_{NL}<0$ in the steady state, the turbulent plasma electric field $\vec{E}$ will be due entirely to dressed clumps. This means that an additional field-particle correlation will develop and produce dynamical friction in addition to diffusion. The equation for $f(\vec{v})$ becomes a Fokker-Planck equation. It contains, in addition to the usual quasilinear diffusion, a drag term which is the reaction of the plasma-shielding cloud on the test clump.

The equation for the average distribution function $f(\vec{v})$ for each species is derived as follows. We use the notation $\varphi_{\vec{k}\omega}$ and $f_{\vec{k}\omega}$ for the Fourier transform (on $\vec{r}$ and t) of the fluctuating piece of the total potential and distribution function and $\tilde{\varphi}_{\vec{k}\omega}$ and $\tilde{f}_{\vec{k}\omega}$ for the corresponding clump portions. We can write for each species

$$f_{\vec{k}\omega}=\frac{q}{m}\varphi_{\vec{k}\omega}g_{\vec{k}\omega}i\vec{k}\cdot\frac{\partial f}{\partial\vec{v}}+\tilde{f}_{\vec{k}\omega}, \tag{1}$$

where $g_{\vec{k}\omega}$ is the Fourier transform (on $\vec{r}-\vec{r}_0$ and t) of the ensemble average (denoted by angular brackets) Green's function operator $\int d^3r_0\int d^3v_0 \times\langle\delta(\vec{r}_0-\vec{r}(-t))\delta(\vec{v}_0-\vec{v}(-t))\rangle s(t)$, where $\vec{r}(t)$ and $\vec{v}(t)$ are the particle orbits with initial values $\vec{r}$ and $\vec{v}$ and $s(t)=1$ for $t>0$, 0 for $t<0$. Equation (1) is solved by treating $\tilde{f}_{\vec{k}\omega}$ as a source and substituting in

$$\frac{\partial f}{\partial t}=\left(\frac{q}{m}\right)\frac{\partial}{\partial\vec{v}}\cdot(2\pi)^{-8}\int d^3k\int d^3k'\int d\omega\int d\omega'\,i\vec{k}'\langle\varphi_{\vec{k}'\omega'}f_{\vec{k}\omega}\rangle.$$

We put $\langle\tilde{\varphi}_{\vec{k}'\omega'}\tilde{f}_{\vec{k}\omega}\rangle=\langle\tilde{\varphi}\tilde{f}\rangle_{\vec{k}\omega}(2\pi)^4\delta(\vec{k}+\vec{k}')\delta(\omega+\omega')$, where $\langle\,\rangle_{\vec{k}\omega}$ denotes a Fourier transform with respect to $\vec{r}_1-\vec{r}_2$ and t_1-t_2, and eliminate the clump self-interaction ($\mathrm{Im}\langle\tilde{\varphi}\tilde{f}\rangle_{\vec{k}\omega}=0$). The resulting equation for the lth species is

$$\frac{\partial}{\partial t}f_l=\frac{q_l}{m_l}\frac{\partial}{\partial\vec{v}}\cdot\int\frac{d^3k}{(2\pi)^4}\int d\omega\frac{\vec{k}\vec{k}}{k^2|\epsilon(\vec{k},\omega)|^2}\sum_j 4\pi n_j q_j\cdot\left\{\left[\int\langle\tilde{\varphi}\tilde{f}_j(\vec{v})\rangle_{\vec{k}\omega}d^3v\right]\frac{q_l}{m_l}g_{\vec{k}\omega}{}^{(l)}\frac{\partial f_l}{\partial\vec{v}}-\langle\tilde{\varphi}\tilde{f}_j(\vec{v})\rangle_{\vec{k}\omega}\frac{q_l}{m_j}\int d^3v\,g_{\vec{k}\omega}{}^{(j)}\frac{\partial f_j}{\partial\vec{v}}\right\}. \tag{2}$$

$\epsilon(\vec{k},\omega)$ is the nonlinear dielectric constant

$$\epsilon(\vec{k},\omega)=1-\sum_j\frac{\omega_{pj}{}^2}{k^2}\int d^3v\,g_{\vec{k}\omega}{}^{(j)}i\vec{k}\cdot\frac{\partial f_j}{d\vec{v}}, \tag{3}$$

and

$$\langle\tilde{\varphi}\tilde{f}(\vec{v})\rangle_{\vec{k}\omega}=2\,\mathrm{Re}g_{\vec{k}\omega}\langle\tilde{\varphi}\tilde{f}(\vec{v})\rangle_{\vec{k}}. \tag{4}$$

If the turbulence level is low $g_{\vec{k}\omega}\approx i(\omega-\vec{k}\cdot\vec{v})^{-1}$ and if the clumps were just discrete particles $\langle\tilde{\varphi}\tilde{f}(\vec{v})\rangle_{\vec{k}\omega}=4\pi qk^{-2}f(\vec{v})2\pi\delta(\omega-\vec{k}\cdot\vec{v})$. In this case it is easy to see that (2) reduces to the Lenard-Balescu equation.[9,10] Of course in the Vlasov limit this discreteness vanishes. The first term in curly brackets in (2) is just quasilinear diffusion due to a random distribution of dressed clumps. The second term is the dynamical friction on each clump. Equation (2) is a clump collision operator.

One can easily show that (2) conserves total particle energy and momentum. From this it follows that the energy and momentum of any species cannot change due to collisions within that species [$l=j$ in (2)]. This ensures that electron-electron scattering will not change T_e, u, and therefore γ_e. Therefore we neglect the e-e terms in (2). (In the Lenard-Balescu collision integral, like-particle collisions do not change f at all in one dimension!)

$\partial f_e/\partial t$ consists of two terms: a diffusion term due to the dressed fields of ion clumps and a drag term describing ion friction on the electron clumps. For a steady state this drag force must just balance the applied field $\vec{E}_0$. From (2) and (3) this drag term is

$$\vec{E}_{\mathrm{drag}}f_e(\vec{v})=-\int\frac{d^3k}{(2\pi)^4}\int d\omega\frac{\vec{k}}{|\epsilon(\vec{k},\omega)|^2}\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}\mathrm{Im}\epsilon_i(\vec{k},\omega). \tag{5}$$

This can be approximately evaluated as follows for the one-dimensional case. Turbulent fields which are large enough to trap ions in acoustic waves will also trap most of the electrons since the electrons can respond to higher frequency fields. In such fields the stochastic electron motion will cause it to sample most of velocity space on a fast (trapping) time scale $(kv_e)^{-1}$. Therefore the velocity dependence of $\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}$ should just be proportional to the probability of finding an electron at $\vec{v}$: $\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}\approx\langle\tilde{\varphi}^2\rangle_{\vec{k}\omega}k^2f_e(\vec{v})/4\pi n_eq_e$. Using this, we can write

$$\int\frac{d^3k}{(2\pi)^4}\int d\omega\, k_{\parallel}\frac{\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}}{|\epsilon(\vec{k},\omega)|^2}\approx\frac{f_e(\vec{v})}{4\pi n_e q_e}\bar{k}\langle E^2\rangle, \tag{6}$$

where $\bar{k}$ is an appropriate average parallel to u. The requirement that the edge of the ion-distribution function is just barely trapped in a wave of phase speed v_s and wavelength $2\pi/\bar{k}$ implies that $\langle E^2\rangle\approx(\bar{k}m_iv_s^2/4\pi q_i)^2$. Using this result along with (6) in (5) and putting $\mathrm{Im}\epsilon_i=-\gamma_i(\partial\epsilon/\partial\omega)\approx-2\gamma_i/\omega_s\bar{k}^2\lambda_D^2$, we obtain

$$E_{\mathrm{drag}}=\left(\frac{\bar{k}m_iv_s^2}{4\pi q_i}\right)^2\frac{\bar{k}}{4\pi n_e q_e}\frac{2\gamma_i}{\omega_s\bar{k}^2\lambda_D^2}. \tag{7}$$

Setting $E_0=-E_{\mathrm{drag}}$ and $\gamma_i\approx-\gamma_e=-\pi^{1/2}u\omega_s/2v_e$ we obtain for the steady-state conductivity

$$\sigma\equiv\frac{n_eq_eu}{E_0}=\frac{(32\pi)^{1/2}\omega_{pe}}{\bar{k}\lambda_D}. \tag{8}$$

This value of σ is the same order of magnitude as those observed in several experiments.[17,18]

The nature of the drag term can be made somewhat more intuitive by writing (5) in a different form. By expanding $\epsilon(\vec{k},\omega)$ around the wave resonance at $\omega=\omega_s$, (5) becomes

$$\int\frac{d^3k}{(2\pi)^4}\int d\omega\frac{\vec{k}\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}\gamma_i}{(\partial\epsilon/\partial\omega)[(\omega-\omega_s)^2+\gamma_{NL}^2]}=-\int\frac{d^3k}{(2\pi)^3}\vec{k}\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}\frac{\gamma_i}{2(\partial\epsilon/\partial\omega_s)\gamma_{NL}}.$$

Since the frequency spectrum of the electron clump density has a width of order kv_e, the trapping frequency, we can write $\langle\tilde{\varphi}\tilde{f}_e(\vec{v})\rangle_{\vec{k}\omega}\approx4\pi q_ek^{-2}f_e(\vec{v})(\langle\tilde{n}_e^2\rangle_k/n)[2kv_e/(\omega^2+k^2v_e^2)]$, where $\tilde{n}_e$ is the clump density. Equation (5) for the drag force on each electron now becomes

$$q_e\vec{E}_{\mathrm{drag}}=\int\frac{d^3k}{(2\pi)^3}\left[\frac{4\pi q_e^2\gamma_i\vec{k}}{(\partial\epsilon/\partial\omega_s)\gamma_{NL}^2k^2}\right]\left(\frac{-\gamma_{NL}}{kv_e}\right)\left(\frac{\langle\tilde{n}_e^2\rangle_{\vec{k}}}{n}\right). \tag{9}$$

The first factor in (9) is the drag force on an electron of speed v_s due to Cherenkov emission of an acoustic wave whose growth rate is γ_{NL}. The second factor is that portion of the ω spectrum of the electron clump charge density lying within the Cherenkov resonance. The third factor is the number of electrons in the clump.

The following picture for the drag force emerges. The electron clumps follow erratic and random orbits which carry them throughout the electron velocity space in a time of order $(kv_e)^{-1}$. The resulting clump charge density $\tilde{\rho}_{\vec{k}\omega}$ induces a field $\vec{E}_{\vec{k}\omega}$ in the background plasma. On the average, this field exerts a force density $\int d^3k\int d\omega\times(2\pi)^{-4}\langle\vec{E}\tilde{\rho}\rangle_{\vec{k}\omega}$ on the clump. The principal contribution to this force is at the resonance $\omega=\omega_s$. The resonance corresponds to Cherenkov emission of the acoustic wave in the direction of u and therefore the force is always in the $-u$ direction. Since particle "mixing" in the electron-velocity space is fairly complete due to the e-e terms in (2), this force applies uniformly to almost all electrons. Of course those electrons which are not resonant (trapped) will run away.

The fact that direct electron-electron interaction cannot change T_e or u plays an essential role in this model. If $\langle E^2\rangle$ is large enough to trap most of the electrons then velocity diffusion will heat electrons at a rate $(d/dt)n_eT_e\gg\sigma E_0^2$ which is, of course, impossible. The problem is how to explain the coexistence of a large $\langle E^2\rangle$ with electrons having a small $(\partial/\partial t)T_e$. In this model the answer is that the inclusion of a dynamical friction force cancels the electron energy gain from diffusion due to other electrons (i.e., conserves energy in electron-electron collisions). T_e can change only through interaction with the ions or, in other words, with that portion of $\langle E^2\rangle$ driven by ion clumps. This reduces $(\partial/\partial t)T_e$ by a factor γ_i/ω_{pe}.

A number of enlightening conversations with various members of the Trieste Plasma Physics Group, especially B. Kadomtsev, T. O'Neil, and

R. Sudan, are gratefully acknowledged. The author is grateful to Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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*Work supported in part by the U. S. Atomic Energy Commission, Contract No. AT(30)-1-3980, and by the National Science Foundation, Grant No. GK-10472.

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EVIDENCE OF PLASMONS IN SECONDARY ELECTRON EMISSION SPECTRA

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(Received 16 June 1970)

New secondary electron emission spectra have been obtained by a rapid analyzer, which presents the full spectra in the range from 0 to 100 eV in 1 msec on an oscilloscope. The spectra show distinct peaks corresponding in energy to the volume and surface plasmons: for Al at 15.2 eV; Al_2O_3 at 31, 22.5, and 17.5 eV; and KCl at 13.6, 9, and 7 eV. It is suggested that these groups of secondary electrons originate from plasmon decay.

The predictions of volume and surface plasmons have been made by several authors.[1-3] For example, the surface-plasmon frequency ω_l of a metallic spherical particle surrounded by a medium of dielectric constant ϵ is $\omega_l = \omega_p\{1+[(l+1)/l]\epsilon\}^{-1/2}$ $(l=1, \cdots)$, ω_p being the well-known plasma frequency $\omega_p = (4\pi e^2 n/m)^{1/2}$, where n is the number density of electrons taking part in the oscillation.[4] Experimental verifications of the existence of volume and surface plasmons have been made by the measurement of distinct energy losses of the primary electrons[5] or by optical spectra in the far uv and computing the energy-loss function $\mathrm{Im}(1/\bar{\epsilon})$.[6,7] On the other hand, when plasmons decay, they may emit photons[8] or they may transfer their energy to conduction electrons. These electrons then can emerge from the solid giving rise to the so-called plasmon secondary emission.[9] As far as we know this has never been established by experiment. We obtained new secondary electron emission spectra of Al, Al_2O_3, and KCl with our rapid analyzer. Figure 1 shows the spectrum of aluminum. The main peak is located at 2 eV and corresponds to the commonly observed value for metals.[10] The peak at 15.2 eV should belong to the volume plasmon, because we get $\hbar\omega_p = 15.8$ eV with three conduction electrons taking part in the oscillation.

Figure 2 shows the spectrum of aluminum oxide. The main peak is located at 31 eV, the second at 22.5 eV, and the third at 17.5 eV. We obtain $\hbar\omega_p = 28$ eV for Al_2O_3 with 24 valence electrons and this should correspond to the experimental value of 31 eV, for ω_p is based on a free-