FIRST PHASE-BAND DESCRIPTION: π^+-p SCATTERING AT 2.50 AND 2.75 BeV/c*

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The first phase-band analysis of actual data is reported. Good solutions are obtained for π^+-p scattering at considerably higher energies than conventional phase shifts have reached.

Partial-wave decomposition of particle reactions remains an interesting analytical tool because resonances continue to appear as the energy of the reactions is raised. It is one of the central questions of particle physics whether resonances exist at any arbitrarily high energy or not. Since higher energy resonances are often partly inelastic and are buried within many other angular-momentum state contributions, they are not apparent without a detailed angular-momentum decomposition of the experimental observables. At the same time, as one goes to higher energies, such a decomposition in the form of the conventional phase-shift analysis becomes increasingly difficult because of the large number of parameters needed.

A method of overcoming this difficulty and thus extending the feasibility of partial-wave studies to arbitrarily high energies was suggested recently.[1] Its viability has been supported by an example on make-believe data, but the actual experimental information was then not sufficiently complete on any reaction so as to permit an application of the method in a real situation. The present paper reports the first use of the method in connection with actual data, which resulted in a good description of the π^+-p system at two energies, 2.50 and 2.75 BeV/c, which have so far been apparently out of reach for the conventional phase-shift work.

The method, called the phase-band analysis, was described in detail in Ref. 1 which therefore should be referred to for particulars. We will only summarize the most pertinent features. For spinless particles, the scattering amplitude in the phase-band analysis is written as

$$f(\theta)=\sum_{l=0}^{l'}(2l+1)(2ik)^{-1}[\eta(l)e^{2i\delta(l)}-1]P_l(\cos\theta)+\sum_{l=l'+1}^{l_0}(2l+1)(2ik)^{-1}(\eta_l e^{2i\delta_l}-1)P_l(\cos\theta), \tag{1}$$

where k is the center-of-mass wave number, $P_l(\cos\theta)$ is the Legendre polynomial, θ is the center-of-mass scattering angle, δ_l and η_l are the angular-momentum parameters in angular-momentum state l, and finally $\delta(l)$ and $\eta(l)$ are certain functions of the angular momentum l which will be discussed below.

The first sum in Eq. (1) represents the collective phase band in which not the individual angular-momentum parameters, but only the description of the characterization of the whole band as a unit are sought. The second sum contains those high angular-momentum states in which we want to determine the individual phases and absorption parameters. The feasibility of the method, which is expected to work the better the more angular momenta are involved, can be made plausible by representing it as a not too rough approximation to the situation in which we want to determine the infinite number of coefficients in the expansion of an errorless differential cross section, known at all angles, into a complete set of orthogonal functions. In that case not only can one separate "bands" of angular momenta, but in fact each angular-momentum contribution can be described completely independently of the knowledge of any other angular-momentum state.

In the present application to π^+-p scattering at 2.50 and 2.75 BeV/c we used $l'=4$ and $l_0=6$. The use of the method at such low energies is certainly not optimal, but relatively complete sets of data recently became available at these energies. The set at 2.50 BeV/c consisted of 70 pieces of data: two total cross sections,[2] 40 differential cross sections,[3] and 28 polarization measurements.[3] The set at 2.75 BeV/c consisted of 100 pieces of data: two total cross sections,[2] 55 differential cross sections,[4,5] and 43 polarization measurements.[6]

Using the traditional notation for the angular-momentum parameters, namely the ± superscripts for the $J = l \pm \frac{1}{2}$ states, the parametrization was as follows: In the collective region we used

$$\eta^+(l) = a_1 + b_1 l + c_1 l^2, \quad \delta^+(l) = a_3 + b_3 l + c_3 l^2, \quad \eta^-(l) = a_2 + b_2 l + c_2 l^2, \quad \delta^-(l) = a_4 + b_4 l + c_4 l^2. \tag{2}$$

Here the δ's are given in radians. Thus the collective region was described in terms of twelve parameters. The conventional phase-shift analysis would have required eighteen. The difference is substantial, though the real savings in terms of parameters of the phase-band method will be much more pronounced at higher energies.

These parametrizations are not based on any particular theoretical scheme, just as the energy dependence of the modified phase-shift analysis of nucleon-nucleon scattering has not always been described in terms of functions originating in theoretical models. Hence their suitability can be judged only a posteriori. It is hoped, however, that eventually parametrizations for such an analysis will also be provided by theoretical models.

The individual band of angular momenta was described by the eight traditional parameters.

The expected χ^2 for 70 pieces of data with 20 parameters is 50, while with 100 pieces of data with 20 parameters it is 80. The two χ^2 values of our solutions at the two energies are 55 and 97, respectively, giving ratios of 1.1 and 1.2, respectively. This is considered excellent, especially if one takes into account the circumstance that when data originate from several different experiments, the goodness of fit is unavoidably degraded by the difference in systematic errors between various laboratories.

The values of the angular-momentum parameters in both the collective and individual bands are listed in Table I. The η and δ in the individual band were determined by the data with statistical errors on the order of 0.03 for the η and 0.01 for the δ. The general behavior of the absorption parameters is as one would have expected. It should be strongly emphasized again that the individual values of the angular momentum parameters in the collective band cannot be expected to be reliable, and thus the negative values of a few of the low absorption parameters (which have a small weight) is not something to worry about. This is especially so since the two relatively large negative absorption parameters are coupled with large enough phase shifts so that the resulting $\eta e^{2i\delta}$ is the same as $\eta e^{2i\delta}$ with a positive absorption parameter and a small phase shift. Thus these negative absorption parameters are no less physical than the positive ones, as far as their effect on the data is concerned. Furthermore, the contributions of those low-l states are suppressed in the data anyway because of their small $2l+1$ factor. The results of interest are the parameters describing the two angular-momentum states on the top individual band.

We have no explanation for the fact that the parameters in the phase-band region are quite different for the two energies, except that 0.25 BeV/

Table I. Values of phase shifts and absorption parameters at 2.5 and 2.75 BeV/c. In the first two sets of columns, the angular momentum states from $l = 0$ to $l = 4$ were treated collectively, and hence, the parameters in these states are not expected to be significant individually. In these sets of columns the angular-momentum states with $l = 5$ and $l = 6$ were treated individually; hence, those phases represent the significant results of this analysis. The third set of columns give the parameters in all angular-momentum states determined individually from the successive approximation and iteration procedure starting from the phase-band values, as described in the text. The parameters in this set of columns therefore all have physical meaning. The δ's are in radians.

	2.5-BeV/c phase band				2.75-BeV/c phase band				2.75-BeV/c final iteration			
l	η_l^+	η_l^-	δ_l^+	δ_l^-	η_l^+	η_l^-	δ_l^+	δ_l^-	η_l^+	η_l^-	δ_l^+	δ_l^-
0	0.401	•••	0.227	•••	−0.442	•••	1.410	•••	0.445	•••	−0.161	•••
1	0.537	−0.400	0.083	−2.518	−0.002	0.790	0.783	−0.105	0.000	0.790	0.837	−0.106
2	0.646	0.037	0.013	−1.033	0.322	0.691	0.347	−0.010	0.320	0.690	0.351	−0.010
3	0.728	0.451	0.015	−0.195	0.530	0.672	0.100	0.023	0.531	0.674	0.098	0.022
4	0.783	0.841	0.090	−0.005	0.621	0.773	0.043	−0.005	0.621	0.734	0.044	−0.005
5	0.959	0.824	0.005	−0.098	0.916	0.904	−0.004	−0.048	0.916	0.905	−0.003	−0.048
6	1.000	0.866	0.034	−0.057	0.911	0.990	0.051	−0.039	0.910	0.991	0.051	−0.039

c is a rather large range in which parameters might change drastically. The parameters of the few other reasonable solutions we found at 2.75 BeV/c resemble much more those of the best 2.75 solution than those of the 2.5 solution, so the conjecture that perhaps the 2.5-BeV/c solution we found "corresponds" to another fairly good solution at 2.75 BeV/c is not likely to be true. On the other hand, since we do not believe that the phase-band parameters necessarily have physical meaning, we are not particularly worried about the whole phenomenon.

The highest available energy for conventional analysis[7] of π^+-p scattering at the moment is at 2.07 BeV/c. Extrapolation from there to 2.50 and 2.75 BeV/c is hardly an unambiguous matter. Furthermore, the highest l used in conventional analysis was 5, and thus the extrapolation is worth doing only in the two states with $l=5$. Argand diagrams for these are shown in Fig. 1. In the absence of error bars it is difficult to make definite statements about these diagrams, but the two extrapolations certainly do not look unreasonable, and might even turn out to be interesting if the apparent reversal of direction can be established as a loop by more thorough analysis.

We have performed at 2.75 BeV/c some 30 random searches to see whether there are other good solutions also. Most χ^2's were prohibitively high. There were, however, two solutions not too far in χ^2 from the 96.9 solution, with respective χ^2's of 103.4 and 112.6. The parameters in these solutions were, however, noticeably different from those in the 96.9 solution. The occurrence of several solutions is quite normal in partial-wave analyses, and the remaining ambiguity is usually resolved by further experiments or by the infusion of theoretical constraints.

We have also investigated the possibility that the small parameters found in the individual bands are perhaps meaningless since setting $\eta_5=\eta_6=1$ and $\delta_5=\delta_6=0$ could perhaps also give a good fit. This possibility turned out to be excluded with a high degree of probability: The best χ^2 we could obtain (after eleven random searches) with η_5 and η_6 fixed at 1 and δ_5 and δ_6 fixed at 0 was 1188 as compared to 96.9. Thus the parameters in the individual band definitely have a significant role in representing the data.

We also investigated to some extent the sensitivity of the results to the values of l' and l_0. We found that both the lowering and the raising of these values make a very striking increase in the χ^2 obtained, the first because too few angular momenta are included, the second because all significant angular momenta are included in the collective band and hence are too constrained. This again serves as a check of the basic feasibility of the method.

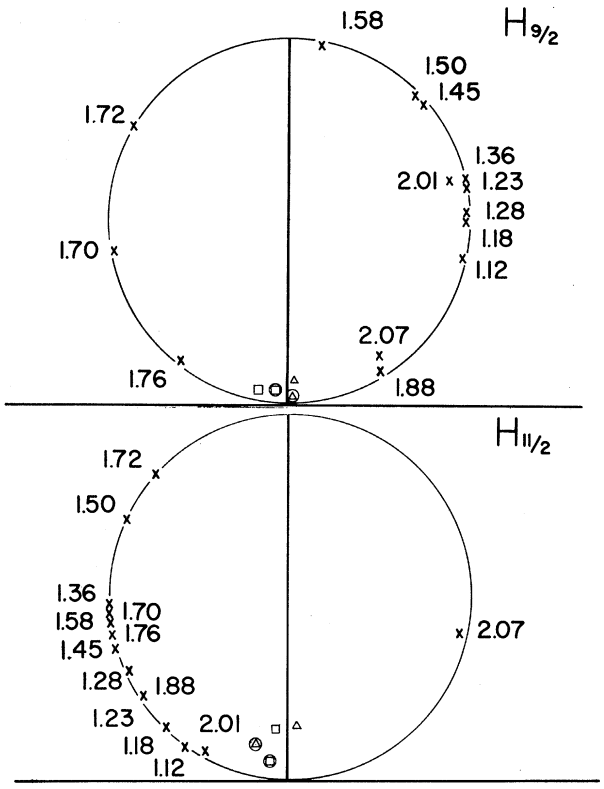

FIG. 1. Argand diagrams for the $l=5$ states. The numbers labeling the points give the energy in BeV/c. The crosses originate from Ref. 7. The encircled triangle is the result of the present paper at 2.5 BeV/c, and the encircled square is the present result at 2.75 BeV/c. The plain triangle and square are the results of Ref. 8 at 2.5 and 2.75 BeV/c, respectively.

The significance of these results for the present is threefold. First, it establishes that the phase-band method indeed can be made to work on real data. The fact that the present examples are rather unfavorable ones for the expected applicability of the method and, nevertheless, good results have been obtained strengthens the point. Second, we have learned something about the pion-nucleon system at energies where such information was not available previously. Finally, the feasibility of the method might serve as an inducement to perform more complete experiments on the pion-nucleon system at higher energies. Completeness would be particularly desirable in terms of the coverage of the whole

angular range, and in terms of data other than differential cross section.

After we had completed most of the work reported here, we received the results of a conventional analysis[8] which includes results at our two energies also. In most respects, that work was done very differently from ours. It is an energy-dependent, conventional phase-shift analysis of π^+-p, π^--p, and charge-exchange scattering (altogether more than 3000 pieces of data), in the energy range from about 1.9 to 2.8 BeV/c, in which various smoothing procedures were used to assure continuity in energy. Only partial waves up to $l=5$ were used, since in much of the energy range $l=6$ was not found to be necessary. Thus the only point of comparison between this work and ours is the four parameters for $l=5$. The agreement, shown in Fig. 1, is excellent, in fact so much so that it might be fortuitous in view of the considerable differences between the two procedures.

Finally, for the present mainly as a check on the method, we have used the phase-band method on one of our solutions in a successive iteration procedure to determine individually all partial wave parameters. This procedure, which is not only a check on the method but also a natural extension of the scheme leading to an easy and complete determination of all angular-momentum parameters at any given energy, went as follows: Using the best solution at 2.75 BeV/c (with the χ^2 of 97), we fixed the parameters for $l=5$ and 6 at their individually determined values, and then searched individually for the $l=3$ and 4 values, still keeping a collective parametrization for the $l=0$-2 partial waves. Having thus determined the $l=3$ and 4 values, now we fixed those also, and searched for $l=2$, and then for $l=1$ and 0. Having thus arrived at a set of individual values for all angular momentum parameters, we made an interative check by now fixing only the $l=5$ and 6 parameters and letting the others vary again. Finally, using as a starting point the results of the previous check, we "unleashed" all parameters simultaneously to find the over-all optimal solution. This final solution is also shown in Table I. It is rather suprising that except for the lowest angular-momentum states, the difference between the phase-band parameters and the final individually treated parameters is very small indeed.

During all these operations, the χ^2 dropped by only 1 (that is, from an initial value of 96.9 to 95.9), and the values of the individually determined parameters remained always the same to two figures. Thus we obtained what we considered a very convincing proof of the basic stability and reliability of the phase-band method and at the same time also demonstrated that it can be easily extended to yield a complete determination of all angular-momentum parameters. Incidentally, by the end of the iteration the two previously negative η have turned non-negative.

One can also compare the complete set of parameters thus obtained with the parameters obtained in Ref. 8. As mentioned before, the highest angular-momentum parameters agree very well. In the lower partial waves the quantitative agreement is not as good, although there is a qualitative correlation. Because of the differences between the two works, however, such a quantitative comparison is of questionable significance in any case.

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