RISING PHASE-SHIFT MODEL AND EVIDENCE FOR DAUGHTERS IN THE REACTION $\bar{p}n \to \pi^-\pi^+\pi^-$†

Austin M. Gleeson
Center for Particle Theory, University of Texas, Austin, Texas 78712

and

William J. Meggs*
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

and

Michael Parkinson
Department of Physics, University of Florida, Gainesville, Florida 32601
(Received 3 February 1970)

It is shown that many of the features of the recent data on the reaction $\bar{p}n \to \pi^-\pi^+\pi^-$ may be explained without recourse to a Veneziano model. A direct fit of particle parameters is obtained by parametrizing the amplitude in terms of the resonance poles in the two nonexotic channels. The existence of daughters is found to be crucial to the fit to the data. No evidence is seen of higher spin resonances; and in all cases, the daughter mass is not degenerate with the parent mass.

Recent data[1] on the reaction $\bar{p}n \to \pi^-\pi^+\pi^-$ display several striking features. There are resonance bands around 0.5 and 1.5 GeV, and a hole in the middle of the Dalitz plot. Furthermore, a strong enhancement is seen at the $\pi^-\pi^-$ threshold.

Lovelace[2] has pointed out that a suitably modified Veneziano model[3] for $\pi\pi$ scattering in conjunction with a pion pole model [Fig. 1(b)] explains the qualitative features of these data. Several aspects of this analysis are difficult to justify. For example, there is the use of an ad hoc factor which is necessary to remove the ρ and f^0 production. The need for this factor indicates that the particle contributions which compose the Veneziano amplitude are relatively independent and are not constrained in the fashion required by the Veneziano model. Furthermore, it is not hard to see that the qualitative features obtained by this fit do not arise from the particular form of the Veneziano model, but rather from the particle mass spectrum of the di-pion resonances predicted by this model and implied by several seemingly unrelated developments.[4] The remainder of this paper will be devoted to this point. In other words, we propose to use these data to determine a particle mass and spin spectrum and compare these predictions with current theoretical prejudices.

We assume the usual type of expression for the $\bar{p}n \to \pi^-\pi^+\pi^-$ amplitude [Fig. 1(a)] which goes under a variety of names (final-state interaction theorem, isobar model, resonance dominance). Then it can be seen that the depletion of events in the middle of the Dalitz plot appears as a natural consequence of unitarity for the $\pi\pi$ scattering amplitude: Consider a partial-wave scattering amplitude $f_l(s)$ (where s is the square of the center-of-mass energy) which has two resonances in the same partial wave. $f_l(s)$ may take the following form $f_l(s) = h(s)/(s-s_1)(s-s_2)$, where h is a real but otherwise arbitrary function and $s_i = m_i^2 - i\Gamma_i m_i$ (m_i and Γ_i being related to the mass and width of the ith resonance). The unitarity constraint $\mathrm{Im} f_l = |f_l|^2$ requires

$$h(s) = s(m_1\Gamma_1 + m_2\Gamma_2) - (\Gamma_1 m_1 m_2^2 + \Gamma_2 m_2 m_1^2).$$

This amplitude will have a zero at $s = (\Gamma_1 m_1 m_2^2 + \Gamma_2 m_2 m_1^2)/(m_1\Gamma_1 + m_2\Gamma_2)$. If $m_1\Gamma_1 \approx m_2\Gamma_2$, the zero occurs at $\frac{1}{2}(m_1^2 + m_2^2)$. Thus if the scattering amplitude for the $\bar{p}n$ annihilation may be well represented by the two-body amplitudes, the depletion in the center of the Dalitz plot is just the overlap of these zeros.[5]

There is another approach to this phenomenon. A rising phase-shift model, in which the nth resonance arises from the phase shift passing through $(2n+1)\pi/2$, will have a zero in the amplitude between each resonance, when the phase shift passes through $n\pi$. We are assuming that the di-pion zeros persist along with the di-pion resonances in the three-pion final state. The only assumption which we shall make which is related to duality[6] is that the background to the resonance amplitudes is small[7] and that all resonances occur in the nonexotic channels.

The strong enhancements at the $\pi^-\pi^-$ threshold (approximately the f^0 band overlap) and at the overlap of the rho bands may be explained as constructive interference between the various reso-

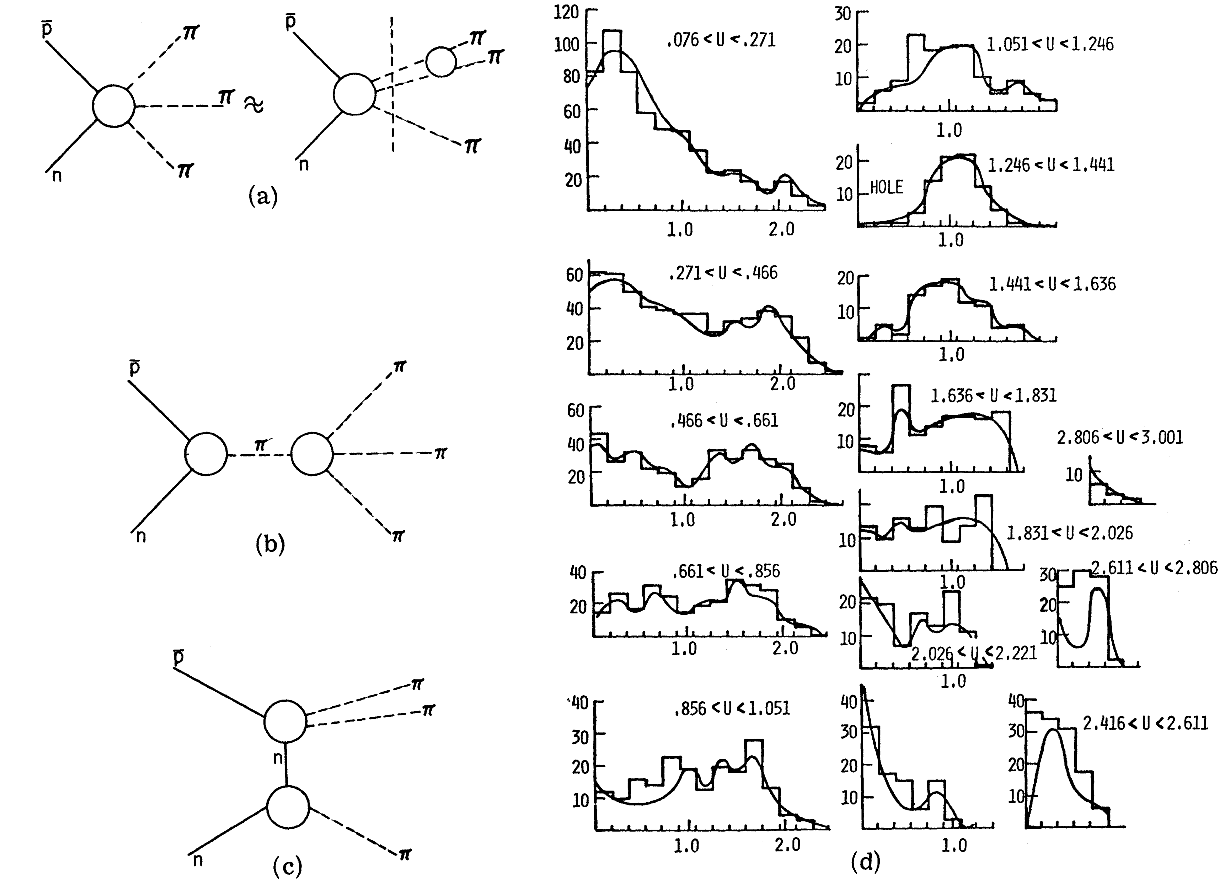

FIG. 1. (a) The diagramatic representation of the model for $\bar{p}+n\to\pi^++\pi^-+\pi^-$. (b) The pion-pole diagram. (c) The nucleon-exchange diagram. (d) The experimental histograms with the best fit of the model in the smooth curves.

nance terms. It has been found that interference effects play an essential role in this model.[8]

In addition to the apparent resonance bands and the depletion of events at the center of the Dalitz plot, the data show a very definite tendency to enhance the corners of the Dalitz plot associated with large values of the di-pion effective mass in which the net charge of the di-pion combination is neutral. This enhancement is smoother than that associated with a nearby resonance in these neutral modes and can be explained by the nearness of the nucleon pole as shown in Fig. 1(c). This diagram, in the case of antiproton capture at rest and for large values of the di-pion mass, would produce, in the zero pion-mass limit, a pole at the appropriate corners of the Dalitz plot. The doubly negative di-pion combination is prohibited in models restricted to only nucleon exchange in Fig. 1(c). The exact mechanism for the prefinal state interaction must be some combination of Figs. 1(b) and 1(c). The effect of the enhancement at these corners is parametrized as a universal linear form factor in the appropriate variables of each amplitude. It was found that this enhancement was essential to obtain the fit.

Neither linear trajectories nor parallel daughter trajectories have been assumed. Instead the values of $M^2(\pi^+\pi^-)$ where $\cot\delta$ is zero or infinite have been used as parameters. A width parameter and parameters which weight the various partial wave amplitudes have been used.

The function which was fitted to the data is

$$A=\frac{\lambda_0(1+\beta s)}{(s-a_1)(s-a_3)/(s-a_2)-i\omega_0}+\frac{\lambda_1(t-u)(1+\beta s)}{(s-b_1)(s-b_3)/(s-b_2)-i\omega_1 s}+\frac{\lambda_2[3(t-u)^2-(t+u)^2](1+\beta s)}{s-c_1-i\omega_2 s^2}+(s\to t),$$

where $s=(p_1+p_2)^2$, $t=(p_2+p_3)^2$, and $u=(p_1+p_3)^2$, and particle 2 is the π^+; a_1 and a_2 are the values

Table I. Fitted parameters and particle masses and widths.

Partial wave	Fitted parameters m_1^2 (GeV^2)	zero (GeV^2)	m_2^2 (GeV^2)	Width parameter	Weight
S	$a_1=0.609$ (ϵ)	$a_2=0.908$	$a_3=1.77$ (ϵ')	$\omega_0=1.12$	$\lambda_0=1.59$
P	$b_1=0.626$ (ρ)	$b_2=0.845$	$b_3=2.08$ (ρ')	$\omega_1=0.22$	$\lambda_1=0.35$
D			$c_1=1.61$ (f^0)	$\omega_2=0.27$	$\lambda_2=0.05$
		$\beta=0.819$			

Resonance masses and widths [a,b]	ϵ	ϵ'	ρ	ρ'	f^0
m (MeV)	780	1330	790	1440	1270
Γ (MeV)	370	640	30	290	550

[a]Some of these parameters contain large errors. In all cases, they are consistent with known values.
[b]These masses and widths are derived from the above parameters; they should be regarded as approximate values. See the text for further discussion.

of s where the S-wave shift goes through an odd multiple of $\pi/2$, and a_3 is the value of s where the amplitude vanishes. The b's are similarly defined parameters for the P-wave contribution. The factors involving t and u in the numerator are the appropriate analytic expressions for $\cos\theta$ and $3\cos^2\theta-1$.[9] ω_i is a width parameter for the ith partial wave, and λ_i weight the contribution of the various amplitudes. In Table I the values obtained for these parameters are listed. In the lower half of Table I are the usual resonance parameters obtained from these data. The Dalitz plot was divided into 152 bins, and a χ^2 of 325 was obtained. Although the χ^2 is not very good, we believe the fit has merit for the following reasons. (a) The fit looks good. In Fig. 1(d), a bin-by-bin plot of the amplitude is compared with the experimental data. (b) A model of the type described here is at best only a rough approximation to the complexities of this process. The work of Amado and Noble[8] on the three-body problem shows that it is probably unwarranted to expect a simple final-state interaction model to do much better than this. One thing that the work of Amado and Noble does indicate is that although the complete three-body interaction tends to complicate the final-state effects, the resonance positions are reasonably clearly indicated, but the widths may be markedly changed. This is one possible explanation for the small ρ^0 and large f^0 widths obtained here.

On the basis of this analysis, we believe the following conclusions can be drawn:

(a) There is evidence for the ϵ (S-wave daughter of the ρ) and the ϵ' and ρ' (S- and P-wave daughters of the f^0). The masses, however, do not appear to be degenerate. If the mass-spin relations between these particles are assumed to lie on trajectories, these trajectories even if assumed to be linear are not parallel. (See Fig. 2 for the Chew-Frautschi plot.)

(b) The daughter trajectories are not linear. Linear extrapolation of the leading trajectory predicts a spin-3 recurrence of the ρ at 2.5 GeV^2. There is no evidence for the presence of this particle. This fact is not too striking since the leading trajectory is somewhat weakly coupled. More striking is the definite absence of any daughters to this state. None of the S, P, or D resonances can be included in the fit. Linear extrapolation of the ϵ and ϵ' trajectories predicts masses larger than 2.0 GeV^2. There is definite absence of the first member of a new spin-zero daughter at this mass. Adding these daughter particles to our assumed amplitude leads to a poor description of the data. The χ^2 minimizer does its best to remove them; when constrained not to do that, the resultant fit is bad, both visually and in terms of χ^2. Thus, it appears that the daughter trajectories besides not being parallel do not continue to appear first at masses degenerate with a leading linear trajectory. There is the possibility that the leading trajectory is not linear but turns down and does not cross an integer before the value of 3.5 GeV^2 is reached. Any particle which has less than 1.87 GeV should be observed in this analysis. Figure 2 shows possible trajectories with minimum curvature consistent with the above.

(c) Production of the S wave dominates the Da-

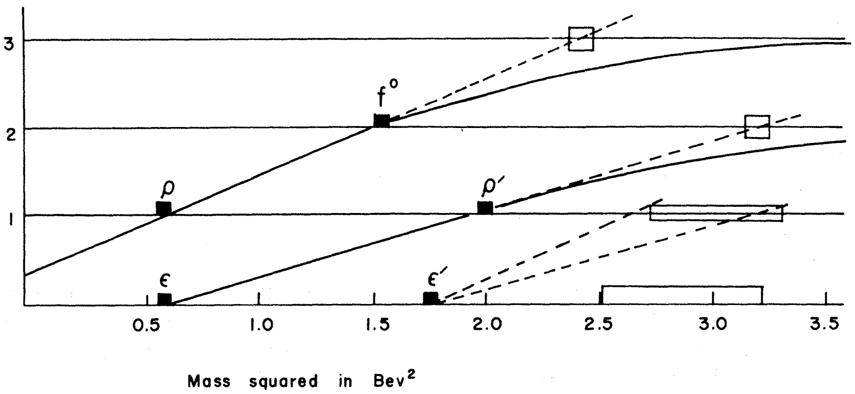

FIG. 2. The Chew-Frautschi plot of the possible trajectories formed by the observed resonances (shown in black boxes). White boxes shown absent but expected particles on the basis of linear daughter theories. Dashed trajectories are linear trajectory predictions. The curved lines represent the minimally curved trajectory consistent with this analysis.

litz plot, followed by the P wave and finally the D wave. Writing $A=\lambda_0 A_0+\lambda_1 A_1+\lambda_2 A_2$, we find

$$\int dsdt|A|^2\approx 27,\quad \lambda_0^2\int dsdt|A_0|^2\approx 30,$$

$$\lambda_1^2\int dsdt\,|A_1|^2\approx 4.5,\quad \text{and } \lambda_2^2\int dsdt|A_2|^2\approx 0.8.$$

The suppression of the P wave (and thus the ρ meson) has long been a mysterious aspect of the $\bar{p}n$ annihilation. Perhaps the explanation is to be found in some kind of centrifugal barrier effect, since we are dealing here with a ${}^1S_0$ state, whereas in $\bar{p}p$ annihilation where ρ production is large, we have a predominantly ${}^3S_1$ state. However, using the expression $pR\approx L$, where p is the center-of-mass momentum of the resonance, and R is the interaction radius, we need R about 0.1 F in order to explain the suppression. This is not as large a radius as one might expect, although the nucleon-pole diagram would lead to a fairly small radius of $R\approx 0.2$ F.

(d) The pion-pion model, which was used by Lovelace[9] [Fig. 2(a)], seems to be ruled out by two arguments: (1) The required linear form factor shows the importance of the nucleon pole, and (2) a strict pion-pole model would require $\lambda_0/\omega_0=\lambda_1/\omega_1=\lambda_2/\omega_2$ in Eq. (1). The resultant parameters do not satisfy this relation.

(e) Although we now have two 1^-, $I=1$ $\pi\pi$ resonances, the prediction for the electromagnetic form factor when the parameters of Table I are used is not materially changed for low q^2 from the simple ρ-dominance prediction. This is because the ρ' is very broad, which may explain why it has not been seen before. Thus, any explanations used to help out the old ρ-dominance predictions are still serviceable and even required.

Finally, we would like to mention points of similarity between our conclusions and those of others: (a) First, a broad ϵ^0 at 780 MeV or so has been repeatedly suggested from a variety of analyses, both theoretical and experimental.[10] (b) A recent analysis of $\bar{p}p\to\omega\pi^+\pi^-$ suggested an S-wave phase shift qualitatively much like the one presented here.[11] Also, an analysis of $\bar{p}p\to\pi^+\pi^-\pi^+\pi^-$ indicated the same kind of phase shift.[12] (c) Theoretically, it is known in at least one model that daughter trajectories do not rise indefinitely.[13] (d) The necessity of including nucleon-pole contributions and the invalidity of the pion-pole model in the analysis of $\bar{p}n\to\pi^+\pi^-\pi^-$ has also been mentioned by Berger.[14] (e) The prediction that the S-wave phase shift goes through π around 890 MeV and through $3\pi/2$ shortly thereafter (approximately at 910 MeV) has been made on the basis of a unitarized Veneziano model.[15]

Much of the work on this calculation was carried out while all three authors were affiliated with the Physics Department of Syracuse University. We benefitted from discussions with all our colleagues there but would especially like to acknowledge useful discussions with Professor G. C. Moneti and T. Kalogeropoulos.

---

†Work supported in part by the U. S. Atomic Energy Commission under Contract No. NYO-3399 and Contract No. ORO-3992.

*National Science Foundation Postdoctoral Fellow.

[1]P. Anninos *et al.*, Phys. Rev. Lett. 20, 402 (1968).

[2]C. Lovelace, Phys. Lett. 28B, 264 (1968).

[3]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[4]Daughter trajectories have been discussed from many points of view: D. Z. Freedman and J. M. Wang,

Phys. Rev. 153, 1596 (1967); M. Toller, Nuovo Cimento 37, 631 (1965), and 53A, 671 (1968), and 54A, 295 (1968); P. Di Vecchia and F. Drago, Phys. Rev. 178, 2329 (1969).

[5]The presence of zeros between overlapping resonances is a phenomenon that periodically is rediscovered. The earliest reference the authors know of is in N. G. Van Kampen, Phys. Rev. 89, 1072 (1953), and a more recent reference is C. Rebbi and R. Slansky, Phys. Rev. 185, 1838 (1969).

[6]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Lett. 19, 402 (1967), and Phys. Rev. 166, 1779 (1968).

[7]There is justification for neglecting diffractive effects. The fact that s- and t- channel poles are added as in the interference model is consistent with duality since a symmetric dual amplitude $V(s,t)$ may be represented in this form by writing $V(s,t)=\frac{1}{2}[V(s,t)+V(t,s)]$.

[8]The importance of interference effects in an exactly soluble model for weak three-body decays has been demonstrated by R. D. Amado and J. V. Noble, Phys. Rev. 185, 1993 (1969).

[9]They are suggested by $\cos\theta=(t-u)/(t+u)$ for $\pi\pi$ scattering, and removal of the pole at $t+u=0$. Actually, the D-wave factor is not quite right near $s=4m_\pi^2$ where it should vanish, but this causes a negligible effect in the work described here because the D wave is so small.

[10]D. D. Carmony, in Proceedings of a Conference on the $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 14-16 May 1969 (unpublished) (referred to as PCKπ below), p. 27; N. N. Biswas, PCKπ, p. 55; N. N. Biswas *et al.*, Phys. Lett. 27B, 513 (1968); E. Malamud and P. Schlein, PCKπ, p. 93, and Phys. Rev. Lett. 19, 1056 (1967); S. Marateck *et al.*, Phys. Rev. Lett. 21, 1613 (1968); V. Hagopian *et al.*, PCKπ, p. 149; L. J. Gutay, PCKπ, p. 241; L. J. Gutay *et al.*, Phys. Rev. Lett. 18, 142 (1967); P. B. Johnson *et al.*, Phys. Rev. 163, 1497 (1967); C. Lovelace, PCKπ, p. 562, and CERN Report No. TH. 1041, 1969 (to be published); E. P. Tryon, PCKπ, p. 665, and Phys. Rev. Lett. 20, 769 (1968); D. Morgan and G. Shaw, PCKπ, p. 726, and Nucl. Phys. B10, 261 (1969). The authors apologize for the probable omission of other relevant papers since the literature concerned with this point is vast.

[11]R. Bizzari, *et al.*, Nucl. Phys. B14, 169 (1969). The parametrization introduced for the S-wave phase shift was $(K/E)\cot\delta=a(b^2-s)(c^2-s)/(d^2-s)$, and the fitted values were $a=0.60\pm0.16$ GeV^{-2}, $b=853\pm19$ MeV, $C=1115\pm68$ MeV, and $d=940\pm3$ MeV.

[12]J. Diaz, CERN Report No. Ph. II/Phys 69-23, 1969 (to be published). We thank Professor G. C. Moneti for calling this paper to our attention.

[13]R. E. Cutkosky and B. B. Deo, Phys. Rev. Lett. 19, 1256 (1967).

[14]E. L. Berger, PCKπ, p. 750.

[15]Lovelace, Ref. 10.

ERRATUM

ANOMALOUS DAMPING OF VOLUME PLASMONS IN POLYCRYSTALLINE METALS. Vinod Krishan and R. H. Ritchie [Phys. Rev. Lett. 24, 1117 (1970)].

It has been brought to our attention that Dr. C. Kunz [Z. Physik 167, 53 (1962)] was the first to notice that volume plasmons are damped anomalously in metals composed of small crystallites. Kunz (Figs. 8 and 9 of the reference cited) displays experimental data exhibiting the same qualitative dependence of plasmon loss width on plasmon momentum for different crystallite sizes as do Festenberg's data. We extend our apologies to Dr. Kunz for having overlooked his fine work in this area.

In addition, some typographical errors have crept into the manuscript. In both Eqs. (3) and (4) the power of the factor ω_{q_0} multiplying the sum should be -3 rather than -2. Similarly, the wave-vector argument of the dielectric function appearing in both Eqs. (3) and (4) is incorrect: The dielectric function should be written $\epsilon_{q_f,\omega_{q_0}}$ rather than $\epsilon_{q,\omega_{q_0}}$.