INVESTIGATION BY SECOND SOUND OF THE INERTIAL MASS OF ^{3}He IN SUPERFLUID ^{4}He AT LOW TEMPERATURES*

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From measurements of the velocity of second sound and the osmotic pressure the inertial mass of ^{3}He in superfluid ^{4}He has been obtained as a function of temperature and concentration. The results indicate that (i) the ^{3}He quasiparticle spectrum is not quite parabolic and (ii) part of the quasiparticle effective interaction is velocity dependent or "nonlocal" and resembles the interaction of spheres moving through a nonviscous classical fluid.

As usually interpreted, the Landau and Pomeranchuk[1] (LP) and Bardeen, Baym, and Pines[2] (BBP) theories of liquid ^{3}He-^{4}He mixtures predict that at low temperatures, where the phonon and roton densities are negligible, the normal-fluid density ρ_n should be given by

$$\rho_n = n_3 m, \tag{1}$$

where n_3 is the ^{3}He number density and the mass m should be independent of temperature and ^{3}He concentration. This result depends on two assumptions: (i) The quasiparticle energy ϵ, which is the energy change on adding one ^{3}He of momentum p to a very weak solution of ^{3}He in ^{4}He, is given by

$$\epsilon = -E_3 + p^2/2m$$

and terms in higher powers of p^2 may be neglected. (ii) In the BBP theory, which takes into account an effective interaction between ^{3}He quasiparticles, the interaction $v(r)$ is assumed to depend only on the interparticle distance r and not on the particle velocities with respect to the superfluid. The interaction energy is thus unchanged when the quasiparticles are accelerated with respect to the superfluid, and therefore $v(r)$ (or its Fourier transform V_k) does not contribute to the normal-fluid inertial mass.

If we define an empirical inertial mass

$$m_i = \rho_n / n_3, \tag{2}$$

determination of the temperature and concentration dependence of m_i allows one to test assumptions (i) and (ii). In this Letter we obtain ρ_n and m_i from measurements of the velocity of second sound.

An equation for the velocity of second sound in the low-frequency, hydrodynamic limit has been derived by Khalatnikov.[3] This rather complicated equation, which depends only on thermodynamic and Galilean-invariance arguments, can be written as

$$m_4 u_2^2 = \frac{(1-f\xi)\{-(\partial\mu_4/\partial\ln\xi)_{T,P} + (\xi T/C)(\partial S/\partial\ln\xi)_{T,P}^2\}}{\rho_n/\rho_s + f^2\xi^2}, \tag{3}$$

where

$$\xi = n_3/n_{40} = X/(1+\alpha X), \tag{4}$$

$$f = 1 + \alpha - m_3/m_4 \cong 0.53. \tag{5}$$

(This result assumes that $n_3 = n_{40}X/(1+\alpha X)$ and it therefore neglects, among other things, thermal expansion. The approximations involved have been investigated and can be justified.) In Eqs. (3)-(5), X is the atomic concentration of ^{3}He, n_{40} is the number density of pure ^{4}He, α is the BBP parameter 0.284 (see, for instance, Edwards, Ifft, and Sarwinski[4]), μ_4 is the ^{4}He chemical potential, and S and C are the entropy and specific heat per atom of ^{3}He. [Using $\xi \ll 1$ and $\mu_4(T,X) = \mu_4(T,0) - \pi/n_{40}$, where π is the osmotic pressure of the solution, Eq. (3) can be transformed to

$$\rho_n u_2^2 \simeq (\partial\pi/\partial\ln\xi)_{S,P}.$$

The analogy between second sound in the mixture and ordinary sound in the quasiparticle gas is quite clear.[5] The derivative $\partial\pi/\partial\ln\xi$ can be regarded as the "osmotic bulk modulus" of the mixture.] To determine ρ_n from the second-sound velocities we used Eq. (3) with $\partial\mu_4/\partial\ln\xi$ and $\partial S/\partial\ln\xi$ determined from an empirical equation fitted to our measurements of the osmotic pressure.[6]

The velocity of second sound was determined by a time-of-flight pulse method over a 2.5-cm path. To avoid difficulties in coupling thermally to the normal component, the second sound was generated mechanically using electrostatic trans-

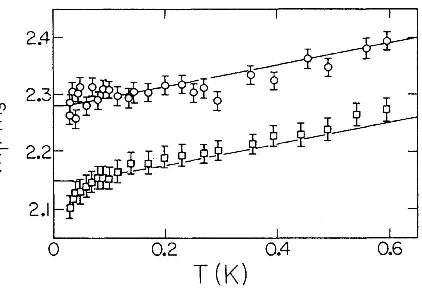

FIG. 1. Inertial mass m_i versus temperature for two ^{3}He concentrations: 0.67% (circles) and 4.47% (squares).

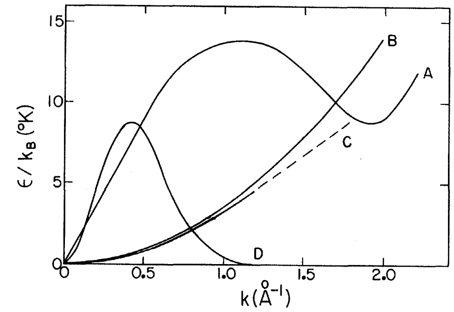

FIG. 2. The excitation spectra of liquid helium. A, the phonon-roton spectrum. The kinetic energy of ^{3}He quasiparticles: B, simple quadratic form; C, with term in p^4 added. Curve D (arbitrary units) shows the Maxwell-Boltzmann distribution in momentum for a temperature of 0.6 K, the highest temperature used in the present measurements.

ducers with porous "superleak" diaphragms.[7] The measurements were made at 0, 10, and 20 atm pressure between 0.03 and 0.6 K in a dilution refrigerator. At zero pressure the concentrations measured ranged from $X = 0.143\,\%$ to $6.278\,\%$.

Typical results for m_i [defined by Eq. (2)] are shown in Fig. 1. Clearly m_i depends systematically on both concentration and temperature. The errors bars in the graph represent only the uncertainties in measuring u_2, X, and T; possible systematic errors in the derivation of the "osmotic compressibility" are about the same size, although not large enough to account for the variation of m_i with X and T. The intrinsic attenuation of the second sound could not be measured, but the signals at very low concentrations in the range 0.2 to 0.5 K were anomalously weak and the results have not been used because of possible dispersion errors. For other temperatures observation of the echoes put upper limits on the attenuation which indicate that the dispersion was small.

The dependence of m_i on the temperature can be explained in the following way. Instead of assumption (i), we continue the expansion of the quasiparticle energy in the LP theory and include one more term:

$$\epsilon = -E_3 + (p^2/2m)[1 - \chi p^2/p_c^2]. \qquad (6)$$

Here, we define the characteristic momentum $p_c = m_4 s$ (so that $p_c/\hbar = 1.5\ \text{Å}^{-1}$), where s is the velocity of first sound and χ is a number which we may expect to be of order unity. An elementary calculation of ρ_n using Eq. (6) gives

$$m_i = m[1 + \tfrac{10}{3}\chi\langle p^2\rangle/p_c^2] \approx m[1 + U_F/R\theta]. \qquad (7)$$

Here we have calculated the mean square of the momentum $\langle p^2\rangle$ using the fact that the distribution of momenta in the quasiparticle gas is very close to that in an ideal Fermi gas of particles of mass m. Tables of the molar internal energy U_F of an ideal Fermi gas have been given by Stoner.[8] The temperature θ is related to χ by $k_B\theta = \frac{3}{20}(m_4 s)^2/m\chi$. At temperatures high compared with the Fermi degeneracy temperature of the mixture, when $U_F \simeq \frac{3}{2}RT$, Eq. (7) predicts a linear dependence of m_i on T. The curves shown in Fig. 1 have been drawn in from Eq. (7) with one value of χ but, instead of a constant m as required by the theory, we need different values of m for each concentration. These values we call m_x. The fitted value of χ is 0.14 ± 0.05 corresponding to $\theta = 17 \pm 6$ K.

The effect of the p^4 term on the excitation energy is illustrated in Fig. 2. The magnitude and sign of the effect is plausible from a number of points of view. For instance, the increase in mass with momentum is in order-of-magnitude agreement with a simple classical model. If we assume that the difference between m and m_3 is due to the inertia of the ^{4}He backflow around the quasiparticle, dilation of the volume displaced by the ^{3}He due to Bernoulli pressure would give[9] approximately the effect observed.

The apparent values of m in Eq. (7) for each concentration m_x are shown in Fig. 3 as a function of $\xi = n_3/n_{40} \approx X$. (Note that m_x is slightly less than the value of m_i at $T = 0$.) The dependence of m_i or m_x on ξ or X is not unexpected, since clearly $\rho_n = \rho$ and $m_i = m_3$ for pure ^{3}He ($X = 1$). The fact that m_i approaches m_3 at very high concentrations has been demonstrated at high temperatures in the recent fourth-sound experiments of Dyumin et al.[10] Our values of m_i/m_3 are quite consistent with this limiting behavior. On the other hand, as we have seen, the dependence of m_x on X is in contradiction to as-

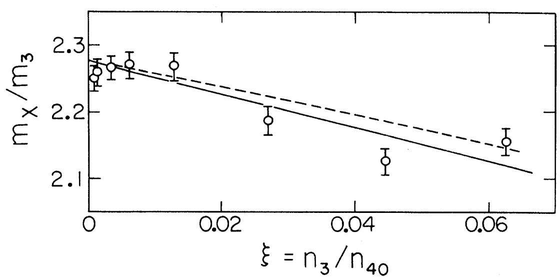

FIG. 3. Dependence of the mass m_x on reduced number density $\xi = n_3/n_{40}$. The full line represents Eq. (10), while the dashed line gives the corresponding inertial m_i at the absolute zero of temperature.

sumption (ii) and is clear evidence that the quasiparticle effective interaction is velocity dependent.

Bardeen, Baym, and Pines included a discussion of a velocity-dependent contribution to the interaction in their paper but they did not include it in any comparison with experiments. This is the quantum analog of the "hydrodynamic" interaction between two spheres (representing the ^{3}He) moving through a classical nonviscous liquid. The corresponding Fourier transform of this interaction in momentum ($\vec{k}$) space has the dipolar form[2]

$$V_k{}^d = -(A_d/n_{40}m_4)(\vec{p}_1\cdot\hat{k})(\vec{p}_2\cdot\hat{k}). \tag{8}$$

Here $\vec{p}_1$ and $\vec{p}_2$ are the momenta of the (spherical) quasiparticles, $\hat{k}$ is the unit vector in the direction of $\vec{k}$, and A_d is a number whose magnitude is determined by the volume ω and mass m of the interacting spheres. BBP give as the classical result $A_d = (\frac{3}{2}\rho_4\omega)^2/m^2$, where $\rho_4 = n_{40}m_4$. Using the empirical mass m and $\omega = (1+\alpha)/n_{40}$, A_d is equal to 1.2. If we choose the volume ω to be appropriate to the hydrodynamic mass so that $\frac{1}{2}\rho_4\omega = (m-m_3)$, then $A_d = 2.9$. BBP have also obtained $A_d = [1+\alpha+(m-m_3)/m_4]^2(m_4/m)^2 = 1.7$ from a quantum mechanical calculation for the virtual exchange of a single phonon between the quasiparticles.

In a recent paper, McMillan[11] has shown that the "nonlocal" potential in Eq. (8) together with the long-range, compressive term $V_0 = -\alpha^2 m_4 s^2/n_4$, calculated by Baym,[12] is sufficient to explain at least semiquantitatively the empirical BBP interaction. When both the interacting particles have momenta on the Fermi surface the potential reduces to the local form:

$$V_k = V_0 + \tfrac{1}{4}A_d k^2/\rho_4 = (m_4 s^2/n_4)[-\alpha^2 + \tfrac{1}{4}A_d k^2/p_c{}^2]. \tag{9}$$

The calculation of ρ_n, m_i, and m_x using the nonlocal potential Eq. (8) (and including the p^4 term in the spectrum) yields to first order[13]

$$m_x = m[1 - \tfrac{1}{6}A_d(m/m_4)(n_3/n_{40})]. \tag{10}$$

This equation expresses the fact that as the concentration is increased the volume of ^{4}He involved in the backflow around each quasiparticle is reduced, thereby decreasing the associated inertial mass. The linear dependence on concentration which is predicted agrees with experiment. The line drawn on Fig. 3 corresponds to $A_d = 3.8$ with an uncertainty of ±1.0 when we allow for possible systematic errors in the osmotic compressibility. This A_d is larger than the theoretical values which we mentioned above, and empirical values for the local approximation [Eq. (9)] which are around 2.5. Nevertheless, in view of the many approximations involved we think that the agreement is satisfactory, and that the true nonlocal form of the interaction is properly represented, at least at large distances, by Eq. (8).

Finally we note that the p^4 term in the energy spectrum gives an additional contribution in the specific heat which is approximately linear in temperature for both degenerate and nondegenerate mixtures. When this contribution is allowed for, the values of m from specific-heat data[14] and from the osmotic pressure[6] are all consistent with the value of $m_{x\to 0}$ from Fig. 3, namely $(2.28 \pm 0.04)m_3$.

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MAGNETIC BUNDLES IN REACTING FLOWING PLASMA*

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An exact theory of plasma filaments observed in coaxial accelerators is derived from a steady-state ($\partial/\partial t = 0$) description of the three-dimensional flow of deuterium plasma. Ionization and recombination reactions are considered. Magnetic structure of the filaments (bundles of helical field lines), density distributions, and flow field fit the experimental evidence.

By considering shock-wave conditions for the current sheath (CS) in coaxial accelerators (CA), it can be proved[1] that the plasma vorticity $\vec{\omega} = \frac{1}{2}\nabla\times\vec{u}$ is large in the region of space where the filaments are located, immediately behind the shock front Σ (Σ is considered, essentially, as the foremost luminous face of CS). Vorticity and filament axis have the same orientation[1] (parallel to the z axis; see Fig. 1; without a relevant loss of generality we take $\partial/\partial z \cong 0$, the filaments are considered as parallel cylinders). The existence of a large ω in a narrow region of space (containing the filaments) is not sufficient to conclude that vortex structures exist in that region. More stringently an analytic description of the plasma can be deduced which depicts the vortex nature of the filaments. The essential assumptions of the theory [see Eq. (1)] and the general results [see Eqs. (2)] are set forth below before specializing formulas to the filament problem. We can show that a strong component B_z (2500 G and larger) of the self-consistent magnetic field exists along the filament axis (orthogonal to $B_\theta \cong B_y$) inside the filaments. This fact was already pointed out by magnetic probe measurements.[2] The density in phase space for ion and electrons $f_\pm$ satisfies

$$df_\pm/dt = S_\pm, \qquad (1)$$

where d/dt is the Vlasov operator with self-consistent fields $\vec{E} = -\nabla\varphi$, $\vec{B} = \nabla\times\vec{A}$; $\partial/\partial t = 0$ in the CS frame of reference (moving with a velocity $u_0 \sim 10^7$ cm/sec in the laboratory system); the source term S accounts for ionization and recombination reactions; the indices $\pm$ are sometimes dropped. The role of the neutral atoms is simply to affect the anisotropy in velocity of the newly born charged particles and to function as a reservoir for ions and electrons. Neutral atoms are further disregarded. S is chosen ad hoc, according to these criteria: (I) A solution of (1) can be obtained at a glance if a solution f_v of the Vlasov equation $df_v/dt = 0$ is known; in our case $f_v = f_v(\epsilon, p_z)$, where ϵ, p_z are ener-

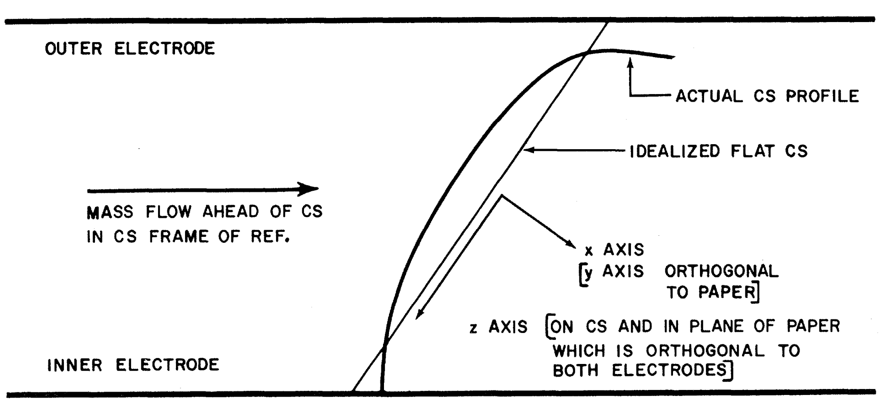

FIG. 1. Profile of the current sheath between the electrodes.