shida[1] and by Moos, Opal, and Huang,[2] but, on the other hand, their methods are not very conducive to the detection of very fine absorption lines. These differences may explain the discrepancies between the present work and the work published by Kushida,[1] although Kushida's spectrum does show some small "kinks" at around 15 000 cm^{-1} in the E perpendicular to c spectrum but very little similarity in the region of 27 500 cm^{-1}. The spectra of Huang and Moos[3] show a weak feature near 3600 Å. See Figs. 6 and 7 of that paper.

Based upon the polarization of the spectra, the clear connection between the lower common state of the two systems with the originating levels of the R_1 and R_2 lines, the fact that the origins consist of a simple pair of lines in each case, and, finally, the energies at which the two transitions occur, we assign the terminating level of the lower-frequency transition as $t_2^{\,2}({}^1E)e$ ${}^2A_1$ and the originating level of the higher-frequency transition as $t_2^{\,2}({}^1E)e$ ${}^2A_2$. The latter might, alternatively, possibly form the E part of the $t_2^{\,2}({}^1T_2)e$ ${}^2T_2$ level since there is some intensity polarized parallel to c as would be expected from the $E \leftarrow E$ transition in this case (C_{3v} notation).

More complete details of the apparatus and related studies will be published elsewhere.

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COMPUTATIONS ON ANOMALOUS RESISTANCE*

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Results of computer simulations of electrostatic "anomalous resistivity" are reported. One- and two-dimensional plasma simulations were performed in which an external (nonself-consistent) spatially constant electric field was imposed to drive a current in the plasma. In these calculations the electron distribution is subject to collective runaway with beaming and thermal energy increasing about equally.

The data described here were produced by two different plasma simulation programs. The GALAXY programs, described in earlier papers,[1-3] perform two-dimensional plasma simulations on a square region with 64×64 cells. The one-dimensional simulations were performed using the finite-sized particle model in a program developed originally at Princeton[4,5] and adapted the Naval Research Laboratory for these calculations. The one-dimensional simulations extend the two-dimensional calculations to include cases having larger mass ratios, more particles, smaller driving fields, and systems containing more unstable modes, but they are less complete in the sense that they are one-dimensional. The earlier two-dimensional calculations indicate that with only a few modes in the system the motions tend to become one-dimensional with only waves parallel to $\vec{E}$ being strongly excited. This tendency toward alignment is a result requiring further investigation but suggests that one-dimensionality may not be a serious limitation.

Figure 1 shows results from a two-dimensional calculation in which the ion and electron masses are equal. Here 8192 ions and 8192 electrons are released on a doubly-periodic system. The initial velocity distributions are Maxwellian with zero relative drift. The electric field is quite strong, accelerating an electron by $0.14V_{te}$ in the plasma time $1/\omega_{pe}$. Figure 1(a) shows the evolution of system electrostatic energy and also the breakdown of this energy into the two most

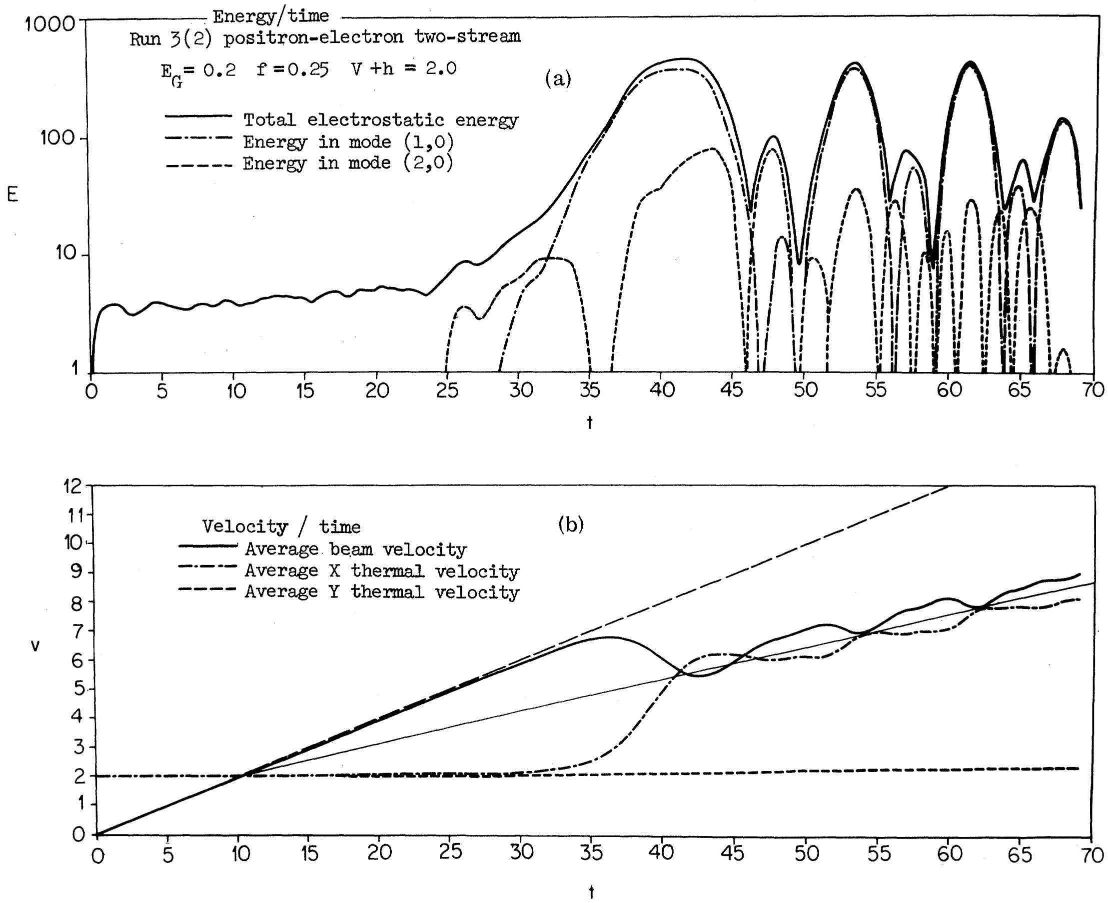

FIG. 1. Evolution of a two-dimensional calculation on anomalous resistance.

important spatial modes as a function of time. For $t \gtrsim 30$ ($\omega_{pe}^{-1} = \sqrt{2}$ in these units) almost all of the electrostatic energy resides in the two modes of longest wavelength which are aligned with the electric field (here taken in the X direction). Alignment appears in this run during the linear-growth phase of the two-stream instability. In other cases involving larger mass ratios, however, one-dimensional mode alignment occurred after the nonlinear saturation of the two-stream instability. Mode alignment may be caused and supported in later times by enhanced thermal damping of off-angle modes, by nonlinear interaction, or by numerical effects arising from finite system length. This aspect of the results merits further investigation. In any case, the alignment as observed suggests the adequacy of a one-dimensional analysis for many aspects of the problem.

Thermal distributions of electrons and positrons were started at rest for the two-dimensional calculations. The electrons accelerate initially at the free-streaming rate as shown by the upper dashed line in Fig. 1(b). When the beams are traveling fast enough, a large two-stream interaction takes place ($t \sim 30$ to 45) during which the space-averaged random "thermal" spread increases until it equals the average beam or drift velocity. The repetitive energy pattern in Fig. 1(a) arises from the repeated interpenetration of particle clusters on the oppositely charged beams. These clusters are evidently dynamically stabilized. Positron-density plots in Fig. 2 show a wavelike structure propagating toward positive X. These regions of enhanced density correlate with the particle clusters shown on the upper of the two phase-space beams. These clusters, furthermore, move with the mean beam velocity and thus are not wave phenomena propagating along the beam. When the particle clusters on the oppositely charged beams overlap (at $t \sim 45$, 57 in Figs. 1 and 2) the energy resides primarily in mode (2, 0) and is rather small. When the charge concentrations become separated ($t \sim 39$ in Figs. 1 and 2) due to the relative motion of the beams, large self-consistent electrostatic fields

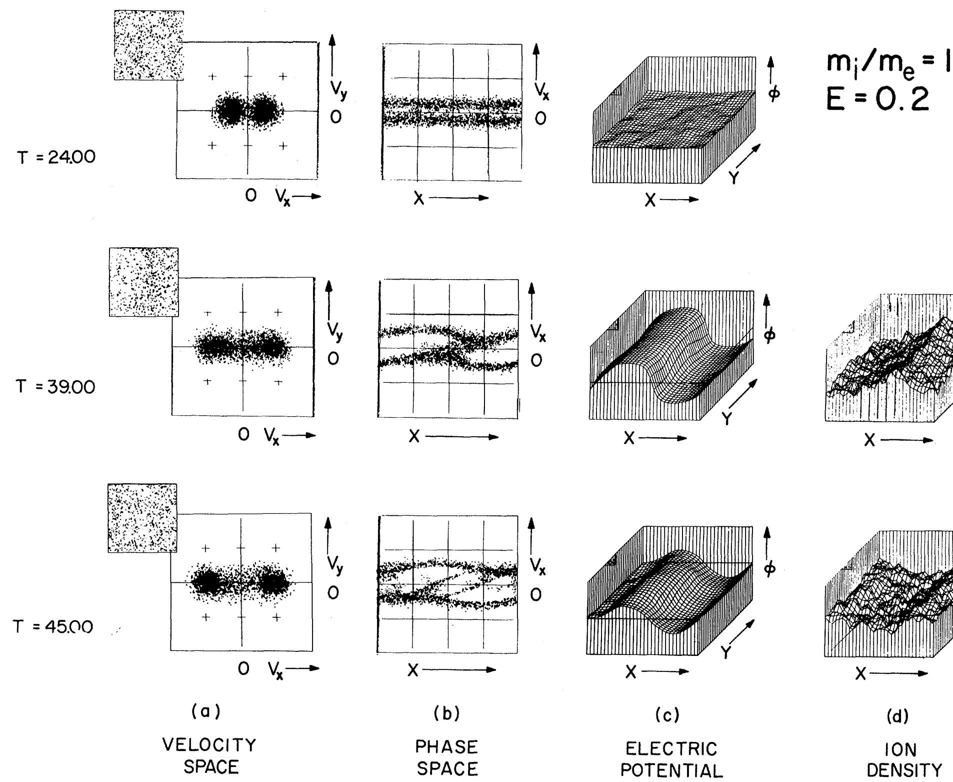

FIG. 2. Physical phenomena in a two-dimensional calculation on anomalous resistance.

build up which slow down the beams and increase their rms spread as shown in Fig. 1(b). Under continued action of the external electric field, which always acts to increase the relative drift, the nonlinear two-stream instability is repeatedly excited and stabilized, each time increasing V_{te} and decreasing the drift velocity V_D until they are again equal.

The lower curve in Fig. 1(b) shows the transverse thermal spread for the beam (the rms spread in the Y direction). For this value V_{teY} increases only slightly during the entire run. This result is obvious from the velocity-space plots in Fig. 2 as well. V_{teY} can increase because of coherent and turbulent processes, particle collisions, or because of the spurious numerical phenomenon called "stochastic heating."[6] The absence of coherent or turbulent heating in the transverse direction is supported by the energy alignment discussed earlier. That numerical stochastic heating and particle collisions also give negligible contributions implies that spurious numerical errors, in the form of enhanced collisional phenomena, are absent from the calculations.

The one-dimensional calculations summarized in Fig. 3 used 10 000 to 20 000 electrons and an equal number of ions. Mass ratios m_i/m_e of 27 and 100 were used and initial temperature ratios T_i/T_e of between 1 and 100 were chosen. At $t=0$ the velocity distributions were Maxwellian and the current was set equal to zero. A current was produced by applying an external electric field. This field started at a high value and was decreased over a time of $10\omega_{pe}^{-1}$ to a steady value to raise the current to the desired level in a short time. The initial field varied between 1 and 40 times its final value.

After an initial transient, the current again satisfied the empirical and intuitively very reasonable law, $V_{te}=V_D$, where V_D is the mean electron velocity and V_{te} is the rms deviation from the mean. This result is illustrated in Figs. 1 and 2. Figure 3(a) shows a plot of V_D and V_{te} versus time for $eE/[m\omega_{pe}V_{te}(0)]=0.005$, $m_i/m_e=27$, and $T_e(0)/T_i(0)=10$. As can be seen, V_D and V_{te} stay very nearly equal though they do wiggle back and forth around each other.[7]

The upper straight line shows how the current would develop if the electrons freely accelerated while the lower straight line shows what would happen if half the electron energy went into ran-

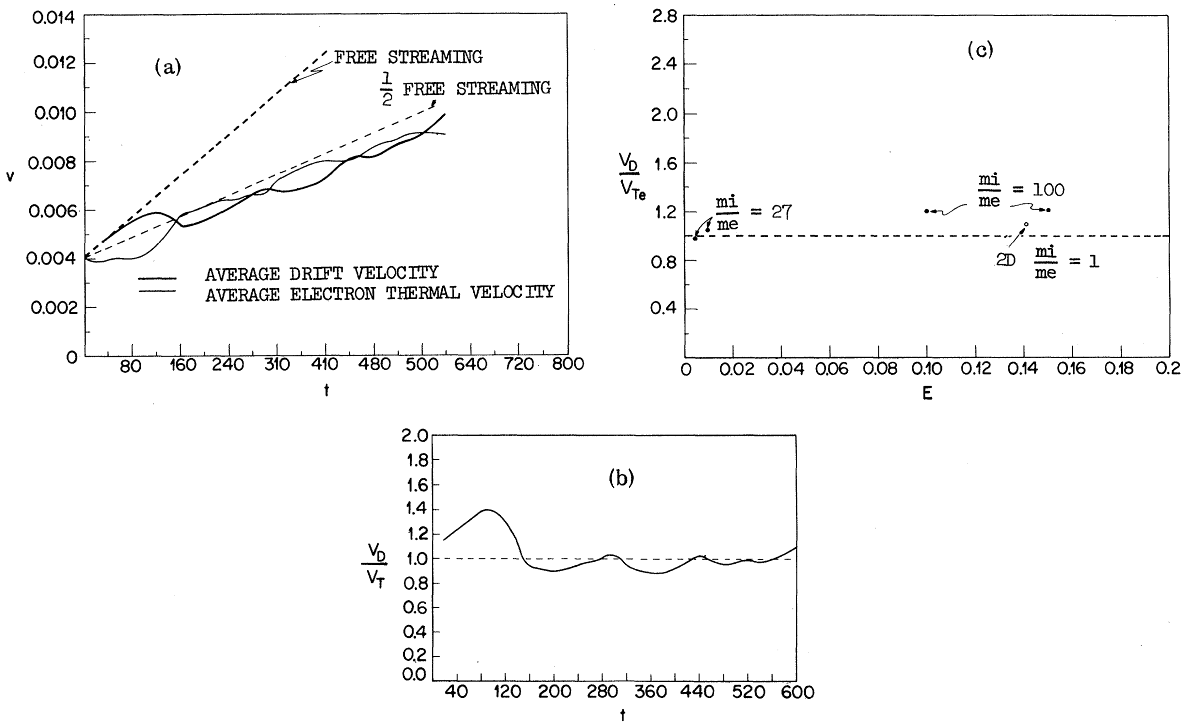

FIG. 3. One- and two-dimensional results on anomalous resistance. (a) Drift and thermal velocity for a typical one-dimensional run follow the same behavior as two-dimensional calculation. (b) V_D/V_{te} oscillated slightly about unity. (c) The average value of V_D/V_{te}.

dom motion and half into streaming. Figure 3(b) shows the ratio of V_D/V_{te} versus time for $eE/m\omega_{De}V_{te}(0)=0.005$, $m_i/m_e=27$, and $T_e(0)/T_i(0)=10$. The initial rapid increase of V_D due to free-streaming acceleration is seen. The drift V_D overshoots V_{te} until the instability gets going at which time it drops back to V_{te} and stays there throughout the run. Both V_D and V_{te} are increasing during this time. Figure 3(c) shows a plot of V_D/V_{te} (after this value has settled down) versus $eE/[m\omega_{pe}V_{te}(0)]$ for mass ratios of 1, 27, and 100. The results for mass ratio 1 are taken from the two-dimensional calculations described earlier.[2,3]

As can be seen, the ratio remains at essentially 1 for all these cases. The slightly larger value for mass ratio 100 may be real and could be explained by the slower growth rate of the instability for large mass ratios. The drift continues to increase during the time needed for the instability to grow to a large level. However, this point needs further investigation.

For one case we attempted to see if the drift could be stabilized at the ion acoustic value $[V_{te}(m_e/m_i)^{1/2}]$ by quickly accelerating the electrons to this speed and then dropping the field back to a smaller value $[eE/m\omega_{pe}V_{te}(0)=0.005]$. It was found, nonetheless, that the current continued to increase until V_D/V_{te} equaled 1. Evidently the ion acoustic wave cannot interact strongly with sufficient numbers of electrons to stop the increase of the current. Instead, the electron distribution function flattened out in the velocity region where ion acoustic waves propagate and the bulk of the electrons freely accelerated until the two-stream instability came into play.

In all cases with large mass ratios it was found that a relatively small amount of energy went into the ions. Most of the ions remained in a distribution which looked Maxwellian and maintained the original temperature. The ion distributions, however, did sprout long energetic tails with particle energies going up to the ion acoustic value $[V_{te}(m_e/m_i)^{1/2}]$.

One can derive the following empirical law for the rate of increase of the electron drift:

$$\dot{V}_D=-\tfrac{1}{2}eE/m_e, \tag{1}$$

as follows. The rate of work being done on the electron is

$$W=-eEV_D. \tag{2}$$

This must equal the rate of increase of the total electron energy (since a negligible amount of energy goes into the ions). Thus,

$$W=\tfrac{1}{2}m_e\frac{d}{dt}V^2=\tfrac{1}{2}m_e\frac{d}{dt}[\overline{V}^2+(V-\overline{V})^2]$$
$$=\tfrac{1}{2}m_e\frac{d}{dt}(V_D{}^2+V_{te}{}^2)=m_e\frac{d}{dt}V_D{}^2 \quad (3)$$

since $V_D \approx V_{te}$; from Eqs. (2) and (3) we obtain Eq. (1). The current increases as if the electron mass were doubled. Since the thermal velocity is also increasing, the effective electric field decreases in time, accelerating a particle by a smaller and smaller fraction of V_{te} in a plasma period. This decrease in effective electric field does not mean that the plasma will eventually become collision dominated because the collision frequency also decreases in time as the thermal and drift velocities increase. In the worst case, one-dimensional calculations where $\omega_{pe}/\nu_c=\beta n\lambda_D$, with β a number of order 2, these two effects just cancel. If collisions are small initially, they become neither larger nor smaller in time. In two- and three-dimensional cases, the situation is even better; collisions decrease in importance with time.

The generalization of Eq. (1) to systems where full equipartition of thermal energy in the transverse directions takes place is straightforward:

$$\dot{V}_D=-eE/(n+1)m_e, \quad (4)$$

where n is the effective dimensionality (number of degrees of freedom involved in the turbulence) of the physical system. This effective dimensionality need not be integral. If, for instance, the transverse thermalization in the full three-dimensions lags a classical collision time behind the dynamically excited longitudinal thermalization, n will be of the order of $1+\alpha$, where α is the ratio of randomized energy in the transverse directions. If may even be that n is less than unity. This can occur if transverse Landau damping forces the ratio V_D/V_{te} to be much larger than unity.[8]

The importance of these results is severalfold. The locking together of V_D and V_{te} over a very large range of electric fields, mass ratios, and initial temperature ratios gives an empirical, intuitively reasonable, semiquantitative law for the anomalous resistivity which is inversely proportional to the current in the mean. This law, $V_D \sim V_{te}$, has several observable consequences and may provide a good basis for computational and experimental comparison. Coppi and Mazzucato,[8] for instance, report experimental results in which $V_D \sim 0.3V_{te}$. It may even be possible to measure n in Eq. (4) to determine experimentally the effective dimensionality of the two-stream interactions. The difference between $n=1$ [no transverse heating as in Figs. 1(a) and 1(b)] and $n=3$ (full, immediate transverse heating) is a factor of 2 in the rate of current increase and should be observable.

The appearance of large-amplitude charge concentrations in the strong-field cases points to a nonlinear dynamic stabilization mechanism which keeps these clusters from exploding under the action of their strong self-fields. This clustering might be invoked in enhanced radiation from turbulent plasmas. For example, enhanced synchrotron emission from pulsars could be attributed to the coherent radiation from groups of particles clustered together by this nonlinear mechanism.

Perhaps the most important aspect of these calculations is the determination of a prescription, $V_D \sim V_{te}$, which seems to hold over a very large range of parameters and which will be easy to implement in magnetohydrodynamic codes. In regions where $V_{te} \sim V_D$ can be expected to hold, for instance, the rather difficult resistive-diffusion equation can be replaced by the simple prescription observed here.

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