opposite signature,[10,11] and (c) difficulty in the cross-section determination for $\rho^0\Delta^{++}$ since both the ρ and Δ have large widths.[12]

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*Present address: Argonne National Laboratory, Argonne, Ill.

‡Present address: Institute for Nuclear Study, University of Tokyo, Tokyo, Japan.

§Present address: University of California, Santa Cruz, Calif.

[1]A preliminary version of these data is presented in *Proceedings of Boulder Conference on High Energy Physics, Boulder, Colorado, August 1969*, edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Univ., Boulder, Colo., 1970), and also in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

[2]A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and W. S. C. Williams, Phys. Rev. Lett. 22, 148 (1969).

[3]A. Boyarski, Columbia University Report No. CONF 690301 (unpublished), and Stanford Linear Accelerator Center Report No. SLAC-PUB-559 (unpublished).

[4]G. E. Fischer and Y. Murata, Nucl. Instrum. Methods 78, 25 (1970).

[5]J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[6]A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and W. S. C. Williams, Phys. Rev. Lett. 21, 1767 (1968).

[7]M. Aderholtz *et al.*, Aachen-Berlin-CERN Collaboration, Nucl. Phys. B8, 45 (1968).

[8]A. Dar, Nucl. Phys. B11, 634 (1969).

[9]See, for example, D. Schildknecht, DESY Report No. 69/10, 1969 (unpublished); W. Schmidt and D. R. Yennie, Phys. Rev. Lett. 23, 623 (1969).

[10]E. Gotsman, Lett. Nuovo Cimento 2, 563 (1969).

[11]F. Gilman, Phys. Lett. 29B, 673 (1969).

[12]M. Walter, Deutsche Akademie der Wissenschaften zu Berlin-Zeuthen Report No. PHE 69-1, 1969 (unpublished).

EXPERIMENTAL STUDY OF $\pi^- + p \to \pi^- + N^*$ AT 8 AND 16 GeV/c*

E. W. Anderson, E. J. Bleser,† H. R. Blieden,‡ G. B. Collins, D. Garelick,§ J. Menes, and F. Turkot
Brookhaven National Laboratory, Upton, New York 11973

and

D. Birnbaum, R. M. Edelstein, N. C. Hien,† T. J. McMahon,‡ J. F. Mucci, and J. S. Russ
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

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We have measured $d\sigma/dt$ for five N^* reactions in the process $\pi^- + p \to n^- + N^*$ at incident momenta of 8 and 16 GeV/c in the range of $|t| < 1.5$ (GeV/c)^2. N^* bumps are observed at masses of 1.24, 1.41, 1.52, 1.69, and 2.19 GeV. Considerable structure is apparent in the $d\sigma/dt$ distributions including leveling off for $|t| < 0.15$ and dips at larger t in both the $N^*(1.24)$ and $N^*(1.52)$ reactions. The logarithmic slopes for the four higher mass N^*'s are identical to those seen in the corresponding reaction in pp scattering and the cross sections exhibit a similar elasticlike behavior with incident energy.

The study of angular distributions of various isobar (N^*) channels in the reaction $p + p \to p + N^*$ in the 6- to 30-GeV/c region[1,2] has proven very useful in delineating one of the dominant dynamical mechanisms underlying hadronic collisions at high energy,[3] viz., the "diffractive" mechanism, which controls the class of quasi two-body reactions that occur without exchange of internal quantum numbers. We report here a companion experiment to our earlier pp study,[1] carried out with incident π^- mesons at the Brookhaven alternating-gradient synchrotron, i.e., the reaction

$$\pi^- + p \to \pi^- + N^*, \tag{1}$$

at momenta of 8 and 15 GeV/c and over a four-momentum transfer (t) range of 0.05-1.5 GeV/c^2. The basic data are a high-statistics (~9×10^6 events), high-resolution study of the high-momentum end of the inelastic π^- spectrum emitted by a monoenergetic π^- beam striking a liquid-hydrogen target. In the resulting missing-mass (MM) spectra the N^* reactions are seen as clear maxima. All five of the N^* channels that were seen in the pp experiment were also observed here.

The magnetic spectrometer system used[4] was an improved version of the on-line wire spark-chamber spectrometer used in the pp experiment. The overall momentum resolution of the system (incident beam and spectrometer) was ±33 MeV/c at 8 GeV/c and ±40 MeV/c at 16 GeV/c; the resolution on the scattering angle of the π^- was ±0.8 mrad. Corrections made to the raw data were as follows: event reconstruction inefficiency, 14%; nuclear absorption, 14%; muon contamination in the beam, 2-5%; and π decay in the spectrometer, 2-4%. The empty-target signal was about 4% of that with full target and had no discernible structure in the inelastic region. The π^-p elastic data, that were measured simultaneously, agreed with previous measurements.[5]

At 8 GeV/c data were recorded continuously in a range of laboratory angle from 21 to 179 mrad and at 16 GeV/c from 16 to 74 mrad. Some representative low-t and medium-t MM distributions are shown in Fig. 1; N^* peaks are easily observed near masses of 1.24, 1.41, 1.52, and 1.69 GeV. A more subtle, broader structure is seen at 2.2 GeV in the 16-GeV/c data. At $-t$ ~0.04 (GeV/c)^2 the dominant feature is the broad peak centered at 1.44 GeV; as $|t|$ increases to 0.25 (GeV/c)^2 the peak moves to higher mass and becomes narrower; above 0.25 (GeV/c)^2 it stays fixed at a mass value near 1.52 GeV. Qualitatively, this behavior is identical to that observed in the pp experiment and is interpreted as being a compound peak of $N^*(1.41)$ and $N^*(1.52)$. Several curves from our pp data at 15 GeV/c at the same t values are also given in Fig. 1; the experimental resolution there was ±53 MeV/c. The similarity between π^-p and pp is striking. As was also the case in pp, no signal can be seen for the $N^*(1.94)$.

Cross sections for the various N^* maxima were extracted from MM distributions like those in Fig. 1 through a least-squares fitting procedure. To fit the N^*'s a modified Breit-Wigner function was used, and for the background a function consisting of a square root plus a polynominal in the variable MM−1.075 GeV; before fitting, the tail of the elastic peak was subtracted

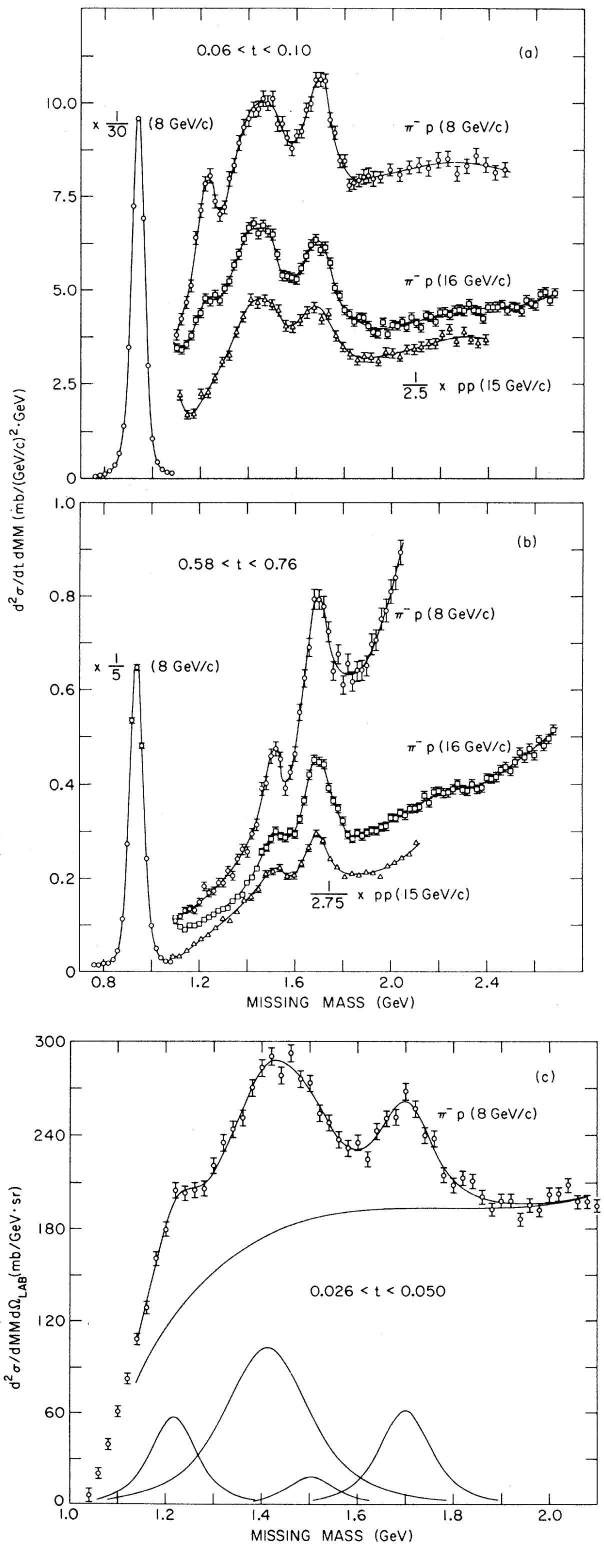

FIG. 1. Missing-mass distributions for $\pi^- + p \to \pi^- + \mathrm{MM}$ at 8 and 16 GeV/c. (a) $d^2\sigma/dtd(\mathrm{MM})$ at $-t = 0.08$ (GeV/c)^2. (b) Same as (a) but $-t = 0.67$ (GeV/c)^2. pp data at 15 GeV/c (Ref. 1) and the same t values are shown for comparison. The curves are hand-drawn fits. (c) Example of a least-squares fit to $d^2\sigma/d(\mathrm{MM})d\Omega_{\mathrm{lab}}$ at 8 GeV/c for the π^- laboratory angle 21-29 mrad.

Table I. Parameters in $\pi^- + p \to \pi^- + N^*$ reactions.

PARAMETER \ REACTION			N^* (1.24)	N^* (1.41)	N^* (1.52)	N^* (1.69)	N^*(2.19)	ELASTIC
FITTED MASS, (MEV)			1217 ± 8	1412 ± 13	1503 ± 6	1691 ± 4	2180 ± 25	-
FITTED WIDTH AT 8 GeV/c, (MEV), (FWHM)			115 ± 5	210 ± 15	120 ± 10	130 ± 10	275 ± 70	65 ± 1
INTRINSIC WIDTH, (MEV)			92 ± 12	200 ± 15	105 ± 9	119 ± 9	265 ± 70	0
SLOPE OF $d\sigma/dt$	-t REGION, $(\mathrm{GeV}/c)^2$		.1 - .31	.05 - .16	.16 - .70	.11 - .85	.1 - .85	.05 - .30
B in Ae^{Bt}	$\pi^- p$	8 GeV/c	10.2 ± .9	13.3 ± 1.3	4.4 ± .2	4.1 ± .1	-	8.2 ± .3
	$\pi^- p$	16 GeV/c	10.0 ± 1.9	15.9 ± 1.3	5.1 ± .15	4.6 ± .1	3.7 ± .6	7.8 ± .3
$(\mathrm{GeV}/c)^{-2}$	pp,	15 GeV/c[b]	14.0 ± 5	16.0 ± 3	4.6 ± .4	4.8 ± .2	5.1 ± .6[c]	9.1 ± .2
σ_{total}	- t REGION, $(\mathrm{GeV}/c)^2$		0.0 - 1.31	0.0 - .25	0.0 - 1.34	0.0 - 1.32	0.0 - .85	0.0 - 1.35
(μb)	$\pi^- p$	8 GeV/c	72 ± 12	187 ± 18	75 ± 6	183 ± 8	-	4700 ± 100
	$\pi^- p$	16 GeV/c	33 ± 10	177 ± 21	62 ± 5	156 ± 6	28 ± 9	4080 ± 120
	pp,	15 GeV/c[a,b]	90 ± 60	400 ± 50	140 ± 25	293 ± 23	64 ± 12[c]	8130 ± 300
	$\sigma_{pp}/\sigma_{\pi p}$(16 GeV/c)		2.7 ± 2.0	2.3 ± .4	2.3 ± .4	1.88 ± .16	2.3 ± .9	1.99 ± .09
n in $\sigma_{total} = C(p)^{-n}$, $\pi^- p$			1.12 ± .49	.08 ± .22	.26 ± .16	.22 ± .08	-	.19 ± .02

[a]For the N^* this is σ_{tot} for the forward peak. [b]Ref. 8. [c]20 GeV/c.

out. The procedure used was to search first for positions and widths of the N^* bumps in regions of t where they are most prominent. The results of this search at 8 GeV/c are given in Table I along with the full width at half-maximum of the elastic peak. The positions found for the four N^* peaks at 8 and 16 GeV/c are in good agreement; the fitted intrinsic widths also agree within 10 MeV, which is the sensitivity of our analysis. Using fixed values of masses and widths, fits were then made over the full range of angles. In general, χ^2's did not improve significantly for background functions higher than cubic. A typical fit is given in Fig. 1(c).

It is attractive to identify the N^* peaks seen here with the $P_{33}(1.24)$, $P_{11}(1.46)$, $D_{13}(1.515)$, $F_{15}(1.690)$, and $G_{17}(2.19)$ resonances of the phase-shift analyses,[6] although this type of experiment cannot exclude the possibility that other states do in fact contribute to a given MM peak. For the isospin-$\frac{1}{2}$ states this assignment is preferred on the basis of the diffraction dissociation model[7] for the reaction mechanism. These states, in addition to having the same isospin as the proton, are consistent with the natural parity-change rule, $\Delta P = (-1)^{\Delta L}$, where ΔP and ΔL are the change in parity and angular momentum in the transformation $N \to N^*$. We note that the N^* masses found here agree to within a few MeV with those found in our pp study[1,8] and those in the ep missing-mass experiments[9] [the $N^*(1.4)$ has not been observed there].

The differential cross sections for the five N^* reactions at 8 and 16 GeV/c are displayed in Fig. 2; the errors shown are statistical only. The systematic error in the overall scale due to the uncertainty of the widths of the N^*'s and the shape of the background is about ±25% for the $N^*(1.24)$ and $N^*(2.19)$, and ±14% for the others. The relative error between the 8- and 16-GeV/c data due to these uncertainties is estimated to be ±6%. Finally, the error in absolute normalization due to uncertainties in system efficiency, incident flux, and solid angle

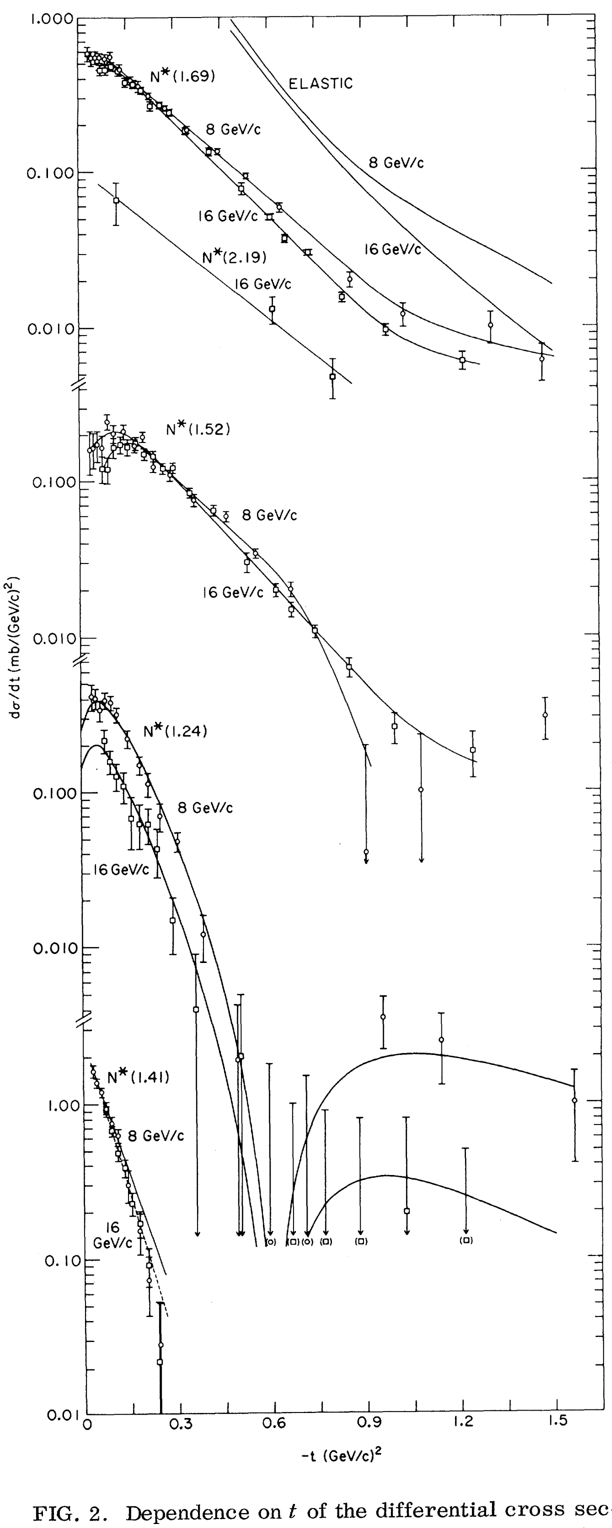

FIG. 2. Dependence on t of the differential cross sections for the various N^*'s in $\pi^- + p \to \pi^- + N^*$ at 8 and 16 GeV/c. Circles represent 8-GeV/c data; squares, 16-GeV/c. The curves of the $N^*(1.24)$ are from Ref. 16; in all other cases the straight-line portion represents the least-squares fit of Table I, and the rest is a hand-drawn fit. The elastic cross sections are also given.

calculation is estimated to be ±8%. Previous measurements[10] of the $N^*(1.41)$ and $N^*(1.69)$ cross sections in the range $-t = 0.0$-0.2 (GeV/c)^2 agree with these cross sections to within 30%.

Prominent features in $d\sigma/dt$ are the following: (1) All but the $N^*(1.41)$ level off for $|t| < 0.1$ GeV/c^2, suggestive of a turnover as $|t| \to 0$. For the $N^*(1.52)$ this effect could possibly be a result of the fitting, as the $N^*(1.41)$ dominates here. (2) At 8 GeV/c the $N^*(1.24)$ has a clear minimum at $-t = 0.6$ (GeV/c)^2 and a second maximum around $-t = 1.0$ (GeV/c)^2. (3) At 8 GeV/c the $N^*(1.52)$ has a dip in the neighborhood of $-t = 0.95$ (GeV/c)^2; at 16 GeV/c the dip is no longer visible and the cross sections in this region of t lie above the 8-GeV/c measurements.

All of the cross sections in Fig. 2 have been fitted by the form Ae^{Bt} in the t intervals listed in Table I. The logarithmic slope B is given there along with the slopes of the π^-p elastic cross sections from this experiment, as well as the slopes for these same reactions found in our pp experiment[8] at 15 GeV/c. We observe: (1) There is a large variation in B, the $N^*(1.41)$ having nearly twice that of the elastic, and the $N^*(1.52)$ and $N^*(1.69)$ about half the elastic. (2) The π^-p slopes at 16 GeV/c are remarkably similar to those observed in pp at 15 GeV/c; for the $N^*(1.69)$ they agree within the statistical error of ±5%. (3) The slopes for the N^*'s at 1.41, 1.52, and 1.69 GeV increase going from 8 to 16 GeV/c. This degree of shrinkage would result from an effective Regge trajectory of slope 0.5 (GeV/c)$^{-2}$.

Total cross sections at 8 and 16 GeV/c, obtained by integrating the curves of Fig. 2, are given in Table I; the errors are mainly statistical but do include an estimate of the uncertainty in extrapolating[11] $d\sigma/dt$ to $t = 0$. The error quoted is statistical only; the ±8% systematic error in absolute normalization mentioned above is common to both N^* and elastic. The corresponding cross sections from the pp experiment at 15 GeV/c are listed as well (the N^* cross sections are for the forward peak only, i.e., $\frac{1}{2}\sigma_{tot}$). The ratios $\sigma(pp)/\sigma(\pi^-p)$ of the total cross sections for N^*'s and elastic scattering are all consistent with the value 2.0. It is interesting to note that the differential cross sections in Fig. 1 above the prominent N^* region, e.g., at MM = 2.0 GeV, exhibit this same ratio, which is close to the value $(\frac{3}{2})^2$ as suggested by the additive quark model. From the viewpoint of Regge theory, the equality of these ratios for the isospin-$\frac{1}{2}$ N^*'s

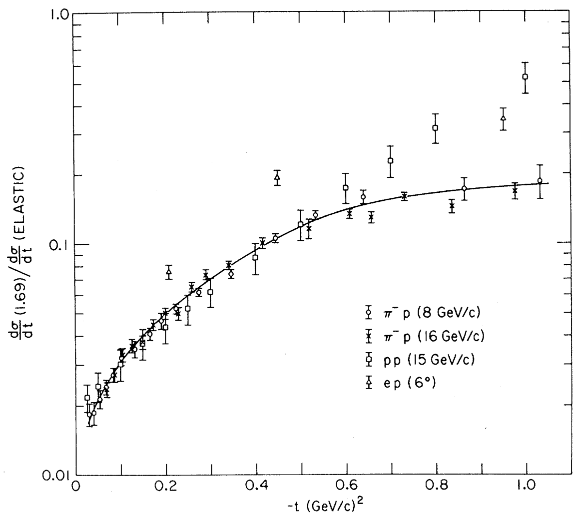

FIG. 3. Dependence on t of the ratio of $d\sigma/dt$ for $N^*(1.69)$ to the elastic scattering for $\pi^- p$ at 8 and 16 GeV/c, pp at 15 GeV/c (Refs. 1 and 8), and from ep scattering data (Ref. 14).

and the elastic scattering is evidence for factorization of the Pomeranchuk singularity.[12]

The dependence of σ_{tot} on incident momentum can be parametrized by the value of n in the formula $\sigma \propto p^{-n}$; the last line of Table I gives the values of n found for the $\pi^- p$ data. All but the $N^*(1.24)$ have a value of n consistent with that of the elastic cross section, viz., $n=0.2$. This same behavior is seen in the pp data.[1,13] The background cross section under the N^* peaks obtained from the fitting process has an energy dependence of $n=0.7\pm 0.1$. This accounts for the fact that the N^* peaks in Fig. 1 appear more prominent at 16 GeV/c than at 8 GeV/c. The $N^*(1.24)$ has n close to 1.0; the secondary maximum in $d\sigma/dt$ at $-t \approx 1.0$ (GeV/c)^2 appears to fall even faster, i.e., $n>2$.

In Fig. 3, we plot the ratio of the $N^*(1.69)$ to the elastic cross section versus $-t$ at both energies; within errors the ratio is observed to be independent of incident energy. Also plotted on Fig. 3 is the same ratio for pp at 15 GeV/c[8]; it agrees well with $\pi^- p$ for $|t|<0.5$ (GeV/c)^2 but lies higher at larger t. This divergence rules out the mechanism of single Pomenanchukon exchange alone. Preliminary results for this ratio from ep scattering[14] are also shown in Fig. 3; the curves are similar but with the ep data lying a factor of 2 higher. Similar but less precise ratios can be plotted for the other N^*'s; the $N^*(1.52)$ and $N^*(1.69)$ will clearly look similar to Fig. 3, whereas the $N^*(1.24)$ and $N^*(1.41)$ will have largely negative slopes for $|t|<0.6$ (GeV/c)^2.

A number of theoretical attempts have been made to explain the features seen in N^* production in πp and pp, on the basis of various models. The $N^*(1.24)$ cross sections are readily explained, in pp by single-pion exchange[15] and in πp by single-ρ exchange. A fit provided by a single Regge ρ-exchange model[16] is shown in Fig. 2. Attempts to explain the production of the isospin-$\frac{1}{2}$ N^*'s belonging to the natural parity series (i.e., specific models for diffraction dissociation) all incorporate diffraction scattering as a basic ingredient, thereby reproducing the near independence on incident energy. Several possibilities have been suggested[17] as to how the large variation in slopes may arise. Finally there has been a series of calculations starting with Drell and Hiida[18] on possible kinematic enhancement effects.

In summary, we have established the dependence on t and incident energy of four N^* peaks seen in $\pi^- p$ in the t range 0.05 to 1.5 (GeV/c)^2. The similarity between $\pi^- p$ and pp scattering for what we think are the isospin-$\frac{1}{2}$ N^*'s has been clearly demonstrated. These results reinforce the ideas of diffraction dissociation for the production mechanism and approximate factorization of the amplitudes.

It is a pleasure to acknowledge our indebtedness to the alternating-gradient synchrotron staff, the On-Line Data Facility, the Instrumentation Division, and the Physics Design Groups both at Brookhaven National Laboratory and at Carnegie-Mellon University. We also acknowledge a useful discussion regarding the data with I. Muzinich.

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†Present address: National Accelerator Laboratory, Batavia, Ill.

‡Present address: State University of New York at Stony Brook, Stony Brook, N. Y.

§Present address: Northeastern University, Boston, Mass.

||Present address: T. J. Watson Research Center, IBM, Yorktown Heights, N. Y.

**Present address: The University of Birmingham, Birmingham 15, England.

[1]E. W. Anderson *et al.*, Phys. Rev. Lett. 16, 855 (1966).

[2]G. Bellettini *et al.*, Phys. Lett. 18, 167 (1965); I. M. Blair *et al.*, Phys. Rev. Lett. 17, 789 (1966); C. M. Ankenbrandt *et al.*, Phys. Rev. 170, 1223 (1963); J. V.

Allaby *et al.*, Phys. Lett. 28B, 229 (1968).

[3]L. Van Hove, in *Proceedings of the International Symposium on Contemporary Physics, Trieste, Italy, June 1968* (International Atomic Energy Agency, Vienna, Austria, 1968), Vol. II.

[4]E. W. Anderson *et al.*, Phys. Rev. Lett. 20, 1529 (1968).

[5]G. Giacomelli *et al.*, CERN Report No. HERA 69-1 (unpublished).

[6]A. Donnachie, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968,* edited by J. Prentki and and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968).

[7]M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960); D. R. O. Morrison, Phys. Rev. 165, 1699 (1968).

[8]Results of the final analysis of the data of Ref. 1: E. W. Anderson *et al.*, to be published.

[9]E. Bloom *et al.*, presented to the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968 (unpublished). See also review talk by W. H. K. Panofsky, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968,* edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 35.

[10]K. J. Foley *et al.*, Phys. Rev. Lett. 19, 397 (1967).

[11]The elastic fit was of the form Ae^{Bt} in the region $|t| \leq 0.2$ (GeV/c)^2; the values of $d\sigma/dt$ ($t=0$) obtained are 38.0 and 31.7 for $\pi^- p$ at 8 and 16 GeV/c and 72 for pp at 15 GeV/c, all in mb (GeV/c)$^{-2}$.

[12]P. G. O. Freund, Phys. Rev. Lett. 21, 1375 (1968).

[13]D. R. O. Morrison, Phys. Lett. 22, 528 (1966).

[14]See Ref. 9, and also P. L. Prichett *et al.*, Phys. Rev. 184, 1825 (1969); models relating N^* production in pp and ep have been examined by A. Rubinstein, Phys. Rev. 182, 1748 (1969). The ep data in Fig. 3 were taken at fixed $\theta_{\rm lab} = 6°$ by changing the incident energy; it is possible that the curve for fixed incident energy may differ.

[15]B. Haber and G. Yekutieli, Phys. Rev. 160, 1410 (1967); B. Margolis and A. Rotsstein, Nuovo Cimento 45A, 1010 (1966).

[16]G. H. Renninger and K. V. L. Sarma, Phys. Rev. 178, 2201 (1969); G. H. Renninger, private communication.

[17]A. W. Hendry and J. S. Trefil, Phys. Rev. 184, 1680 (1969); S. Frautschi and B. Margolis, Nuovo Cimento 57A, 427 (1968); M. Jacob and S. Pokorski, Nuovo Cimento 61A, 233 (1969).

[18]S. D. Drell and K. Hiida, Phys. Rev. Lett. 7, 199 (1961); J. G. Rushbrooke, Phys. Rev. 177, 2357 (1969); E. L. Berger, Phys. Rev. 179, 1567 (1969).

ERRATA

RANGE OF VIRTUAL PHOTONS IN DEEP INELASTIC *ep* SCATTERING. Jean Pestieau, Probir Roy, and Hidezumi Terazawa [Phys. Rev. Lett. 25, 402 (1970)].

On page 402, in the first equation, read M_p^{-1} instead of M_μ^{-1}. Page 403, column 2, line 1 should read: "However, if $F_2(\omega)$ goes to zero as ω^n when $\omega \to 0$ and if m is the order of the highest derivative of $F_2(\omega)$ that exists in $0 < \omega \leq 1$ and that satisfies $F_2^{(m)}(\omega) = 0$ at $\omega = 1$, then $f_2(x \cdot P)$ falls off as $(x \cdot P)^{-(n+1)}$ when $[n] + 1 \leq m$ and at least as fast as $(x \cdot P)^{-(m+2)}$ when $[n] + 1 > m$ as $x \cdot P \to \infty$."

ANOMALOUS REAL PARTS IN THE T MATRICES OF UNSTABLE PARTICLES. Theodore Bauer [Phys. Rev. Lett. 25, 485 (1970)].

The second mathematical expression in the first column on p. 487 should read

$$f(0)_{\gamma N \to \rho^0 N} \simeq g_{\gamma p}(iK/4\pi)\sigma_{\rm tot}(\rho N)[1 + i\alpha_1 + i\alpha_n].$$