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PHOTOPRODUCTION OF $\pi^{\pm}\Delta(1236)$ FROM HYDROGEN AND DEUTERIUM AT 16 GeV†

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Cross sections for $\gamma p\rightarrow\pi^-\Delta^{++}$, $\gamma n\rightarrow\pi^-\Delta^+$, $\gamma p\rightarrow\pi^+\Delta^0$, and $\gamma n\rightarrow\pi^+\Delta^-$ have been measured at 16 GeV for momentum transfers $-t\leq 2$ GeV^2. The charge-symmetric cross sections differ by typically a factor of 2, indicating interference between isoscalar and isovector photon amplitudes. The ratio of $\pi^+\Delta^0$ and $\pi^+\Delta^-$ cross sections is not consistent with pure isospin $I=1$ exchange in the t channel, appreciable $I=2$ exchange being required. A factor-of-5 discrepancy is found for a vector-dominance plus line-reversal prediction.

Measurements of the differential cross sections for $\pi^{\pm}\Delta(1236)$ photoproduction off hydrogen and deuterium have been made at a photon energy of 16 GeV for the following reactions:

$$\gamma+p\rightarrow\pi^-+\Delta^{++}, \quad (1)$$

$$\gamma+p\rightarrow\pi^++\Delta^0 \quad (2)$$

from hydrogen, and

$$\gamma+d\rightarrow\pi^-+\Delta+N_s, \quad (3)$$

$$\gamma+d\rightarrow\pi^++\Delta+N_s \quad (4)$$

from deuterium, where N_s is the spectator nucleon. By subtraction of the above, the two-body reactions

$$\gamma+n\rightarrow\pi^-+\Delta^+, \quad (5)$$

$$\gamma+n\rightarrow\pi^++\Delta^- \quad (6)$$

are obtained. These measurements[1] were made at π production angles such that the square of the four-momentum transfer t ranged from 0.0 to -2.0 GeV^2.

These measurements are of particular interest because of the sensitivity of these cross sections to the possible exchange of particles with isospin >1 (exotic exchanges). The Δ cross section can have a large contribution from the interference of a small exotic exchange amplitude with the $I=1$ amplitudes while cross sections for processes involving double charge exchange ($\pi^-p\rightarrow\pi^+\Delta^-$, for example) are only sensitive to the square of the $I>1$ amplitude.

The method used was the same as in our previous work.[2,3] The Stanford Linear Accelerator Center 20-BeV/c spectrometer measured the angle and momentum of pions produced from either the 30-cm liquid-hydrogen target or the 30-cm liquid-deuterium target by a bremsstrahlung beam with 16-GeV peak energy. The pion was identified by a threshold and a differential Cherenkov counter, a shower counter, and a range counter. The bremsstrahlung beam flux was monitored with a Cherenkov cell in front of the target and a secondary-emission quantameter downstream of the target. These monitors were periodically calibrated against a precision silver calorimeter which served as the absolute standard for the photon beam.[4] In order to minimize irregularities in yields from variations in bin size and efficiency in the momentum counter hodoscope, the spectrometer was operated in a scanning mode by recording many short runs, each separated by one bin width or ~0.2% in the momentum setting of the spectrometer.

At a given angle six different runs were taken: hydrogen, deuterium, and empty target, with the spectrometer set for either positive or negative particles. The full-target minus the empty-target yields were then used as input to the least-squares fitting program.

The processes considered in fitting the data were similar to those used in Ref. 2, and included the single-pion reaction $\gamma N \to \pi N$, the Δ reaction $\gamma N \to \pi\Delta$ with the shape of the Δ given by the Jackson relativistic Breit-Wigner form,[5] the ρ process $\gamma N \to \rho^0 N \to \pi^+\pi^- N$, the three-body phase space $\gamma N \to \pi\pi N$, and the Drell mechanism $\gamma N \to \pi$ + anything. For the Drell case, the measured values of $\sigma_{tot}(\pi^+ p)$ and $\sigma_{tot}(\pi^- p)$ were used in the Drell formalism for π^- and π^+ production off hydrogen, respectively, and the sum $\sigma_{tot}(\pi^+ p) + \sigma_{tot}(\pi^- p)$ was used for either π^+ or π^- production off deuterium. The $\Delta(1236)$ contribution was removed from the measured values of $\sigma_{tot}(\pi^\pm p)$ in order that only the one-Δ process be used in fitting for the Δ contribution in the data.

All of the processes were folded with a bremsstrahlung photon-energy spectrum calculated for a $0.03X_0$ radiator. Smearing due to Fermi momentum was also included for the deuterium case using the Hulthén momentum distortion. Good χ^2's were obtained in most cases with only the single π, Δ, and ρ fits, one set of fits being illustrated in Fig. 1. Fits using the single π and Δ with various combinations of ρ and/or Drell process gave similar delta cross sections to within 5%, with little change in χ^2. The ρ process was chosen in the final fits. The phase-space contribution to the yields was estimated from lower-energy bubble-chamber data. Bub-

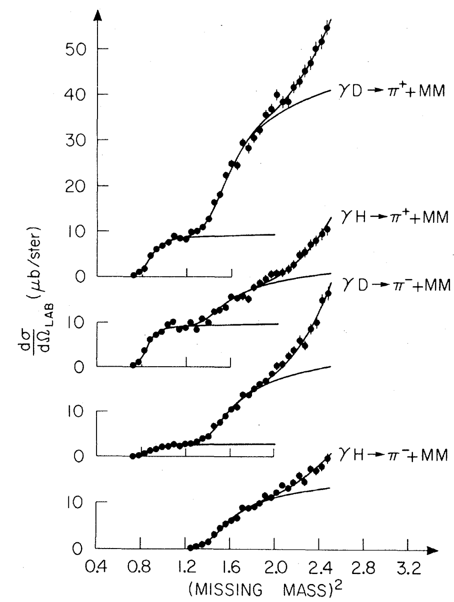

FIG. 1. Fits to the π^+ and π^- data from hydrogen and deuterium for $d\sigma/d\Omega_{lab}$ vs MM^2 at 1.4°. The curves show the single nucleon, the Δ, and the ρ processes folded with a $0.03X_0$ bremsstrahlung spectrum including a Fermi smearing for the deuterium case as given by the Hulthén momentum distribution.

Table I. Cross sections $d\sigma/dt$ in $\mu b/GeV^2$ for $\pi\Delta$ photoproduction off hydrogen, off deuterium, and (by their subtraction) off neutrons at a photon energy of 16 GeV, obtained from fits using the single nucleon, the Δ, and the ρ processes, an estimate for the phase-space process, and inclusion of a ±5% error in quadrature for the hydrogen and deuterium fits to account for errors in the phase-space estimate. An overall normalization error of ±5% has not been included.

Spectrometer Angle (deg)	$-t$ (GeV^2)	$\gamma p \to \pi^-\Delta^{++}$	$\gamma D \to \pi^-\Delta N_s$	$\gamma n \to \pi^-\Delta^+$	$\gamma p \to \pi^+\Delta^0$	$\gamma D \to \pi^+\Delta N_s$	$\gamma n \to \pi^+\Delta^-$
0	0.0010 ± 0.0006	0.74 ± 0.06	0.83 ± 0.15	0.09 ± 0.16	0.47 ± 0.11	1.00 ± 0.14	0.54 ± 0.18
0	0.0028 ± 0.0022	0.81 ± 0.05	1.07 ± 0.07	0.26 ± 0.09	0.28 ± 0.05	0.99 ± 0.07	0.71 ± 0.09
0.30	0.014 ± 0.006	1.07 ± 0.07	1.24 ± 0.09	0.17 ± 0.11	0.35 ± 0.04	1.33 ± 0.10	0.98 ± 0.10
0.70	0.038	0.66 ± 0.03	0.82 ± 0.05	0.16 ± 0.06	0.30 ± 0.03	0.98 ± 0.06	0.68 ± 0.06
1.00	0.077	0.42 ± 0.02	0.52 ± 0.03	0.09 ± 0.04	0.202 ± 0.012	0.70 ± 0.04	0.50 ± 0.04
1.40	0.149	0.169 ± 0.009	0.220 ± 0.012	0.051 ± 0.015	0.138 ± 0.009	0.408 ± 0.022	0.269 ± 0.024
2.30	0.40	0.060 ± 0.003	0.081 ± 0.005	0.020 ± 0.006	0.064 ± 0.006	0.176 ± 0.011	0.112 ± 0.012
3.28	0.80	0.0274 ± 0.0016	0.0393 ± 0.0028	0.012 ± 0.003	0.026 ± 0.003	0.076 ± 0.004	0.050 ± 0.005
3.68	1.00	0.0150 ± 0.0010	0.0192 ± 0.0016	0.0041 ± 0.0019	0.0168 ± 0.0017	0.038 ± 0.003	0.021 ± 0.003
4.20	1.29	0.0048 ± 0.0004	0.0085 ± 0.0008	0.0036 ± 0.0009	0.0065 ± 0.0008	0.0152 ± 0.0012	0.0087 ± 0.0014
4.68	1.58	0.0020 ± 0.0003	0.0036 ± 0.0006	0.0017 ± 0.0006	0.0019 ± 0.0005	0.0047 ± 0.0007	0.0028 ± 0.0009
5.30	2.00	0.00033 ± 0.00009	0.00086 ± 0.0003	0.0005 ± 0.0003	0.0007 ± 0.0003	0.0016 ± 0.0004	0.0009 ± 0.0005

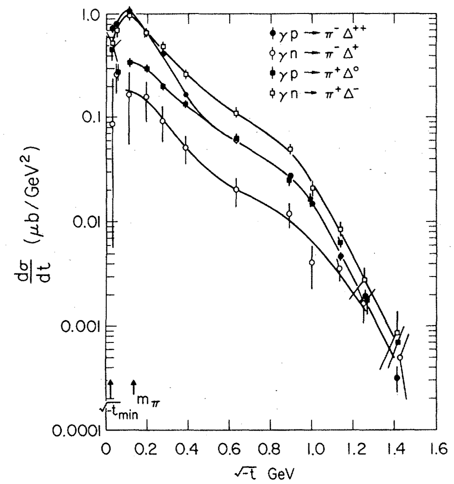

FIG. 2. Measured cross sections for $\pi^{\pm}\Delta$ photoproduction versus momentum transfer $(-t)^{1/2}$. The γn cross sections result from deuterium minus hydrogen subtractions. Fits made to missing-mass spectra included the single-nucleon process, the Δ, background from $\gamma N \to \rho N$, and a correction for phase space $\gamma N \to \pi\pi N$. A Jackson-type Breit-Wigner form was used to calculate the shape of the Δ contribution. Smooth curves are drawn to guide the eye.

ble-chamber data from 3 to 7.5 GeV for $\gamma p \to \pi^+\pi^- p$ give cross sections for the Δ^{++}, ρ^0, and the remainder (which we call phase space). We scaled the phase-space cross section to 16 GeV with that of the Δ^{++}; such scaling agrees with the bubble-chamber data from 3.5 to 7.5 GeV. An e^{3t} dependence was used for the π distribution from the phase-space process, as seen (approximately) in other forward photoproduction processes. Finally, to estimate the π^0 final states not measured in the bubble-chamber data, we assumed the undetected πN systems could be in some combination of isospin $I=\frac{1}{2}$ or $\frac{3}{2}$ states. Various combinations of $I=\frac{1}{2}$ or $\frac{3}{2}$ were tried, and the one that agreed best with the bubble-chamber data was used. If a flat t distribution for phase space were used instead of e^{3t}, the phase-space contribution would be less for $|t|<1.5$ and larger for $|t|>1.5$, resulting in changes in the $\pi\Delta$ cross sections which are less than the quoted errors. The estimated phase-space correction decreased the Δ cross section for Reactions (1) to (4) by 2, 5, 4, and 3%, respectively, with a resulting decrease of ~12 and ~2% for Reactions (5) and (6), respectively. A ±5% error was added quadratically to each of the fits for Reactions (1) to (4) to account for uncertainties in our phase-space estimate.

Comparisons of deuterium and hydrogen cross sections involve several possible systematic problems such as Glauber shadowing of one nucleon by the other in deuterium and the relative target densities (we have used 0.0705 g/cm^2 for H_2 at 20.8°K and 0.1687 g/cm^2 for D_2 at 21.1°K). To get a feeling for these effects we have compared our cross sections for $\gamma p \to \pi^+ n$ from free protons and from those bound in deuterium nuclei [only data with $(-t)^{1/2} \geqslant 0.4$ GeV were used in order to avoid exclusion-principle effects]. These gave a deuterium-to-hydrogen ratio of 1.02 ± 0.02 (statistical errors only). Comparison of our $K^+\Lambda$ production from deuterium to hydrogen for $-t \approx 0$ to 2 GeV^2 gave a ratio 1.02 ± 0.04. The product of the systematic effects is thus consistent with unity and we have taken the neutron cross sections to be simply the difference between deuterium and hydrogen cross sections. The results are shown in Table I and Fig. 2. These results do not include experimental uncertainties common to all points, of about ±5%.

Comparisons of different charge states allow sensitive tests for amplitudes which are usually assumed to be small. In terms of t-channel quantum numbers the four cross sections can be written as

$$\frac{d\sigma}{dt}\begin{pmatrix}\gamma p \to \pi^-\Delta^{++}\\ \gamma n \to \pi^+\Delta^-\end{pmatrix} = \sum_{i=1}^{8}\left|A_{1-}{}^{i} + \begin{pmatrix}-1\\ 1\end{pmatrix}A_{1+}{}^{i} + A_{2-}{}^{i}\right|^2, \quad (7a)$$

$$\frac{d\sigma}{dt}\begin{pmatrix}\gamma n \to \pi^-\Delta^{+}\\ \gamma p \to \pi^+\Delta^0\end{pmatrix} = 3\sum_{i=1}^{8}\left|A_{1-}{}^{i} + \begin{pmatrix}1\\ -1\end{pmatrix}A_{1+}{}^{i} - \tfrac{1}{3}A_{2-}{}^{i}\right|^2, \quad (7b)$$

where the summation is over the eight helicity amplitudes and the subscripts 1 and 2 refer to the t-channel isotopic spin; the + and − subscripts denote the G parity of the t channel and correspond to isoscalar and isovector photons, respectively. Figure 3 shows the ratios of the charge-symmetric cross sections; in general, the cross sections are not equal and interference terms between exchanges of opposite G parity are required, as observed in single pion photoproduction.[6]

If $I=2$ exchange is negligible, the cross sections are uniquely predicted to be in the ratio

$$\frac{\gamma p \to \pi^-\Delta^{++}}{\gamma n \to \pi^-\Delta^{+}} = \frac{\gamma n \to \pi^+\Delta^-}{\gamma p \to \pi^+\Delta^0} = 3. \quad (8)$$

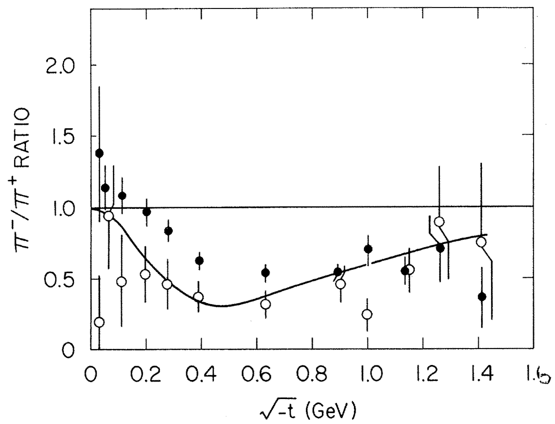

FIG. 3. The ratios $(\gamma p\to\pi^-\Delta^{++})/(\gamma n\to\pi^+\Delta^-)$ (closed circles with error bars) and $(\gamma n\to\pi^-\Delta^+)/(\gamma p\to\pi^+\Delta^0)$ versus $(-t)^{1/2}$ (open circles with error bars). These ratios, where different from 1, indicate an interference between isoscalar (ω-like photon) and isovector (ρ^0-like photon) amplitudes. For comparison the curve shows the ratio $(\gamma n\to\pi^-p)/(\gamma p\to\pi^+n)$, Ref. 6.

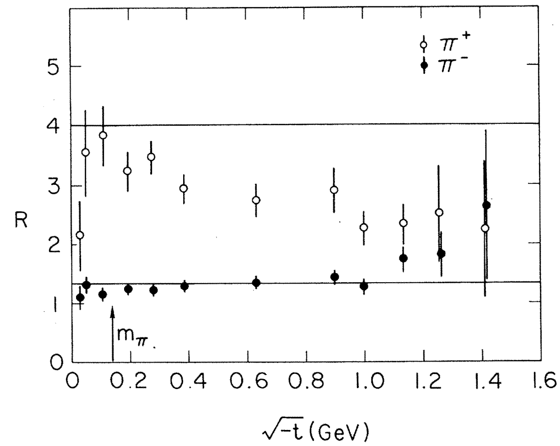

FIG. 4. The deuterium-to-hydrogen ratios versus $(-t)^{1/2}$. Neglecting absorption effects in deuterium, which were found to be negligible in our $\gamma d\to\pi^+nn$ data, dominance of isospin-1 exchange implies $R=4$ for π^+ and $R=\frac{4}{3}$ for π^-. The π^+ data indicate that isospin-1 exchange alone does not fit the data for $|t|\sim 0.15$ GeV^2.

The deuterium-to-hydrogen ratios should then be $\frac{4}{3}$ for π^- and 4 for π^+. These ratios are shown in Fig. 4 as a function of $(-t)^{1/2}$. The π^+ ratio shows clear deviations from the prediction over most of the t range studied, a typical value being 3, in which case the $I=2$ exchange amplitude must be at least 16% of the $I=1$ exchange amplitude. Although this result can be interpreted in terms of exotic meson exchange, it could also be explained by double Regge pole exchange, giving Regge cuts. The measured π^- ratio is not as sensitive a test for $I=2$ exchange since the errors are large in comparison with the smaller ratio.

The vector-dominance model suggests the relation

$$\frac{1}{2}\left[\frac{d\sigma}{dt}(\gamma p\to\pi^-\Delta^{++})+\frac{d\sigma}{dt}(\gamma n\to\pi^+\Delta^-)\right]=g_{\gamma\rho}{}^2\left(\rho_{11}{}^{\mathrm{hel}}\frac{d\sigma}{dt}\right)_{\pi^+p\to\rho^0\Delta^{++}}+g_{\gamma\omega}{}^2\left(\rho_{11}{}^{\mathrm{hel}}\frac{d\sigma}{dt}\right)_{\pi^+p\to\omega\Delta^{++}},\qquad(9)$$

where the two photoproduction cross sections are averaged to eliminate $\rho\omega$ interference terms. In addition to the usual vector-dominance assumptions the derivation assumes line-reversal invariance. Using the $\pi^+p\to\rho^0\Delta^{++}$ data from the Aachen-Berlin-CERN collaboration,[7] the two sides of Eq. (9) are plotted in Fig. 5 for 8 GeV, assuming the ratio of Δ^- to Δ^{++} cross sections to be independent of energy. The 10% ω contribution has been ignored and $g_{\gamma\rho}{}^2=3.5\times10^{-3}$ $(\gamma_\rho{}^2/4\pi=0.52)$ was used. The figure shows a factor-of-5 discrepancy in the vector-dominance relation, which also disagrees with the calculations of Dar.[8] Three possible sources can be found for the factor-of-5 discrepancy: (a) the vector-dominance model which has had troubles elsewhere,[9] (b) the line-reversal assumption which will not be valid if interferences exist between amplitudes corresponding to t-channel exchanges of

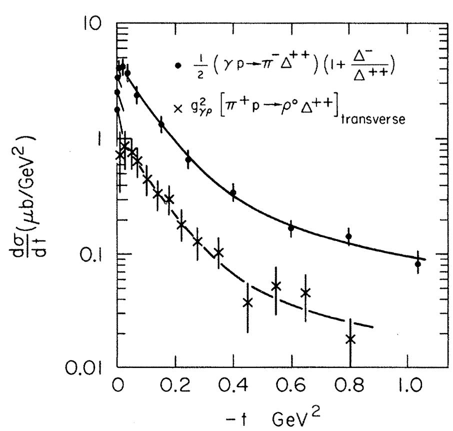

FIG. 5. Test of the vector-dominance model. The average of Δ^{++} and Δ^- is compared with that predicted from the $\pi^+p\to\rho^0\Delta^{++}$ data at 8 GeV by vector dominance. There is disagreement by about a factor of 5. Smooth curves are drawn to guide the eye.

opposite signature,[10,11] and (c) difficulty in the cross-section determination for $\rho^0\Delta^{++}$ since both the ρ and Δ have large widths.[12]

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[1]A preliminary version of these data is presented in *Proceedings of Boulder Conference on High Energy Physics, Boulder, Colorado, August 1969*, edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Univ., Boulder, Colo., 1970), and also in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

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EXPERIMENTAL STUDY OF $\pi^- + p \to \pi^- + N^*$ AT 8 AND 16 GeV/c*

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We have measured $d\sigma/dt$ for five N^* reactions in the process $\pi^- + p \to n^- + N^*$ at incident momenta of 8 and 16 GeV/c in the range of $|t| < 1.5$ (GeV/c)^2. N^* bumps are observed at masses of 1.24, 1.41, 1.52, 1.69, and 2.19 GeV. Considerable structure is apparent in the $d\sigma/dt$ distributions including leveling off for $|t| < 0.15$ and dips at larger t in both the $N^*(1.24)$ and $N^*(1.52)$ reactions. The logarithmic slopes for the four higher mass N^*'s are identical to those seen in the corresponding reaction in pp scattering and the cross sections exhibit a similar elasticlike behavior with incident energy.

The study of angular distributions of various isobar (N^*) channels in the reaction $p + p \to p + N^*$ in the 6- to 30-GeV/c region[1,2] has proven very useful in delineating one of the dominant dynamical mechanisms underlying hadronic collisions at high energy,[3] viz., the "diffractive" mechanism, which controls the class of quasi two-body reactions that occur without exchange of internal quantum numbers. We report here a companion experiment to our earlier pp study,[1] carried out with incident π^- mesons at the Brookhaven alternating-gradient synchrotron, i.e., the reaction

$$\pi^- + p \to \pi^- + N^*, \tag{1}$$

at momenta of 8 and 15 GeV/c and over a four-momentum transfer (t) range of 0.05-1.5 GeV/c^2. The basic data are a high-statistics (~9×10^6 events), high-resolution study of the high-momentum end of the inelastic π^- spectrum emitted by a monoenergetic π^- beam striking a liquid-hydrogen target. In the resulting missing-mass (MM) spectra the N^* reactions are seen as clear maxima. All five of the N^* channels that were