^{49}Cr. (Centroid analysis of these two states —assumed to be equally populated—yields a mass excess for ^{49}Mn differing from that in Ref. 7 by ≈50 keV.) Average differential cross sections to the summed ground and first excited states are ≈1.5 μb/sr (for tritons) and ≈3 μb/sr (for ^{3}He).

These results demonstrate the practicability of direct mass measurements of $Z > N$ nuclei above Ti using heavy-ion-induced reactions. By extension of these investigations to the use of ^{14}N and ^{16}O projectiles as well as to more exotic reactions such as ^{40}Ca(^{12}C, ^{6}He)^{46}Cr, ^{40}Ca(^{12}C, ^{8}He)^{44}Cr, etc., it should be possible to determine nuclear masses and their agreement with theoretical prediction in regions of high Coulomb energy very far from the valley of stability.

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STRUCTURE OF THE POTENTIAL ENERGY SURFACE AT LARGE DEFORMATIONS*

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The two-center shell model has been generalized to the shape of two overlapping spheroids with equal mass. In this model shell corrections have been calculated and the potential energy surface of two heavy nuclei has been investigated. The influence of fragment shells in the model gives rise to structure in this surface which supports assumptions of earlier models for the scission point.

In recent years much new effort has been put into the calculation of shell effects in the nuclear potential-energy surface.[1,2] These calculations begin with well-known shell models and extend them to the inclusion of deformed shapes of the fissioning nucleus by simply deforming the equipotential surfaces in a uniform way. Only very recently a new model has been proposed[3,4] which, in contrast to these, can describe the entire course of a fissioning nucleus from its ground state to stages beyond scission.

In this paper the schematic shapes of Ref. 3 have been generalized to the configuration of two overlapping spheroids with equal mass. This same family of shapes has been investigated very extensively and carefully by Nix and Swiatecki[5] in the liquid-drop model (LDM).

The Hamiltonian operator of the model is

$$H = T + \tfrac{1}{2}m\omega_\rho{}^2\rho^2 + \tfrac{1}{2}m\omega_z{}^2(|z|-z_0)^2 + V(\vec{l}, \vec{s}), \qquad (1)$$

with

$$V(\vec{l}, \vec{s}) = \begin{cases} C\vec{l}_1\cdot\vec{s} + D[\vec{l}_1{}^2 - \tfrac{1}{2}N(N+3)], & z > 0, \\ C\vec{l}_2\cdot\vec{s} + D[\vec{l}_2{}^2 - \tfrac{1}{2}N(N+3)], & z < 0. \end{cases} \qquad (2)$$

Here $\vec{l}_1$ and $\vec{l}_2$ describe the angular momenta with respect to the two centers at $z = z_0$ and $z = -z_0$, respectively. As a direct generalization of the Nilsson model to two centers we have been content here with the $\vec{l}\cdot\vec{s}$ term instead of the usual gradient-dependent expression. The separation parameter z_0 in Eq. (1) is equal to half of the distance of the centers of the two spheroids, while the ratio ω_ρ/ω_z describes the deformation of the spheroids. In Eq. (2) N is given by $N = n_z + N_\rho$, where n_z is an integer only for $z_0 = 0$ and $z_0 \to \infty$,[3] thus describing the continuous transition of one initial nucleus to two final nuclei. The two parameters C and D have been made z_0 dependent and vary between the values

$$C_p = -2\hbar\dot{\omega}_0 \times 0.0577, \quad D_p = \tfrac{1}{2}C_p \times 0.65,$$

$$C_n = -2\hbar\dot{\omega}_0 \times 0.0635, \quad D_n = \tfrac{1}{2}C_n \times 0.325,$$

for the actinides,[6] and

$$C_p = -2\hbar\dot{\omega}_0 \times 0.0688, \quad D_p = \tfrac{1}{2}C_p \times 0.558,$$

$$C_n = -2\hbar\dot{\omega}_0 \times 0.0638, \quad D_n = \tfrac{1}{2}C_n \times 0.491,$$

for the region around mass 100.[7] The transition is made according to the number of particles already in the new fragment shells and ensures that the shell structure of the fragments as well as that of the fissioning nucleus is described properly. For $\hbar\dot{\omega}_0$ the usual value $\hbar\dot{\omega}_0 = 41A^{-1/3}$ MeV has been used.

With this model shell corrections δU have been calculated by using the Strutinsky prescription.[8] The eigenvalues of the Hamiltonian (1) have been obtained by diagonalizing H in the basis of eigenstates of the local part of (1) which can be obtained by a simple matching procedure.[3] For the LDM part we have used the mass formula of Myers and Swiatecki.[9] The final potential-energy surface (PES) is then

$$E(z_0, \beta) = E_{\mathrm{LDM}}(z_0, \beta) + \delta U(z_0, \beta) + E_p(z_0, \beta). \qquad (3)$$

Here β stands for ω_ρ/ω_z. The pairing energy E_p has been calculated in the BCS formalism by taking into account 24 levels at the Fermi surface and by using a surface-dependent pairing force.[1] The strengths at ground-state deformations are $G_p = 32.0/A$ MeV and $G_n = 29.0/A$ MeV; these fit the empirical odd-even mass differences.[10] The usual constraint of volume conservation has been applied for that equipotential surface which coincides with the surface of the nucleus. It is well known that this assumption is not critical for the calculation of δU.[8,4]

Calculations of the potential energy surface $E(\beta, z)$ have been performed for the two nuclei ^{198}Pb and ^{236}U; for the former without inclusion of the pairing interaction. From the extensive investigations of Nix[7] it is known that the barrier heights in the LDM have values which are too large because of the inaccurate description of the neck in the two-spheroid model. However, the two-spheroid model contains all general properties of a fissioning nucleus. In addition, the shell corrections in the two limiting cases of no "neck-in" (pure one-center deformation) and of almost separated fragments do not depend on the specific properties of the neck (in the latter case the wave functions vanish at its location). Therefore, the investigation of shell effects in this model should give, at least qualitatively, some new insight into the structure of the potential-energy surface, especially at large deformations, that can be quantitatively investigated in more refined calculations.

For ^{198}Pb we have limited ourselves to a calculation of the PES near to scission since in this light nucleus the scission and saddle points coincide. In Fig. 1 we show only one example of the results, namely the energy at scission (tangent spheroids) as a function of fragment deformation in comparison with the LDM prediction. This energy curve mainly describes the "stretching mode" in the notation of Ref. 5. One sees that the change due to shell effects is quite appreci-

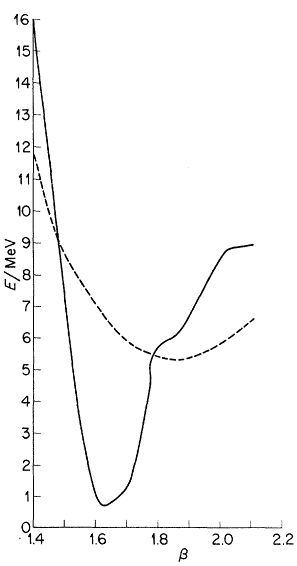

FIG. 1. Potential-energy surface of the nucleus ^{198}Pb at scission as a function of the fragment-deformation parameter β. The relative energy is given in MeV. The dashed curve describes the prediction of the LDM only while for the full-line curve shell corrections have been added.

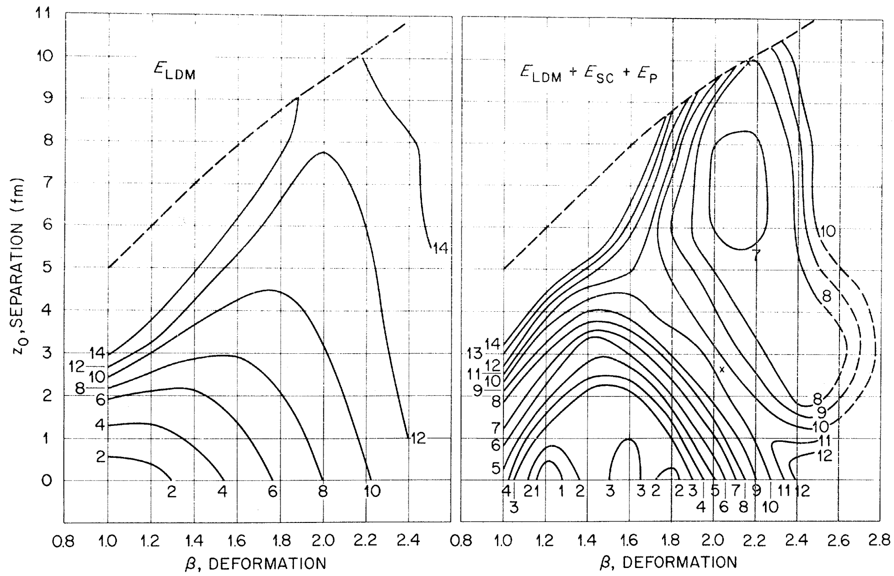

FIG. 2. Potential-energy surface for ^{236}U as a function of the deformation parameter β and the fragment-separation parameter z_0. The numbers at the contour lines give the relative energies in MeV. The energy of the ground state has been normalized to zero. While the left part shows the LDM PES the right part contains the shell and pairing energies also. The straight dashed line marks the scission configuration (two tangent spheroids). Dashed portions of the contour lines indicate extrapolations according to the behavior of the LDM.

able and that the energy of the saddle point is lowered by about 5 MeV. Simultaneously, the minimum is shifted to smaller deformations and becomes much stiffer.[11] The main contribution to the shell correction comes from the neutron configuration of the fragments ($N = 58$). The analysis of the obtained single-particle energies at scission shows that the levels of both fragments are already degenerate within about 0.5 MeV. Also, a calculation with two independent Nilsson models with $\beta = 1.6$ [the minimum position of $E(\beta)$] for the mass $A = 198/2$, $Z = 41$ has confirmed that δU in Fig. 1 is very near to the sum of the shell corrections for these two independent fragment nuclei. This means that the structure of the PES at this large deformation is nearly completely determined by the shell structure of the fragments.

A general conclusion for light nuclei is that all characteristics of the saddle point are determined by the fragment structure, while for heavy nuclei the structure of the fissioning nucleus is important since in these cases the saddle and scission points are well separated. The explanation for the fact that the LDM has worked so well for lighter nuclei is that the PES of the fragments around $A \approx 100$ is rather soft[7] so that their shell effects do not change the LDM PES drastically.

In Fig. 2 the PES for the nucleus ^{236}U is shown together with the pure LDM PES. It is interesting to note that the second minimum appears for $z_0 = 0$, i.e., for pure spheroidal deformation. Therefore, its structure is completely determined by the fissioning nucleus. One also sees that, in agreement with a discussion by Nix and Walker,[12] the fission barrier can be reached in two different ways, i.e., either directly from the ground state or via the second minimum. The saddle point that coincides with the scission point in the LDM is shifted to a position where the fragments still overlap appreciably and has a height of about 10 MeV. The interesting feature of the whole PES is that beyond this very broad barrier, favored by the smooth structure of the LDM PES in this region, a broad and very flat third minimum at about 6 MeV appears, and that the scission point lies again higher at ca. 8 MeV. A check of the single-particle energies at $\beta = 2.1$ and $z_0 = 7$ fm showed that the fragment levels are already almost degenerate with an average perturbation at the Fermi surface of ~0.8 MeV. This means that this third minimum, in contrast to the second, is due to the fragment shells, i.e., it represents the effect of the nuclear interaction of the fragments before scission.

It has been suggested in the literature that this nuclear interaction, in analogy with ion-ion interaction potentials,[13] could lead to some kind of scission minimum.[14] The present calculation has yielded no evidence for such an effect in the case of ^{198}Pb since here the LDM PES is too steep. Whether for ^{236}U the influence of the fragment shells is strong enough to produce a scission minimum depends mainly on the structure of the relative variations of the LDM and shell-correction terms in the region of scission.

In the framework of the two-spheroid model these results may help to justify previous calculations for the scission point by Schmitt and Vandenbosch,[15] and Dickmann and Dietrich.[16] Both calculations assume a scission process sufficiently slow that the two fragments in contact can adjust their deformations in order to retain the lowest energy.

In conclusion we may say that the results obtained in the present two-center shell-model calculation have added to the understanding of the fission process in its latest stages. Models similar to the one presented here are being developed by Dietrich and Dickmann[17] and, for very general shapes, also by Nix[18], so that hopefully more quantitative information about this part of the fission process will soon be developed.

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