proximation when the soft coupling g vanishes. Equation (2) differs from previous Bloch-Nordsieck forms by the inclusion of the factor $1-i\gamma(p+q)/m$ which is kinematically the simplest way to produce a nonzero W_2.

[9]The mass renormalization of the internal nucleon line appears as the exponential of the term $-g^2\xi v^2(2\pi)^{-4}\int d^4k\,(k^2+\mu^2)^{-1}(k\cdot v+i\epsilon)^{-1}$, where v denotes the four-velocity of the BN model, here set equal to $(p+q)/m$. Both this and the expression for Z_2 require a cutoff of the SVNVM momenta, which may be conveniently defined as in I: Insert a factor $e^{-i\alpha k^2}$ into each integrand and take the limit $\alpha \to i\mu_c^{-1}$ at the end of the computation, to produce covariant forms with the proper phase and in which, effectively, $k<\mu_c$. The cutoff necessary for the integral (4) of this calculation will be handled in a somewhat different way, as explained just before Eq. (5).

[10]As in III, this simple estimate of the virtual nucleon is not sufficient to generate a W_1 large enough to reproduce the experimentally small value of $R=(W_2/W_1)(1+\nu^2/q^2)-1$.

[11]D. Yennie, S. Frautschi, and H. Suura, Ann. Phys. (New York) 13, 379 (1961).

[12]These curves have been drawn using $g^2/4\pi=2.0$ and $\xi_0=0.33$; other choices give somewhat different shapes. Essentially, the strength of the coupling determines how fast W_2 vanishes as $x\to 1$, while its intercept at $x=0$ is controlled by ξ_0.

PRECISION MEASUREMENT OF THE MAGNETIC MOMENT OF THE MUON*

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The ratio of muon to proton magnetic moment μ_μ/μ_p has been measured to high precision in three chemical environments. The agreement shows that the "Ruderman correction" is not applicable. The result is $\mu_\mu/\mu_p=3.183\,347(9)$ (2.8 ppm); in terms of the muon mass, this implies $m_\mu/m_e=206.7683(6)$.

The ratio of muon to proton magnetic moment, $\mu_\mu/\mu_p=(g/m)_\mu(g/m)_p^{-1}$ is needed to extract the muon's anomalous magnetic moment, $(g-2)/2$, from the observed frequency in a "$g-2$" experiment, $\omega_{g-2}=(g-2)eB/4mc$. Of more immediate interest, it enters in the relation between the muonium hyperfine splitting ν_m and the fine-structure constant α. The three most recent measurements,[1-3] which have errors of 13 to 22 ppm, are not sufficiently precise to take advantage of the accurate muonium results now available.[4,5] Ruderman[6] suggested that the substantial discrepancy between α determined from hydrogen hyperfine spectra (hfs) and from the then-current muonium hfs and from the Columbia value[1] of μ_μ/μ_p could be partially reconciled by applying to the latter a chemical correction amounting to $\approx$15 ppm. We report[7] new high-precision measurements of μ_μ/μ_p which are 10 ppm below the Columbia result; we show that the Ruderman correction[6] is not applicable; and we find that two newly reported muonium results[4,5] bracket the value for ν_m predicted by our ratio and the currently accepted value of α.

The method is to use the muon decay asymmetry to observe the precession frequency, $geB/2mc$, of a sample of polarized positive muons at rest in a magnetic field, and to observe the resonance frequency of protons in the same field. A 200-MeV/c muon beam was obtained from pions produced at the Lawrence Radiation Laboratory 184-in. cyclotron. Figure 1 represents the arrangement of counters and target in the magnet. The stopped-muon logic was (beam)$HM_{\text{dynode}}\overline{S1S2A1A2}$, and the decay electron was Se($E1$ or E_2)$_{\text{dynode}}\overline{S4S1A1A2M}_{\text{dynode}}$. Timing signals from the muon counter M and the electron counters E were presented to fast discriminators with thresholds set at $\frac{1}{4}$ the trigger thresholds; the output signals were then passed by gated discriminators that were gated on (in a few nsec) if the logic requirements had been met. These gated timing signals then opened (M) and closed (E) the gates of fast scalers which scaled a free-running oscillator. The timing between the muon and electron signals was done by two independent systems: a "digitron" with an effective least count of 1.25 nsec obtained from a 400-MHz clock and two suitably phased scaling systems, and a Hewlett-Packard timing counter (HP5360A) based on a 10-MHz clock and internally converted analog interpolation. The digital information on each event included the two time-interval measurements and records of extra counts which

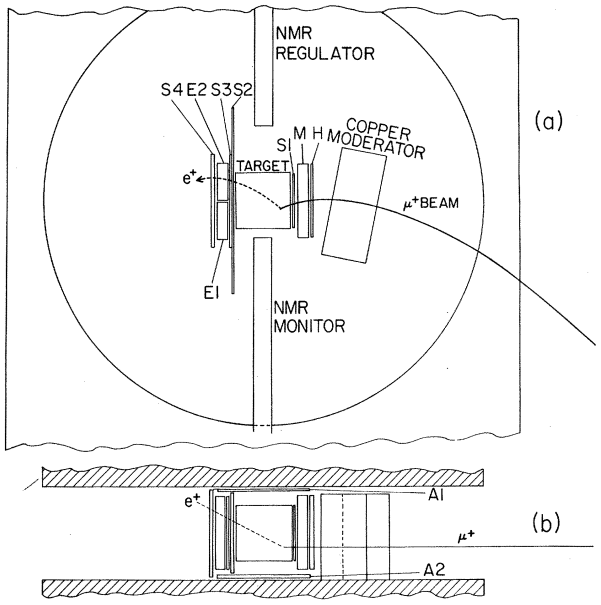

FIG. 1. (a) Plan view and (b) elevation of the apparatus. The 24-in. circle is a special pole-tip assembly fitting inside the 29×36-in.^{2} main gap of the magnet. Collimators, etc., have been omitted.

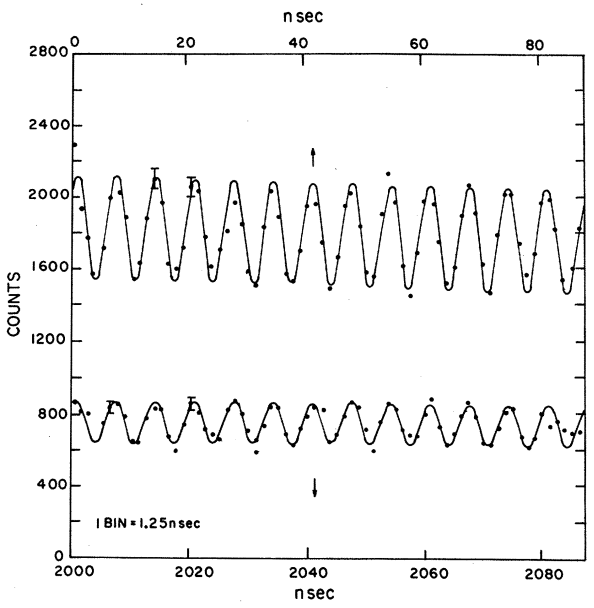

FIG. 2. Two short sections of the data for one target material (0.5N NaOH), for which there were 3.4×10^6 analyzed events. The smooth curve is the maximum-likelihood fit.

could affect the data: second counts in either the E channel or the M channel during the time the gate was open, and any count in an E counter during the 5 μsec preceding the gate opening. This information was stored by an on-line computer, and every few seconds was transferred to magnetic tape along with the digital record of the proton NMR frequency of the "monitor" probe. Details of the method and the many checks on the system will be published elsewhere. The most important point is that the elapsed time for each muon-electron event is recorded with a simple and direct method by counting cycles of a free-running crystal-controlled oscillator. Such a system was used on the muon $g-2$ experiment[8]; it has many internal checks, and can be made highly reliable.

The accumulated data represent the number of events versus elapsed time; it is an exponential, modulated with the frequency we seek. Figure 2 shows a part of the data for one stopping substance.

The stopping material was liquid in a 3-in. cube made of 5-mil Mylar. The container accounted for 1% of the total counting rate; the target-out rate was 2.5%. The decay-electron rate was 60 sec^{-1}, with an asymmetry of 0.16 in water. The target-out asymmetry was 0.05. We measure, and correct for, the frequency of this signal. A large bending magnet with special pole tips and shimming coils[9] gave a field with weighted average of 0.3 ppm above the value at the center of the gap, and a rms deviation of 2 ppm. The field was 11 kG, corresponding to about 149 MHz for muons and 46.8 MHz for protons.

Two separate proton magnetic-resonance systems were used; one was part of the magnetic-field regulation and the other served to monitor the field during running (as shown in Fig. 1) and to map the field (four field maps were made). Proton resonance was observed in a small cylindrical sample of $H_2O+0.005M$ $Fe(NO_3)_3$. Frequency at the monitor position was continuously recorded by a crystal-controlled counter. The field at the center of the gap (target out) was measured every few hours. A small bulk-susceptibility correction exists because the NMR sample and the stopping volume do not have the same shape.

The correct average over the magnetic field map involved an auxiliary experiment. The stopping distribution and decay asymmetry were measured as functions of position in the stopping volume, and the final weight at each point was the product of asymmetry and counting rate.

We have made measurements in NaOH solution, distilled water, and methylene cyanide, $CH_2(CN)_2$. A maximum-likelihood fit was made to the data from each of the two timing systems leaving frequency, phase asymmetry, and (uniform) back-

Table I. Corrections to ω_μ/ω_p, and systematic-error assignments.

Effect	Correction (ppm)	Error (ppm)
Proton resonance frequency at magnet center		0.9 (H_2O) 1.3 [$CH_2(CN)_2$]
Weighted average over field map		1.0
Bulk susceptibility correction	+1.5	0.3
Target-out contribution	−0.4	0.4
Container-wall contribution		0.1
Frequency comparisons		0.02
Root-sum-square of systematic effects		1.4 (H_2O) 1.7 [$CH_2(CN)_2$]

ground as free parameters.[10] The frequency was determined, in each case, to about 2-ppm statistical accuracy. The overall agreement between results from the two independent systems was 0.5 ppm. Starting or ending the analysis interval at different times had no significant effect. The corrections and systematic errors are summarized in Table I.

Results are in Table II. We see no significant difference between the NaOH solution and distilled water. The effect suggested by Ruderman[6] requires the presence of the muon as a positive ion. However, OH^- is known to recombine with H^+ in water at an extremely rapid rate, and it can be shown[11] that the μ^+ ions would become neutralized, in 0.1N NaOH solution, in $<10^{-10}$ sec. The frequency in NaOH solution, expected according to Ruderman to be ~15 ppm lower than in H_2O, is in fact 1.6 ppm higher.

Several lines of evidence lead to the conclusion that μ^+, H^+, and T^+ (tritons) when slowing down in matter do not reach thermal energy as ions.[12] Below a few hundred eV a positive muon has, with high probability, permanently captured an electron. Losing energy by molecular collision, it becomes a "hot atom"[13] which, at a few eV, may become part of a molecule, thus retaining its polarization, or may thermalize, probably depolarizing.

A proton in (liquid) H_2O experiences a magnetic field weakened, due to atomic electrons, by 25.6 ppm. When a muon replaces a proton, it should generally experience approximately the same shielding. Fortunately, most neutral hydrogen-containing molecules have nearly the same shielding effect as water. We list in Table III the species expected on the basis of hot-atom work with tritium, and the shift (with respect to protons in water at room temperature) a proton experiences in each case. There is a muon-proton difference because the muon, with zero-point energy three times as large, sits higher in its anharmonic potential well and moves away from its neighbor. We estimate the effect to be about 0.2 ppm in ordinary molecules.[14] However, the muon in a μHO molecule takes part in hydrogen bonding to neighboring molecules, and the higher zero-point energy should lead to a larger hydro-

Table II. Results for ω_μ/ω_p, including corrections from Table I. The final ratios include the chemical corrections and their errors in ppm from Table III.

Water comparison:	
H_2O	3.183 350 (8) (2.5)
NaOH solution	3.183 355 (8) (2.5)
Final ratios:	
H_2O and NaOH solution combined	3.183 350 (9) (2.8)
$CH_2(CN)_2$	3.183 344 (10) (2.8)
Final result:	
μ_μ/μ_p	3.183 347 (9) (2.8)

Table III. Muons in water and $CH_2(CN)_2$. Composition estimated from tritium hot-atom chemistry. δ is the increase in shielding, in ppm, relative to protons in water. Average shifts: Water, -1.8 ± 2.0 ppm; $CH_2(CN)_2$, $+0.5 \pm 1.5$ ppm.

Species	Fraction	δ (proton)	δ (muon)
Water and NaOH solution			
μHO	0.9	0	−2.0
μH	0.1	0.4	0.2
μH_2O	≈0	−11	−15
Methylene cyanide			
μH	0.7	0.4	0.2
$\mu HC(CN)_2$	0.3	1.5	1.3
$\mu H_2C(CN)$	<0.1	3.0	2.8

gen-bond effect. An estimated upper limit to the additional shielding decrease caused by the hydrogen-bonding effect in water is 4 ppm.[15] We assign 2 ppm for this shift, and an error of ±2 ppm in the net H_2O shift. $CH_2(CN)_2$ does not have a comparable hydrogen-bond problem, but it has a large number of possible species; we assign ±15 ppm error.

The results in Table II for water (combined NaOH and H_2O data) and for $CH_2(CN)_2$ are in gratifying agreement: 1.9 ppm difference, compared with individual errors of 2.8 and 3.1 ppm. We take the average, and since systematic uncertainties contribute over half the error, we leave the error of the average as 2.8 ppm. The final result is thus $\mu_\mu/\mu_p = 3.183\,347(9)$ (2.8 ppm). The previously reported results were: Columbia,[1] 3.183 380(40); Berkeley,[2] 3.183 369(70); Princeton-Penn,[3] 3.183 330(44).

We now put our results and the recent[16] value α into the evaluation by Taylor, Parker, and Langenberg[17] of the muonium hyperfine splitting ν_m. The predicted value proves to be 4463.289(19) MHz. This is very close to the weighted average of the two most recent results: Ehrlich et al.,[4] $\nu_m = 4463.317(21)$ MHz; and Crane et al.,[5] $\nu_m = 4463.249(31)$ MHz. The old discrepancy between hydrogen hfs and muonium hfs, discussed by Ruderman[6] and others, was 40 ppm; it was based on the Columbia muon moment[1] and the 1964 high-field muonium results.[18] Our result brings the muon moment down 10 ppm; the new muonium results account for the remaining 30 ppm; and the muonium hfs is now in satisfactory agreement with theory, using the Josephson-effect α. It is interesting that a more precise value for muonium hfs would lead to a value of α of accuracy comparable with that of the Josephson effect.

Finally, one obtains the muon-electron mass ratio from $g_\mu\mu_e/g_e\mu_\mu$. The result (we follow Taylor, Parker, and Langenberg[17]) is $m_\mu/m_e = 206.7683(6)$.

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[1]D. P. Hutchinson, J. Menes, G. Shapiro, and A. M. Patlach, Phys. Rev. 131, 1351 (1963).

[2]G. McD. Bingham, Nuovo Cimento 27, 1352 (1963).

[3]D. P. Hutchinson, F. L. Larsen, N. C. Schoen, D. I. Sober, and A. S. Kanofsky, Phys. Rev. Lett. 24, 1254 (1970).

[4]R. D. Ehrlich, H. Hofer, A. Magnon, D. Stowell, R. A. Swanson, and V. L. Telegdi, Phys. Rev. Lett. 23, 513 (1969).

[5]P. Crane, J. J. Amato, V. W. Hughes, D. M. Lazarus, G. zu Putlitz, and P. A. Thompson, Bull. Amer. Phys. Soc. 15, 45 (1970).

[6]M. A. Ruderman, Phys. Rev. Lett. 17, 794 (1966).

[7]A preliminary report has been given: J. F. Hague *et al.*, Bull. Amer. Phys. Soc. 15, 608 (1970).

[8]J. Bailey, W. Bartl, G. von Bochman, R. C. A. Brown, F. J. M. Farley, H. Jostlein, E. Picasso, and R. W. Williams, Phys. Lett. B 28, 287 (1968).

[9]This system was used by K. M. Crowe and by Bingham, Ref. 2. See G. McD. Bingham, Lawrence Radiation Laboratory Report No. UCRL-10107, 1963 (unpublished).

[10]Background was 1 to 1.5% of initial counting rate. Leaving the lifetime as an additional free parameter does not change other results, and gives a lifetime within ≈1 standard deviation of the world average. Our value will be discussed in a future publication.

[11]We are grateful to Professor L. Slutsky for elucidating the experimental evidence for this statement; the argument will be included in our detailed publication.

[12]Muonium is known to be formed in Ar and Kr. Observation on proton beams shows that as the beam slows down to a few keV it is increasingly neutralized; in various substances the fraction is up to 0.85-0.90 at the lowest energies observable, and still rising. See S. K. Allison and M. Garcia-Munoz, in *Atomic and Molecular Processes*, edited by D. R. Bates (Academic, New York, 1962), Chap. 19. The evidence from hot-atom chemistry is summarized by R. Wolfgang, Progr. React. Kinet. 3, 99 (1965).

[13]See, e.g., Wolfgang, Ref. 12; and F. S. Rowland, J. Amer. Chem. Soc. 90, 4767 (1968). We are indebted to Professor Rowland for helpful advice on hot-atom chemistry.

[14]The first vibrational state of an OH system will have the same stretching as $O\mu$. From studies of H_2O spectra this is found to be ≈1%. Ruderman (Ref. 6) finds a rate of change of shielding with covalent-bond distance which gives 0.2 ppm for 0.01 Å. He also points out the importance of hydrogen bonding; our argument follows his, but with reference to neutral molecules rather than ions.

[15]We are indebted to Professor J. A. Pople for advice on this question.

[16]T. F. Finnegan, A. Denenstein, and D. N. Langenberg, Phys. Rev. Lett. 24, 738 (1970).

[17]B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[18]W. E. Cleland, J. M. Bailey, M. Eckhause, V. W. Hughes, R. M. Mobley, R. Prepost, and J. E. Rothberg, Phys. Rev. Lett. 13, 202 (1964).