

direction of the beam (target) particle in the vector-meson ( $\Delta^{++}$ ) rest frame, the  $y$  axis is along  $\vec{\pi}_{\text{beam}} \times \vec{V}$ , where  $V$  stands for vector meson, evaluated in the overall center-of-mass system, and  $\hat{x} = \hat{y} \times \hat{z}$ . The polarization analyzer is the decay  $\pi^+$  for the  $\rho^0$ , the decay proton for the  $\Delta^{++}$ , and the normal to the decay plane for the  $\omega^0$ ,  $\vec{\pi}^+ \times \vec{\pi}^-$ , each direction defined in the respective resonance's rest frame.

<sup>5</sup>See the recent review by J. D. Jackson, *Rev. Mod. Phys.* **42**, 12 (1970), for a survey of the literature on models for high-energy processes. A useful glossary of Regge-pole terminology is included in the appendix.

<sup>6</sup>A recent review of the Regge-pole model is given by G. E. Hite, *Rev. Mod. Phys.* **41**, 669 (1969). This reference and Ref. 5 may also be used as glossaries to the Regge-pole effects discussed.

<sup>7</sup>G. Wolf, *Phys. Rev.* **182**, 1538 (1969).

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<sup>9</sup>F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, *Phys. Rev.* **182**, 1579 (1969).

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<sup>12</sup>J. P. Ader, M. Capdeville, G. Cohen-Tannoudji, and Ph. Salin, *Nuovo Cimento A* **56**, 952 (1968). While this separation of the cross section into natural- and unnatural-parity contributions is strictly valid only at infinite energies, we plot the suggested combinations of density-matrix elements to investigate the perseverance of asymptotic properties at our momentum, 3.7 GeV/c.

<sup>13</sup>H. Högaasen and H. J. Lubatti, *Phys. Lett. B* **26**, 166 (1968); G. S. Abrams, B. Eisenstein, and H. Gordon, *Phys. Rev. Lett.* **23**, 673 (1969).

<sup>14</sup>A. S. Goldhaber, G. C. Fox, and C. Quigg, *Phys. Lett. B* **30**, 249 (1969).

### $\pi$ -B EXCHANGE-DEGENERATE MODEL FOR $V^0\Delta^{++}$ PRODUCTION\*

G. S. Abrams and U. Maor<sup>†‡</sup>

Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California 94720

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The forward differential cross section of  $\rho\Delta$  production is shown to be reproduced quantitatively by an evasive Reggeized one-pion-exchange model which smoothly reduces to the Born term at the pion pole. Assuming exact  $\pi$ -B exchange degeneracy leads to predictions for  $\omega\Delta$  production which are in agreement with the experimental  $s$  and  $t$  dependences. Similar agreement is found for  $K^*\Delta$  production.

The  $t$ -channel one-pion exchange (OPE) appears as the dominant forward-production mechanism at medium energies in many two-body reactions. There is, however, a considerable controversy as to the detailed features of this process. The proximity of the pion pole to the physical  $t$  region suggests the importance of the Born term. Nevertheless, it is clear that the experimental momentum-transfer distributions in the relevant reactions force rather severe modifications of the Born amplitude. Such modifications have been suggested (and applied) taking into account off-mass-shell corrections,<sup>1</sup> absorption,<sup>2</sup> and a variety of Regge-pole and cut models.<sup>3-5</sup>

In this Letter we offer evidence that the OPE contribution to the reaction

$$\pi^+p \rightarrow \rho^0\Delta^{++} \quad (1)$$

is consistent with the exchanged pion lying on an evasive ( $M=0$ ) Regge trajectory with a slope near the "universal" value of  $1 \text{ (GeV}/c)^{-2}$ . This result is suggested by a recent high-statistics

experiment<sup>6</sup> at 3.7 GeV/c which provides differential cross sections and spin density matrix elements to a  $|t|$  value as large as  $1.5 \text{ (GeV}/c)^2$ . A similar analysis of the reaction

$$\pi^+p \rightarrow \omega^0\Delta^{++} \quad (2)$$

then provides a stringent test of  $\pi$ -B exchange degeneracy which is well satisfied by the data.<sup>6</sup> Our results are readily extended to the reaction

$$K^+p \rightarrow K^*\Delta^{++} \quad (3)$$

but cannot properly describe either charged-pion photoproduction or  $n\bar{p}$  charge-exchange scattering.<sup>7</sup> However, our approach when coupled to models such as the one suggested by Jackson and Quigg<sup>5</sup> may provide an overall description which is quite satisfactory.

It has been noted<sup>1,3</sup> that the differential cross section of Reaction (1) exhibits shrinkage of the forward peak with increasing primary energy. This behavior suggests a simple Regge-pole parametrization with a steep evasive  $\pi$  trajec-

tory.<sup>3</sup> However, it has been argued<sup>1</sup> that the shrinkage may originate kinematically from the large widths of the measured  $\rho$  and  $\Delta$  resonances. The availability of the new 3.7-GeV/c data<sup>6</sup> makes it possible to examine the structure of the unnatural-parity  $t$ -channel exchanges away from the very forward direction. The crucial observation is that a zero seen in  $\rho_{00}d\sigma/dt$  at  $t'=t-t_{\min}=-0.75$  (GeV/c)<sup>2</sup> would enable us to discriminate against models that are quite satisfactory at lower  $t'$  values. Such a zero seems to be suggested by the experimental data.<sup>6</sup>

We limit our study to  $\sigma_0^- = \rho_{00}d\sigma/d|t'|$  to which only unnatural-parity [ $P=(-1)^{J+1}$ ] exchanges can contribute. The Born OPE cross section for the process  $a+b \rightarrow c+d$  is calculated<sup>8</sup> to be

$$\sigma_0^- = \frac{2\pi}{3sq^2} G^2 \left| \frac{1}{t-m_\pi^2} \right|^2 \frac{a_c^2 b_d^2}{m_\pi^2} [(m_b+m_d)^2 - t], \quad (4)$$

where

$$\begin{aligned} q^2 &= [s - (m_a - m_b)^2][s - (m_a + m_b)^2]/4s, \\ a_c^2 &= [t - (m_a - m_c)^2][t - (m_a + m_c)^2]/4m_c^2, \\ b_d^2 &= [t - (m_b - m_d)^2][t - (m_b + m_d)^2]/4m_d^2. \end{aligned}$$

The product of the coupling constants  $G^2 = (g_{\pi\pi\rho}^2/4\pi)(g_{\pi\pi\Delta}^2/4\pi) \approx 0.9$  if widths of 120 MeV are assumed for both the  $\rho$  and  $\Delta$  resonances. We reggeize this expression by replacing the pion propagator

$$\begin{aligned} \frac{1}{t-m_\pi^2} &\rightarrow \pi\alpha'(0) \frac{1+e^{-i\pi\alpha}}{2\sin\pi\alpha} \\ &\times \frac{(1+2\alpha)(1+\frac{2}{3}\alpha)\Gamma(\alpha+\frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(\alpha+1)} \left(\frac{s-u}{2s_0}\right)^\alpha, \quad (5) \end{aligned}$$

where  $\alpha = \alpha'(0)(t-m_\pi^2)$ . The model thus consists of an evasive pion with kinematical factors determined by the behavior of the Born amplitude at thresholds and pseudothresholds rather than the factors suggested by the crossing matrix.<sup>9</sup> The reduced residue is smoothly continued from the coupling constants calculated at the pion pole. The only arbitrariness in the functional dependence in Eq. (5) is introduced when we suppress the poles of  $\Gamma(\alpha+\frac{1}{2})$ . We choose to multiply by the lowest order polynomial in  $\alpha$  which has the required zeroes in the measured  $t$  region. Alternatively, one may employ a Veneziano-type amplitude where  $\Gamma(\alpha+\frac{1}{2})$  is eliminated altogether.<sup>10</sup>

We have performed a fit to the data for reaction (1) at 3.7 GeV/c, minimizing the  $\chi^2$  as a function of  $\alpha'(0)$ ,  $s_0$ , and  $G^2$ . The curve shown in

Fig. 4(a) of the preceding experimental paper<sup>6</sup> is the best-fit result with a  $\chi^2$  of 20.1 for 22 degrees of freedom. The best-fit parameters are

$$\alpha'(0) = 1.16 \pm 0.03 \text{ (GeV/c)}^{-2},$$

$$2s_0 = 1.08 \pm 0.11 \text{ GeV}^2,$$

$$G^2 = 0.46 \pm 0.05.$$

The zero observed near  $|t'|=0.75$  (GeV/c)<sup>2</sup> is interpreted as a nonsense wrong-signature zero in the OPE Regge amplitude corresponding to  $\alpha_\pi = -1$ . The model is seen to reproduce quantitatively both this zero and the detailed shape of  $\sigma_0^-$  over a large range in  $t$ . In particular, the apparent change in slope near  $|t'|=0.2$  (GeV/c)<sup>2</sup> is well reproduced by our calculation. The fitted  $G^2$  value corresponds to  $\rho$  and  $\Delta$  widths of about 90 MeV each, which may be regarded as satisfactory in view of the simplicity and small number of parameters introduced in this model.

Having fixed the parameters from the fit at 3.7 GeV/c, we compare in Fig. 1 the model predictions with the results of other experiments at different energies.<sup>11,12</sup> We wish to stress that we have not attempted to fit the overall data as different experimental procedures have been applied both to cross-section normalization and to background estimation. In our opinion systematic discrepancies as large as 20% may be present in

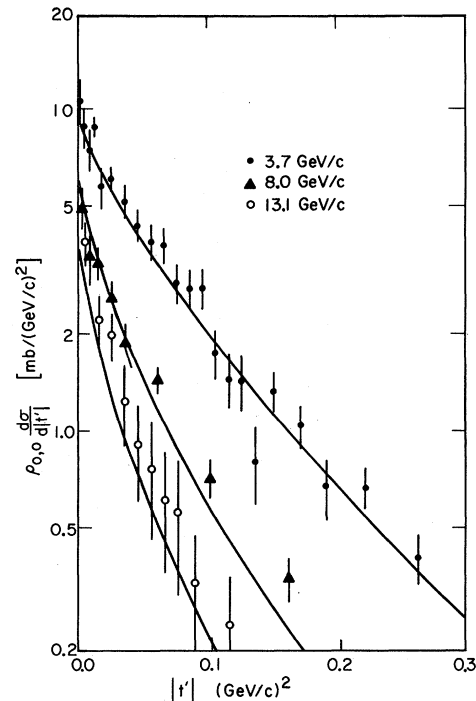


FIG. 1.  $\sigma_0^- (\rho^0 \Delta^{++})$  as a function of  $|t'|$  at 3.7, 8.0, and 13.1 GeV/c.

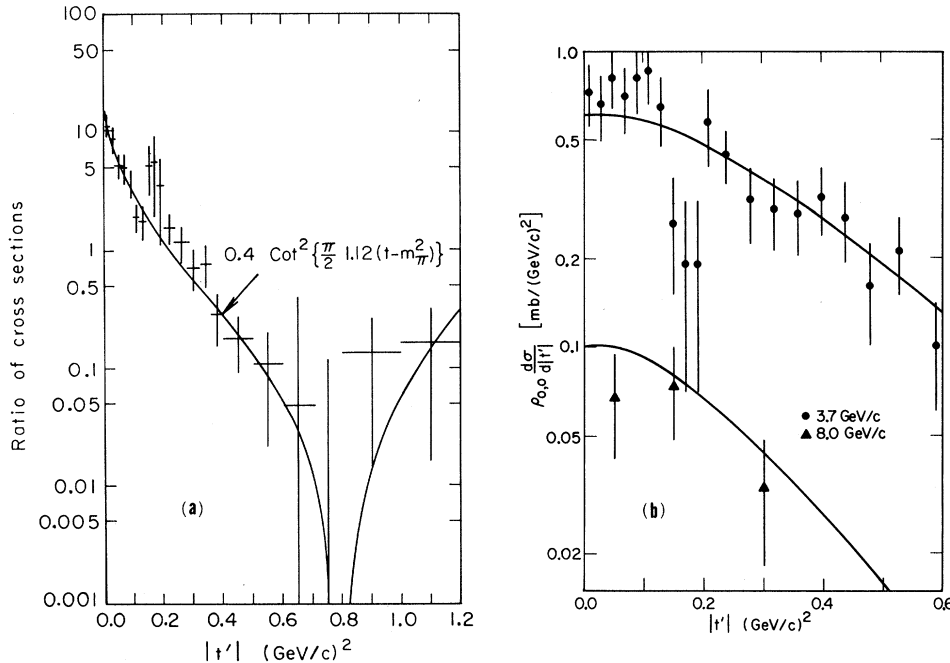


FIG. 2. (a)  $\sigma_0^-(\rho^0\Delta^{++})/\sigma_0^-(\omega\Delta)$  as a function of  $|t'|$ . (b)  $\sigma_0^-(\omega^0\Delta^{++})$  as a function of  $|t'|$  at 3.7 and 8.0 GeV/c.

comparing two different experiments. It may be seen, however, that our calculation provides quantitative agreement with both the  $s$  and the  $t$  dependence of the data.

We next consider the unnatural-parity-exchange contribution to Reaction (2) for which the nearest  $t$ -channel singularity is the  $B$  meson with  $J^{PC}=1^{++}$ . From the absence of strongly coupled resonances in the  $s$ -channel  $K^+p$  system, one can argue that Reaction (3) proceeds via the exchange of exchange-degenerate pairs such as  $\pi+B$ . Assuming then that  $\alpha_B=\alpha_\pi=\alpha$  and that the residues are equal (strong exchange degeneracy), one finds that

$$\sigma_0^-(\rho\Delta)/\sigma_0^-(\omega\Delta)=A \cot^2 \frac{1}{2} \pi \alpha, \quad (6)$$

where  $A=1$  in the limit of exact SU(3) symmetry [assuming that  $\Gamma(B \rightarrow \varphi\pi)=0$ , as expected for the ideal nonet].

In Fig. 2(a) we show the data<sup>6</sup> and the best-fit curve from an independent fit of  $\sigma_0^-(\rho\Delta)/\sigma_0^-(\omega\Delta)$  as a function of  $t'$ . The parameters at the minimum value of  $\chi^2$  (14 for 19 degrees of freedom) are  $A=0.41 \pm 0.08$  and  $\alpha'(0)=1.12 \pm 0.08 (\text{GeV}/c)^{-2}$  when a linear trajectory,  $\alpha=\alpha'(0)(t-m_\pi^2)$ , is assumed. This fitted value of  $\alpha'$  agrees well with the value we have obtained using Eq. (4) and the  $\rho\Delta$  data alone. The experimental distribution is well reproduced over a wide range of  $t$  values, but the significant dip in  $\sigma_0^-(\omega\Delta)$  [see Fig. 2(b)] at  $|t'|=0.17 (\text{GeV}/c)^2$  is unexplained by our model.

A possible explanation in terms of a nonsense wrong-signature zero of the  $B$  amplitude appears to be unattractive both on experimental grounds<sup>6</sup> and the grounds of the rather good general agreement with  $\pi$ - $B$  exchange degeneracy. The fit of our model to Reaction (2) at 3.7 GeV/c is shown in Fig. 2(b) [see also Fig. 4(b) in Ref. 6]. The parameters  $\alpha'(0)$  and  $s_0$  are fixed to the values found for Reaction (1), so that the only parameter varied in the fit is  $G^2$  [the sign of  $\exp(-i\pi\alpha)$  is reversed in Eq. (5) for  $B$  exchange]. This fit was performed over the range  $0.2 \leq |t'| \leq 1.4 (\text{GeV}/c)^2$  to eliminate complications due to the dip and possible  $\rho$ - $\omega$  interference.<sup>6</sup> While some systematic deviations are seen at the largest  $t'$  values, the overall agreement is good ( $\chi^2=16.3$  for 15 degrees of freedom), as expected from the agreement shown in Fig. 2(a).

From the fit to the  $\sigma_0^-(\omega\Delta)$  distribution at 3.7 GeV/c we find that  $G^2=0.94 \pm 0.15$ . Combining this result with the previous one for  $\rho\Delta$  production we get  $A=0.49 \pm 0.06$  which suggests possible breaking of either SU(3) or exchange degeneracy.<sup>13</sup> A further consequence of our analysis is that the  $\omega\Delta$  cross section is expected to fall more rapidly with energy than the  $\rho\Delta$  cross section. This effect arises kinematically from the decrease of  $|t_{\min}|$  as  $s$  increases, and is evident from the form of Eq. (6). The predicted differential cross section for  $\omega\Delta$  at 8 GeV/c is compared with the data<sup>11</sup> in the lower part of Fig. 2(b), and is seen

to reproduce the experimental energy falloff.

An independent experimental test of our model may be made by utilizing available data on Reaction (3). To incorporate possible symmetry breakings we have chosen to use the experimental coupling constants determined by the pionic decay of  $K^{*0}$  and  $\Delta^{++}$ , and take  $G^2=0.38$ . The only changes in Eqs. (4) and (5) are thus the appropriate different masses and signatures, retaining the values of  $\alpha(t)$  and  $s_0$  determined in Reaction (1). These predictions, shown in Fig. 3, are seen to agree rather well with the data,<sup>14,15</sup> especially so in view of the lack of arbitrariness in the model.

The model proposed for  $V^0\Delta^{++}$  production is seen to be in quantitative agreement with experiment. We have considered alternative models to analyze the uniqueness of our conclusions. If the exchanged pion were a fixed pole, or had a trajectory which is almost flat,<sup>7</sup> then a zero at  $|t'| \cong 0.75$  (GeV/c)<sup>2</sup> is not expected. In particular we note that a form factor<sup>1</sup> correction does not produce such a dip. In our opinion the data analysis of Wolf<sup>1</sup> is somewhat misleading as he considers the full differential cross sections which contain contributions from natural-parity exchanges which cannot be neglected.<sup>6</sup> Such contributions cannot be reproduced by an OPE amplitude as long as it is modified by a form factor only. Additionally, a flat trajectory does not motivate a possible  $\pi$ - $B$  exchange degeneracy which is strongly indicated by the data. Hence, we argue that the shrinkage observed in  $\rho\Delta$  production is a consequence of a  $\pi$  trajectory with a slope  $\sim 1$  (GeV/c)<sup>-2</sup>, rather than a pure kinematic effect.<sup>1</sup>

Another suggested model<sup>4</sup> with only Regge poles considers contributions from an  $M=1$  pion, its conspirator, and a strongly coupled  $A_1$ . While this model provides a good fit to data from Reaction (1) at small  $t'$  values, no structure near  $t' \cong -0.75$  (GeV/c)<sup>2</sup> is suggested. Furthermore the fitted  $A_1$  trajectory is quite different from the  $\pi$  trajectory, so that it may be difficult to construct an exchange-degenerate scheme for this model. It is worth noting that we have neglected an  $A_1$ -exchange contribution in the present model, indicating that the  $\bar{p}A_1\Delta$  coupling may be rather small.

We have also considered the possibility that the zero observed in  $\sigma_0^-(\rho\Delta)$  is due to the optical properties of the scattering amplitude.<sup>16</sup> Such a zero would be expected near  $t' = -0.6$  (GeV/c)<sup>2</sup> if the net  $s$ -channel helicity-flip is one. Present

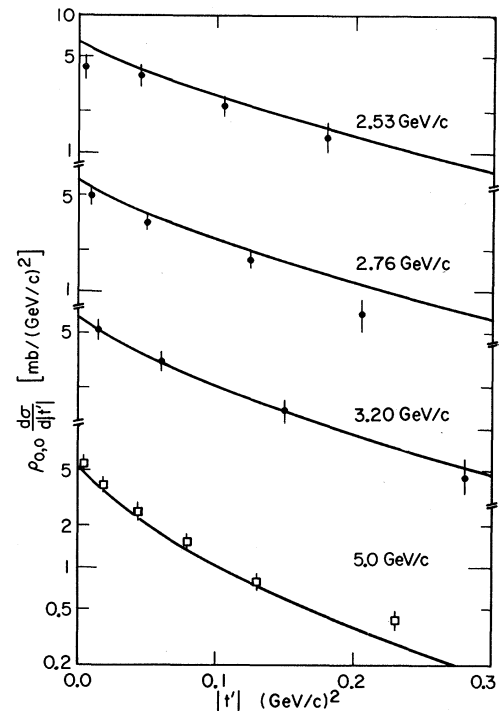


FIG. 3.  $\sigma_0^-(K^{*0}\Delta^{++})$  as a function of  $|t'|$  at 2.53, 2.76, 3.20, and 5.0 GeV/c.

data cannot rule out such a correlation. However, this interpretation leads to some difficulties. One expects that the small pion mass would shift the dip to  $|t'|$  values smaller than  $0.6$  (GeV/c)<sup>2</sup>, and not larger as seen experimentally. It is possible to circumvent this complication by associating the dip with  $A_1$  exchange rather than with  $\pi$  exchange. However, one would then expect the  $B$  contribution to  $\omega\Delta$  to show a similar dip which is not observed experimentally.<sup>6</sup> We comment also that the dip observed in  $\rho_{11} + \rho_{1-1}$  near  $t' = -0.65$  (GeV/c)<sup>2</sup> for Reaction (2) contradicts the simple optical picture.

The present model offers no explanation for the observed forward peaks in the photoproduction of charged pions and  $n\bar{p}$  charge exchange. Modification of the OPE Regge amplitude as used in our calculation by appropriate cuts (absorption) does provide a good fit<sup>5</sup> to these problematic reactions. It is not entirely clear why such cut corrections are not required in the analysis of Reactions (1)-(3), although we note that there is no  $s$ -channel structure in  $K^+p$  reactions and relatively little in  $\pi^+p$  reactions. A better understanding of the role of Regge cuts is therefore crucial to a complete parametrization of the OPE amplitude.

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<sup>11</sup>Aachen-Berlin-CERN Collaboration, Phys. Lett. **22**, 533 (1966); Nucl. Phys. B **8**, 45 (1968); K. Barnham, private communication.

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<sup>13</sup>Our analysis supports a strong exchange degeneracy in which the  $\pi$  and  $B$  lie on the same trajectory, have the same threshold (and pseudothreshold) factors, and have the same  $t$  dependence for their residues. The breaking of exact exchange degeneracy [or SU(3) invariance] occurs only via an overall  $t$ -independent scale factor. While the existence of such a scale factor may be associated with SU(3) breaking for the coupling constants of the exchanged  $\pi$  and  $B$  (retaining exact exchange degeneracy), the comparison is not straightforward due to the different dimensions of the two couplings.

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## SUPEREIKONAL MODEL OF DEEP-INELASTIC ELECTRON-PROTON SCATTERING\*

H. M. Fried and Hector Moreno†

Department of Physics, Brown University, Providence, Rhode Island 02912

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A field-theoretic model is written to describe the structure function  $W_2$  of deep-inelastic  $e-p$  scattering. Strict Bjorken scaling is found not to hold; nevertheless, the model provides a modest fit to the experimental data. A variant of the Drell-Yan-West relation between the elastic limit of  $W_2$  and the asymptotic falloff of the electromagnetic form factors is obtained.

We would like to present a calculation of deep-inelastic  $e-p$  scattering, in particular the structure function  $W_2$ , based upon a model refinement of a previous eikonal attempt.<sup>1</sup> Soft, virtual, neutral, vector mesons (SVNVM) are here exchanged between the proton legs of an off-shell Compton amplitude<sup>2</sup>; and in addition, similar exchanges are inserted into the model to provide vertex and nucleon propagator structure, as in Fig. 1. The vertex corrections can be thought of as due to soft-meson exchange in the elastic region,  $\omega^{-1}x = q^2/2m\nu \sim 1$ , where they have the nice property of providing agreement with the experimental data<sup>3</sup> and a connection with previ-

ous theoretical efforts.<sup>4</sup> The calculation exhibits the following features: (1) The absorptive part of the elastic scattering amplitude displays the

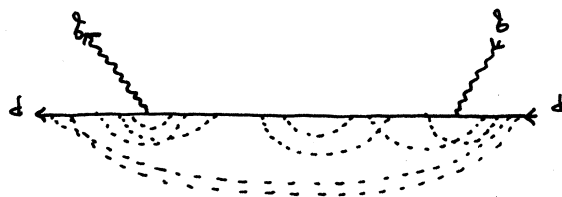


FIG. 1. A typical graph included in the soft summations of this model Compton amplitude.