Report No. CEA R-2786.

[6] J. R. Pizzi, thesis, University of Lyon, 1970 (unpublished).

[7] J. V. Meboniya, Phys. Lett B 30, 153 (1969).

[8] V. V. Balashov and J. V. Meboniya, Nucl. Phys. A 107, 369 (1968).

[9] V. G. Neudachin Y. F. Smirnov, At. Energy Rev. 3, 157 (1965).

[10] K. Bähr, T. Becker, O. M. Bilaniuk, and R. Jahr Phys. Rev. 178, 1706 (1969).

[11] K. Wildermuth and W. McClure, *Cluster Representations of Nuclei, Springer Tracts in Modern Physics* (Springer, Berlin, 1966), Vol. 41, p. 15.

[12] V. K. Dolinov *et al.*, Nucl. Phys. A 129, 577 (1969).

---

# PARTICLE-VIBRATION DOORWAY RESONANCE IN $Pb^{209}$

William P. Beres
Physics Department, Wayne State University, Detroit, Michigan 48202

and

M. Divadeenam† *
Prairie View A & M College, Prairie View, Texas 77445, and Triangle Universities
Nuclear Laboratory, Durham, North Carolina 27706

(Received 6 July 1970)

With no free parameters the particle-vibration model is used to describe the $\frac{1}{2}^+$ doorway resonance observed in $Pb^{209}$ at 500 keV with a width of 58 keV. We find that the $(2g_{9/2}, 4^+)$ state which occurs at nearly the experimental energy has a width of about 1 to 2 times the measured value.

While the idea of a particle-vibration weak-coupling model has been widely used with good results, continuum levels have not been studied to any great extent. Mottelson[1] has, in fact, emphasized a need for the evaluation of radial matrix elements when the one-particle state is at a known energy in the continuum. The purpose of this paper is to show that the particle-vibration model provides a good explanation for an $s$-wave resonance observed in $Pb^{209}$ by Farell et al.[2] The weak-coupling model has been quite successful in describing inelastic scattering exciting vibrational states in the Pb region.[1,3] Since this $\frac{1}{2}^+$ resonance is not very high in the continuum, it is reasonable to expect an extension of the successful predictions of the particle-vibration model for bound states. An additional impetus for this work is the interpretation in Ref. 2 of the $s$-wave resonance as a possible doorway state leading to complex structure in adjacent nuclei. The doorway concept has become rather important in recent years.[4]

Specifically, Ref. 2 observed the high-resolution elastic scattering of neutrons from $Pb^{206,207,208}$ yielding resonances in $Pb^{207,208,209}$. In $Pb^{209}$ only one $\frac{1}{2}^+$ resonance is observed in the energy range from 0 to 1.7 MeV above the neutron threshold. This occurs at about 500 keV (4.38 MeV above the $Pb^{209}$ ground state) with a width of 58 keV. The experimental cross sections for the $Pb^{207}$ and $Pb^{208}$ compound nuclei show much fine structure in the general region of 500-keV incident neutron energy. In particular, there are 11 $s$-wave resonances in $Pb^{207}$ up to the inelastic threshold (800 keV), the sum of whose reduced widths compares remarkably well with that of the single $s$-wave resonance in $Pb^{209}$.

The usual doorway mechanism is that of the two-particle–one-hole (2p-1h) state. However, while it is possible to form a large number of $\frac{1}{2}^+$ 2p-1h states in $Pb^{209}$, prior to mixing there are none as low as 500 keV. In fact there are none below 1 MeV and only a few between 1 and 2 MeV. Shakin[5] has suggested that a possible explanation of the $Pb^{209}$ resonance is the coupling of the $4^+$ collective core vibrational state at 4.31 MeV[6] in $Pb^{208}$ with the $2g_{9/2}$ zero-energy single-neutron level. Since the $4^+$ state probably has 2p-2h components, the decay to the $Pb^{208}$ ground state would just go through the ground-state correlations, and Stein[7] indicates that these are non-negligible. The proposed particle-vibration state nearly coincides in energy with the observed resonance, and according to Auerbach and Stein[8] loses only a few percent of its strength in mixing with the bound $4s_{1/2}$ single-neutron level. It is conceivable that other excited core-plus-neutron levels mix with the $(2g_{9/2}, 4^+)\frac{1}{2}^+$ state; however, these are fairly far away. [The closest configuration is $(1i_{11/2}, 6^+)$ at 5.2 MeV relative to the $Pb^{209}$ ground state, and other available particle-vibration states are even higher.] We point out that the $(2g_{9/2}, 4^+)\frac{1}{2}^+$ state in its microscopic form probably contains 2p-1h components. This is qualitatively consistent with the interpretation in Ref. 2. A quantitative study in Pb via the 2p-1h scheme is very difficult; however, we would expect it to yield results similar to those presented here. The first stage in the evolution of the fine structure observed in $\mathrm{Pb}^{207}$ and $\mathrm{Pb}^{208}$ presumably results from the coupling of the available neutron holes to the doorway.

Hamamoto[9] calculated the distorted-wave Born approximation cross sections for the $\mathrm{Pb}^{209}$ states excited in $\mathrm{Pb}^{208}(d,p)\mathrm{Pb}^{209}$ by Ellegaard, Kentele, and Vedelsby.[10] Some of these states are unbound but are treated as just bound in Ref. 9 by deepening the Woods-Saxon potentials. In the present paper we do not use this mechanism, and the $\frac{1}{2}^+$ state is treated explicitly as a resonance.

The Hamiltonian in the weak-coupling model is

$$H = H_{\mathrm{core}} + H_{\mathrm{part}} + V_c. \tag{1}$$

Here $H_{\mathrm{core}}$ and $H_{\mathrm{part}}$ are the vibrational and single-particle Hamiltonians and $V_c$ is the coupling interaction. The width of the neutron elastic-scattering resonance is given by

$$\Gamma = 2\pi \sum\nolimits_{m_s} \int d\Omega_K |\langle X_{1/2}{}^+ \times | V_c | 0^+, \chi^{(+)}(\vec{K}, m_s)\rangle|^2 Km/(2\pi)^3\hbar^2, \tag{2}$$

where $K$ is the wave number related to the energy $E$ by $E = \hbar^2K^2/2m$, $m$ is the neutron reduced mass, and $m_s$ is the $z$ projection of intrinsic spin. The wave function $X_{1/2^+}$ represents the unperturbed particle-vibration state $\langle (2g_{9/2}, 4^+)\frac{1}{2}^+|$, and $\chi^{(+)}(\vec{K}, m_s)$ is the outgoing neutron wave function. Since $V_c$ is a scalar and the resonance is $s$ wave, only the $l=0$, $j=\frac{1}{2}$ partial-wave component of $\chi^{(+)}$ contributes and Eq. (2) becomes

$$\Gamma = 4m/K\hbar^2 |\langle (2g_{9/2}, 4^+)\tfrac{1}{2}^+ | V_c | 0^+, u(s_{1/2}, Kr)/ r\Phi(s_{1/2})\rangle|^2, \tag{3}$$

where $u(s_{1/2}, Kr)/r$ is the continuum radial wave function for the state whose angular wave function is $\Phi(s_{1/2})$. The matrix element of $V_c$ is taken to be that described by Mottelson,[1] so that,

$$\langle (2g_{9/2}, 4^+)\tfrac{1}{2}^+ | V_c | 0^+, u(s_{1/2}, Kr)/r\Phi(s_{1/2})\rangle = \langle 2g_{9/2} | k(r) | u(s_{1/2}, Kr/r\rangle \langle g_{9/2} \| Y_4 \| \Phi(s_{1/2})\rangle 2^{-1/2}(\hbar\omega_4/2c_4)^{1/2}. \tag{4}$$

The vibrational amplitude is represented by the last factor in Eq. (4), and it may be extracted from the measured $4^+$ excitation cross section. The reported collectively enhanced $B(E4)$ experimental values range from 15 to 25 single-particle units.[11-12] Following the reasonable calculations of Refs. 8 and 9, we use the value of 15. This leads to a value of $2.4 \times 10^{-2}$ for $(\hbar\omega_4/2c_4)^{1/2}$. The radial form factor of the deformation is given in terms of the real radial one-body potential $V(r)$ as

$$k(r) = -r dV(r)/dr. \tag{5}$$

Using the computer code ABACUS[13] the radial wave functions for the bound $2g_{9/2}$ and continuum $s_{1/2}$ states were generated from Woods-Saxon potentials with the addition of a spin-orbit interaction of the Thomas form. In order to study the dependence of the radial integral in Eq. (4) on the Woods-Saxon depth $V_0$, diffuseness $a$, radius $R = R_0A^{1/3}$, and spin-orbit strength $V_{\mathrm{so}}$, three different sets of values were tried for $2g_{9/2}$, all having proved relatively successful in diverse calculations, viz., those of Blomquist and Wahlborn,[14] Hamamoto,[9] and Dover and Dietrich.[15]

We emphasize that our calculation contains no free parameters. Therefore, the real-well depth, diffuseness, and radius used in generating the continuum $s_{1/2}$ wave function must be obtained by satisfying certain physical criteria. (Although a complex optical potential might be more appropriate, the imaginary part is expected to be rather small and should introduce only a small correction.[16]) Two different physically reasonable criteria are applied in order to determine these parameters. Firstly, they should yield the $4s_{1/2}$ bound state at its appropriate energy in $\mathrm{Pb}^{209}$, and secondly, they should reproduce the neutron-scattering data over a wide range of energy and nuclei. The first criterion is satisfied by wells given in Refs. 14, 9, and 15. However, Buck and Perey[17] through extensive work have satisfied the latter criterion by using a somewhat different local well derived from a nonlocal potential. Consequently, we calculate the width using the parameters that satisfy each criterion separately. A summary of the parameters and results is given in Table I.

Table I shows that the $s_{1/2}$ parameters of Ref. 17 yield a width of the order of the observed value of 58 keV. The other $s_{1/2}$ parameters result in higher values, however, and set a reasonable upper limit to the width in our simple model.

Table I. Results.

| Ref. No. for $2g_{9/2}$ | Ref. No. for $s_{1/2}$ | State | $V_0$ (MeV) | Well parameters<br>$a$ (fm) | $R_0$ (fm) | $V_{so}$ (MeV)[a] | $\Gamma$ (keV) |
|---|---|---|---|---|---|---|---|
| 14 | | $2g_{9/2}$ | 44 | 0.67 | 1.27 | 7.75 | |
| | 14 | $s_{1/2}$ | 44 | 0.67 | 1.27 | ⋯ | 122 |
| | 17 | $s_{1/2}$ | 48 | 0.65 | 1.27 | ⋯ | 51 |
| 9 | | $2g_{9/2}$ | 45.6 | 0.65 | 1.25 | 7.75 | |
| | 9 | $s_{1/2}$ | 47.0 | 0.65 | 1.25 | ⋯ | 92 |
| | 17 | $s_{1/2}$ | 48 | 0.65 | 1.27 | ⋯ | 55 |
| 15 | | $2g_{9/2}$ | 46.37 | 0.6334 | 1.238 | 5.87 | |
| | 15 | $s_{1/2}$ | 47.38 | 0.6334 | 1.238 | ⋯ | 94 |
| | 17 | $s_{1/2}$ | 48 | 0.65 | 1.27 | ⋯ | 56 |

[a] Units of $(\hbar/m_\pi c)^2$.

We therefore predict a width of between about 51 and 122 keV. Actually, the width will be somewhat smaller than this because (i) the $2g_{9/2}$ spectroscopic factor in $Pb^{209}$ is about 0.95,[7] and (ii) the $(2g_{9/2}, 4^+)\frac{1}{2}^+$ configuration loses 4% of its strength via coupling to the bound $4s_{12}$ state.[8]

The effects of antisymmetrization of the microscopic core ingredients and the odd particle in the core-plus-particle wave function have not been considered here. Mottelson[1] and Hafele[18] have discussed this vibration dissociation effect, and the resultant energy shifts of bound states in $Bi^{209}$ are found to be significant in certain special cases. A continuum calculation including this effect is planned for the future.

An additional test of the applicability of the weak-coupling model is to look for resonances based on the other members of the $(2g_{9/2}, 4^+)J^\pi$ multiplet. Parity conservation allows only even-$l$ values for the outgoing neutron. Penetrability considerations at about the centroid energy eliminate the possibility of observing compound resonances with spins greater than $\frac{5}{2}^+$ $(l>2)$. But there is a good chance of observing the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ resonances below a neutron energy of 1 MeV if the reduced widths are not too small. In fact two $\frac{3}{2}^+$ and three $\frac{5}{2}^+$ resonances have been observed[19] in the 0.5- to 1.0-MeV neutron energy range in $Pb^{209}$. A theoretical study of these levels is in progress.

In summary our results indicate that the $(2g_{9/2}, 4^+)$ particle-vibration state is a reasonable candidate for the observed $\frac{1}{2}^+$ doorway resonance in $Pb^{209}$.

We thank Professor Henry Newson for stimulating our interest in these neutron experiments and appreciate his helpful comments and suggestions. We have greatly benefitted from conversations with Professor Carl Shakin and Dr. Morton Weiss. We also thank Professor William Dorenbusch for reading and commenting on the manuscript.

---

*Work at Triangle Universities Nuclear Laboratory supported by the U. S. Atomic Energy Commission.

†Now at North Carolina Central University, Durham, N. C. 27707.

[1] B. R. Mottelson, J. Phys. Soc. Jap. 24, 87 (1968).

[2] J. A. Farell, G. J. Kyker, Jr., E. G. Bilpuch, and H. W. Newson, Phys. Lett. 17, 286 (1965).

[3] J. C. Hafele and R. Woods, Phys. Lett. 23, 579 (1966).

[4] H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (New York) 41, 230 (1967).

[5] C. M. Shakin, private communication.

[6] J. Saudinos, G. Vallois, O. Beer, M. Gendrot, and P. Lopato, Phys. Lett. 22, 492 (1966).

[7] N. Stein, in *Proceedings of the International Conference on Properties of Nuclear States, Montreal, Canada, 1969,* edited by M. Harvey *et al.* (Presses de L'Université de Montréal, Montréal, Canada, 1969), p. 337.

[8] N. Auerbach and N. Stein, Phys. Lett. B 25, 628 (1969).

[9] I. Hamamoto, Nucl. Phys. A 141, 1 (1970), and A 126, 545 (1969), and A 135, 576 (1969).

[10] C. Ellegaard, J. Kantele, and P. Vedelsby, Nucl. Phys. A 129, 113 (1969).

[11] J. Alster, Phys. Lett. B 25, 459 (1967).

[12] J. F. Ziegler and G. A. Peterson, Phys. Rev. 165, 1337 (1968).

[13] E. H. Auerbach, BNL Report No. 6562 (unpublished).

[14] J. Blomquist and S. Wahlborn, Ark. Fys. 16, 545 (1960).

[15] C. B. Dover and K. Dietrich, Nucl. Phys. A 135,

481 (1969).

[16] C. M. Shakin and M. S. Weiss, Phys. Rev. (to be published).

[17] B. Buck and F. G. Perey, Phys. Rev. Lett. 8, 444 (1962).

[18] J. C. Hafele, Phys. Rev. 159, 996 (1967).

[19] H. W. Newson, private communication; J. L. Fowler, Phys. Rev. 147, 870 (1966).

---

# STUDY OF NUCLEAR STATES OF SEVERAL ODD-$A$ NUCLEI $68 \leq Z \leq 79$ THROUGH ELECTROMAGNETIC EXCITATION FROM 2.3 TO 3.6 MeV†

W. T. K. Johnson*‡ and B. T. Chertok*
The American University, Washington, D. C. 20016

and

C. E. Dick
National Bureau of Standards, Washington, D. C. 20234

(Received 23 June 1970)

34 excited states have been studied between 2.3 and 3.5 MeV in $^{167}$Er, $^{179}$Hf, $^{191}$Ir, and $^{197}$Au by observing the decay of the isomeric levels in these nuclei following electron and photon activation. 28 of these states are observed for the first time. Of the 34 observed transitions, $M1+E2$ multipolarity assignment is made to 20, two are identified as $E0$ transitions, and one is assigned $E1$ multipolarity. Accurate values for the isomeric half-lives are also presented.

Measurements reported here provide the first systematic investigation of states by inelastic electron scattering and photoexcitation from 2.3 to 3.6 MeV in the region $Z = 68$ to 79. Further, the method employing the electromagnetic interaction yields the multipolarity and radiative strength of the transition which will provide important confirmation of quantum numbers, determined by nuclear reactions such as $(d,p)$ and $(d,t)$. Coulomb excitation measurements in this nuclear structure region have only explored states up to ~1.5 MeV. Finally, the use both of electron and of photon excitation allows a search for electric monopole ($E0$) transitions which are, of course, forbidden for photons. $E0$ transitions had not been observed in the region explored prior to the present work.

The sytematic study of radiative-transition probabilities has helped delineate the regions of applicability of various nuclear models. The Nilsson model, with additions such as pair correlation and quasiparticle-phonon interactions, has had some success in describing low-lying states, below 1.5 MeV, in deformed nuclei.[1] Pertinent to this investigation the strength of $M1$ $\gamma$ rays ($4\ \mathrm{MeV} < E_\gamma < 9\ \mathrm{MeV}$), following neutron capture by deformed nuclei, has been quantitatively explained by a calculation employing pairing and spin-spin effects, together with the standard Nilsson Hamiltonian.[2] In the transition region $76 < Z < 82$ the model of de-Shalit[3] for odd-$A$ nuclei, where the odd nucleon is weakly coupled to excitations of the even-even core, has met with good success in predicting electromagnetic moments and transition probabilities for low-lying states, <0.8 MeV in several nuclei.[4,5]

The experimental method dictated investigation of only stable nuclei with low-lying isomeric states. Accordingly, the odd-$A$ nuclei $^{167}_{68}\mathrm{Er}_{99}$, $^{179}_{72}\mathrm{Hf}_{107}$, $^{183}_{74}\mathrm{W}_{109}$, $^{191}_{77}\mathrm{Ir}_{114}$, and $^{197}_{79}\mathrm{Au}_{118}$ were investigated with electrons and bremsstrahlung from the National Bureau of Standards' 4-MeV electron Van de Graaff. All but $^{183}$W had sufficient cross sections for study. Targets of natural isotopic abundance, in the thickness ranges of ~5g/cm$^2$ for photoexcitation and ~18 mg/cm$^2$ for electroexcitation, were placed in a fast shuttle to transport them from the irradiation position to the counting position without a significant decay of the 2- to 20-sec isomeric states. The excitation functions were measured by incrementing the bombarding energy and measuring the subsequent isomeric decays with NaI(Tl) scintillators. A state was identified by comparing the two curves produced where a break corresponded to a new higher-lying state populating the isomer.[6] This method is shown in detail for a 200-keV portion of $^{179}$Hf excitation in Fig. 1. The photoexcitation yield curve corresponded to a series of straight lines produced by the intermediate-thickness bremsstrahlung target (0.001 in. platinum, backed by 1 in. water). The elec-