wider frequency range to attempt to observe the normal-liquid structural relaxations at frequencies below those used here.

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VORTICITY IN HELIUM FILM CREEP

Edward H. Takken
Naval Research Laboratory, Washington, D. C. 20390

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The viscous drag of the normal-fluid component on moving superfluid vortex-line cores is shown to have two effects: to push the vortex lines toward the film surface and to enhance the film-creep velocity. A class of experiments in the millidegree temperature range is proposed in order to test for the presence of vorticity in helium film creep.

A common facet of the variety of models[1-5] designed to explain the helium film-creep velocity is the assumed existence of quantized vorticity in the superfluid.[6,1] In spite of the quantitative and even conceptual[7] problems associated with these vortex models, alternatives such as intrinsic fluctuation effects[8] are lacking; and the initial ideas of Feynman[1] and Richards and Anderson[9] still hold as relevant for an explanation of the film-creep velocity.[4] In the present paper a new factor is introduced in a state-of-the-art vortex model, and predictions are given for a new class of experiments designed to test for the presence of vorticity in helium film creep.

The core region of a superfluid vortex line can scatter quasiparticles as it moves through the normal-fluid component of the film, the resulting viscous drag per unit length of vortex line being $2v(m/h)\alpha(T)$, where $\alpha(T)$ has been defined and measured by Rayfield and Reif,[10] m is the helium atom mass, h is Planck's constant, and v is the film-creep velocity. One effect of this viscous drag follows from the condition that it must be counteracted by a Magnus force associated with the motion of the vortex line outward with velocity u toward the film surface.[11] Equating the Magnus force, $\rho_s(h/m)u$,[10] with the magnitude of the viscous drag gives

$$u/v = Id/Z, \tag{1}$$

with I defined as

$$I = (2/\rho_s)(Z/d)(m/h)^2\alpha(T), \tag{2}$$

and d/Z as the ratio of the film thickness to the film height. If $I \ll 1$, $u/v \ll d/Z$, and the vorticity can exit into the bulk liquid at the base of the film before it is forced into the film surface.

Including this viscous loss in the energy conservation condition used by Donnelly[4] leads to the prediction of enhanced film-creep velocities by a factor of $(1-I)^{-1}$. Anderson's phase-slippage formula,[9,12]

$$\dot{n} = gzm/h, \tag{3}$$

for the average rate at which quantized vortex lines move across an orifice, is not obviously applicable to the case of helium film creep. However, the criterion for applying Eq. (3) is that the vortex-line motion be parallel to equipotential surfaces in the liquid. Hence, we assume that this equation gives the average rate at which horizontal[13] vortex lines are first created in the liquid and then translated as part of the superfluid through the film. Here z is the height difference of the film, while g is the acceleration due to gravity. Then with e_0 the energy per unit length of a vortex line and n the average number of vortex lines in the film, conservation of energy for a unit width of film requires

$$\rho_s gzdv = e_0\dot{n} + 2(m/h)\alpha(T)nv^2. \tag{4}$$

Provided the vortex lines are stable and long-lived, the average number of these lines is

$$n = Z\dot{n}/v, \tag{5}$$

or one-half this value depending on whether one assumes all the vortex lines to be created at the top of the film or uniformly through the down side of the film.[14] For e_0 one can use

$$e_0 = \epsilon(\rho_s/4\pi)(h/m)^2, \tag{6}$$

where ϵ is a parameter on the order of unity. In particular, $\epsilon = \ln(d/a) \approx 5$ for a classical vortex line with core and outer radii of a and $\frac{1}{2}d$, while for the interacting Bose gas model,[3] $\epsilon \approx 2$ or $\epsilon = \frac{1}{2}$ for a vortex line located either one core radius from a boundary or infinitesimally close to the boundary. With Eqs. (3), (4), and (5) one finally has

$$v = \epsilon(\tfrac{1}{4}\pi d)(h/m)(1-I)^{-1}, \tag{7}$$

where I given by Eq. (2) now takes on the meaning of a velocity enhancement factor.

Equation (6) may be compared with a disparity of experimental results.[15] Pickar and Atkins'[5] Doppler measurements of third sound agree well with the formula $(\frac{1}{4}\pi d)h/m$, which suggests $\epsilon \approx 1$ and $I \approx 0$. One might expect the viscous drag on a vortex line and hence I to reduce significantly when the separation between the vortex line and a fluid boundary becomes comparable with or less than the wavelength of the quasiparticles to be scattered by the vortex core. Thus when I, as determined by Eq. (2) without surface shielding effects, is larger than unity, it is not unreasonable to picture a vortex line initially positioned in the center of the moving film as being nudged toward the film surface according to Eq. (1) for u/v until it reaches an equilibrium position in close proximity to the surface.[16] With $\epsilon \approx 1$ and the average effective I being nearly zero, this is essentially the case suggested by Pickar and Atkins. However, until the thickness-dependent critical velocities of unsaturated films[17] can be included at least qualitatively in a self-consistent manner, this point need not be pursued further.

We examine instead the possible physical significance of the velocity enhancement term $(1-I)^{-1}$ for the case $I \ll 1$. With $Z = 1$ cm and the data of Rayfield and Reif, Eq. (2) for I becomes approximately

$$I = 4.0\times10^6 e^{-8.65/T} + 6.6\times10^5 n_3 T^{1/2} + 13T^4, \tag{8}$$

where T is in °K and n_3 is the concentration of He^3 impurities in cm^{-3}. The plot of this expression in Fig. 1 shows that, depending on the He^3 impurity concentration, the condition $I < 1$ should hold for temperatures considerably less than 0.5°K, that is, in the millidegree temperature range.[18] In this case, then, the functional dependence of v on T, n_3, and Z is predicted by Eq. (8) to be

$$v = v_0/(1 - cZn_3T), \tag{9}$$

where v_0 is a constant, while $c \approx 6\times10^5$ cgs units may depend weakly on Z and whether the film flow is into or out of the beaker.[14]

Basic assumptions used in deriving Eq. (9) are that vorticity is responsible for limiting the superfluid film-creep velocity, that the creation of the vorticity occurs at a position other than at the film surface, and that average energy conservation on a macroscopic scale rather than a microscopic Landau criterion is sufficient for describing the velocity limiting process. The velocity enhancement factor that results from this theory is analogous to an ac Josephson junction phenomenon[19] and can be understood intuitively as follows. A là Eq. (1), the rate at which horizontal vortex lines must be created is a function of z. In the absence of viscous effects this means that the energy supply, $\dot{n}e_0$, required from the van der Waals and gravitational fields is also dictated by the film height difference z without any dependence on v and only a relatively small effect from the position of the vortex cores

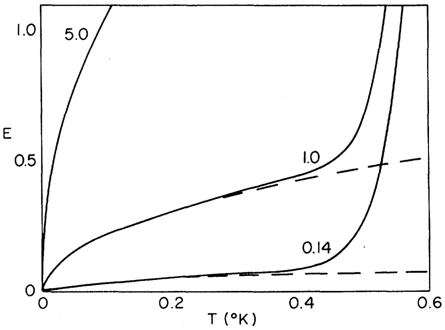

FIG. 1. I values for various n_3 in ppm. The dashed lines show the contribution of He^3 scattering only.

relative to boundaries. But the rate of energy supply to the film is proportional to its downward velocity v in the effective gravitational field. Hence, v must be large enough to supply the energy required, and any extra energy dissipation necessitates a corresponding increase in v. Such enhanced flow rates over contaminated or roughened surfaces have, in fact, been observed for many years.[20]

Experimental confirmation of the functional dependences expressed in Eq. (7) for saturated film-creep experiments in the millidegree temperature range would give considerable support to the oft-discussed but qualitative concepts concerning the roll of vorticity in helium films. Also informative would be experiments in which variable electric forces are applied to the supposed vortex lines when they contain trapped ions in the film, as well as all the corresponding but even more difficult experiments on unsaturated films.

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[12] An alternative but conceptually distinct point of view for describing the physics expressed in this formula for $\hbar$ follows from the assumption that the substrate over which the film passes should be responsible for restraining the superfluid flow. One can argue that v is not related to film height [which would relate v to $(2gZ)^{1/2}$] or to viscous effects (which would be markedly temperature dependent). Since this assumption implies that a force couple acts on the film, it necessitates the creation of vorticity as a means of absorbing the angular momentum imparted to the film. With a restraining force located along the wall and the average motivating force of the van der Waals and gravitational force fields located at the film surface, the classical torque equation gives an answer eight times larger than Eq. (3) for $\hbar$.

[13] The vortex lines must be created horizontally, but it is interesting to note that the viscous drag on a vortex line should be independent of its orientation because the size of a vortex core, a, is very much smaller than the mean free path, λ, of the normal-fluid quasiparticles. In Eqs. (2) and (4) it is assumed that $d \ll \lambda$ does not change $\alpha(T)$ from the bulk values measured by Rayfield and Reif.

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[20] Many experiments are summarized on p. 217 of

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STRONG SHOCK WAVES*

R. A. Gross, Y. G. Chen, E. Halmoy, and P. Moriette
Plasma Physics Laboratory, Columbia University, New York, New York 10027

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Shock waves traveling at speeds in excess of 10^8 cm/sec (Mach number ≥ 1000) in deuterium gas at 50 mTorr and room temperature ($n_1 = 1.3 \times 10^{15}$ cm^{-3}) have been created in a 3-m-long, electromagnetically driven coaxial shock tube, and neutrons have been observed.

Strong ionizing shock waves propagating through room-temperature deuterium at shock speeds $u_s \gtrsim 10^8$ cm/sec should, according to theoretical calculations, heat the gas to kilovolt energies.[1,2] Kurtmullaev et al. previously reported[3] shock speeds of nearly 10^8 cm/sec in low-density deuterium ($n_1 \sim 10^{12}$ cm^{-3}) in which the shock wave traveled a distance of several centimeters and the wave structure was sufficiently thin to be considered as collisionless. At the Seventh International Shock Tube Symposium in June of 1969, Levine, Vitkovitsky, and Kolb reported[4] experiments with collisionless shock waves propagating several centimeters at speeds $\gtrsim 10^8$ cm/sec into low-density ($n \sim 10^{12}$ cm^{-3}) ionized argon. Both these groups employed small, fast, theta-pinch devices.

No measurements of collisional shock waves traveling faster than about 4×10^7 cm/sec have been reported.[2] High-speed collisional shock waves are of special interest for controlled fusion because such waves are expected to heat the ions, a process necessary to achieve thermonuclear reactions in the resulting plasma. We report here the operation of a new device, a coaxial electromagnetically driven shock tube, which has produced collisional shock-wave speeds $\geq 10^8$ cm/sec over distances of ~100 cm, into relatively dense ($n \sim 10^{15}$ cm^{-3}) deuterium.

A coaxial electromagnetic shock tube has been constructed at Columbia University. This device is 3 m long, has an outer diameter of 22.5 cm, and an annular gap of 5 cm. The coaxial tube, whose outer conductor is aluminum and inner conductor is copper, is pumped to approximately 10^{-7} Torr and then filled with room-temperature deuterium to a pressure p_1 between 10 and 100 mTorr. An initial azimuthal magnetic field $B_1 \approx 7200$ G is produced by the current from a 300-kJ capacitor bank whose quarter period is 100 μsec. At the time of maximum field a shock wave is driven into the cold gas by an azimuthal magnetic field (piston) produced by a current ($\sim 2 \times 10^6$ A) which flows axially along the inner conductor, radially through the plasma, and returns axially along the outer conductor. For the experiments described here the inner conductor was positive with respect to the grounded outer conductor. Both the initial field and the shock-driving field had the same azimuthal direction. The shock-wave drive current was produced by a 300-kJ, 100-kV capacitor bank, which has a quarter period of 4 μsec. The shock-wave drive current is transmitted through six SF_6-gas-filled low-inductance switches, each of which is triggered by a 180-kV Blumlein pulse initiated from a single master switch. Further details of this device have been described elsewhere.[5]

Some of the initial results obtained from operation of this new electromagnetic shock tube are described below. In Table I are shown shock speeds measured in deuterium with initial pres-

Table I. Shock-wave speeds in deuterium (cm/sec) ($p_1 \approx 50$ mTorr, $T_1 = 298$°K).

		0-50 cm	0-100 cm	0-150 cm
Magnetic field probes	foot	$3.6 \pm 0.5 \times 10^8$	$9.2 \pm 1.4 \times 10^7$	
	slope	•••	$7.6 \pm 0.6 \times 10^7$	$4.0 \pm 0.2 \times 10^7$
Radiation detectors	foot	$2.5 \pm 0.7 \times 10^8$	$8.8 \pm 0.5 \times 10^7$	
	slope	$1.7 \pm 0.1 \times 10^8$	$6.4 \pm 0.4 \times 10^7$	$3.6 \pm 0.5 \times 10^7$