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INTERMEDIATE-ANGLE PION-PROTON ELASTIC SCATTERING FROM 3.0 TO 5.0 GeV/c*

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Differential cross-section measurements for π^+p and π^-p elastic scattering at 3.0, 3.5, 4.0, and 5.0 GeV/c are presented for center-of-mass scattering angles between 55° and 130°. A shoulder near $t=-0.9$ and a dip near $t=-3.0$ (GeV/c)^2 are seen in the data throughout this momentum range.

An experiment to measure elastic differential cross sections for pion-proton elastic scattering was carried out at the Argonne National Laboratory zero-gradient synchrotron (ZGS) accelerator. The measurements were made at incident momenta of 3.0, 3.5, 4.0, and 5.0 GeV/c for π^- and π^+ and covered an angular range of $55° < \theta_{\text{c.m.}} < 130°$. At momenta both above and below this momentum range, recent elastic differential cross-section measurements[1-3] indicate structure in this intermediate angle region. Present at all momenta above 2.5 GeV/c, for example, are a forward nonshrinking diffraction peak, a dip or shoulder at $t=-0.9$ (GeV/c)^2, and a deep minimum[4] near $t=-3.0$ (GeV/c)^2. Data from the present experiment show that these features also occur throughout the 3- to 5-GeV/c interval.

Theoretically, the intermediate angular region provides a stringent test of elastic-scattering models. Several mechanisms, including diffraction and t-channel Regge-pole exchange,[5] have successfully described forward scattering, while backward scattering has been fitted using direct channel resonances[6] or the exchange of Reggeized baryon trajectories.[7] Consistent with duality, the intermediate region could be interpreted either in terms of resonances or in terms of a combination of t- and u-channel exchanges including cuts at large values of both $|t|$ and $|u|$.

In the present experiment pions were incident upon a 12-in.-long liquid-hydrogen target. The angles of the two final-state particles were measured in a two-arm counter and wire spark-chamber system. This system was designed to measure p-p elastic scattering at large angles and is described by Brabson et al.[8] Pions in the momentum-selected ZGS beam No. 1 were identified by a Freon-12–filled threshold gas Cherenkov counter (CG in Ref. 8). Negative-pion flux at 4.0 GeV/c was typically 3×10^4 π^- per 600-msec beam spill. The corresponding positive-particle flux was 4×10^4 particles containing approximately 40% π^+. A trigger of the system consisted of a beam telescope signal, no signal from the anticounter array surrounding the target, and a signal from one and only one of the counters in each of the two arms (see Fig. 1 in Ref. 8).

Since no final-state momenta were measured, there are two kinematic constraints for an elastic event. These events were separated from background by first imposing the requirement of coplanarity. Events with $|\hat{P}_1\times\hat{P}_2\cdot\hat{P}_{\text{beam}}| \leq 0.012$ were accepted as coplanar ($\hat{P}_i$ is a unit vector along the ith final-state particle). Figure 1(a) shows a histogram of coplanarity for reconstructable π^+p events at 3.0 GeV/c. Defining θ_1 as the smaller laboratory scattering angle and θ_2

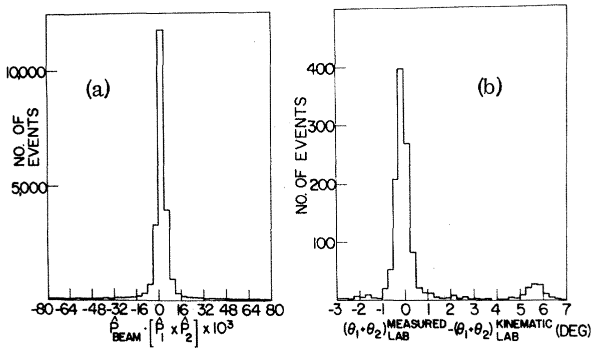

FIG. 1. (a) Coplanarity distribution for all reconstructable events at 3.0 GeV/c for the reaction $\pi^+p\to\pi^+p$. $\hat{P}_{\rm beam}$, $\hat{P}_1$, and $\hat{P}_2$ are unit vectors along the directions of the beam, and the two final-state particles. (b) Distribution of $(\theta_1+\theta_2)_{\rm lab}^{\rm meas}-(\theta_1+\theta_2)_{\rm lab}^{\rm kinematic}$ for a bin $1.090\le|t|\le1.190$ (GeV/c)^2 at 3.00 GeV/c for the reaction $\pi^+p\to\pi^+p$, illustrating relative magnitudes of the background and elastic events. In addition to the main peak at 0°, there is a secondary peak corresponding to a reversal of the pion-proton track assignment.

as the larger scattering angle, coplanar events were sorted into bins of $\theta_2-\theta_1$ intervals. For each interval the distribution of events versus the quantity [opening angle $(\theta_1+\theta_2)$–kinematically predicted opening angle] was made assuming θ_1 to be the pion direction. Then similar plots were made assuming θ_1 to be the proton direction. Figure 1(b) displays such a distribution for a particular interval in $\theta_2-\theta_1$, where θ_1 was taken to be the pion direction. Background under these opening-angle distributions was estimated by a simple linear interpolation from regions outside of the elastic-peak regions. The second peak in the opening-angle distribution reflects the absence of information concerning the identification of the pion and proton. The 1-mm resolution for the spark location in space made it possible to separate the pion from the proton down to c.m. scattering angles within 7° of the equal-angle ambiguous case cleanly. Background subtractions varied from typically 6% for cross sections of 100 μb (GeV/c)$^{-2}$ to 50% for 1-μb (GeV/c)$^{-2}$ cross sections. At 3.0 GeV/c, for example, a total of 100 000 π^- events were collected of which 12 000 were elastic. At 5 GeV/c 125 000 π^- events contained 2600 elastic events.

Angular-dependent corrections to the data include the effects of nuclear interactions of the final-state particles (3 ± 2)% and chamber inefficiencies (2-5 ± 2)%. Normalization corrections applied to the data were: counter inefficiencies (1 ± 1)%, beam attenuation in the hydrogen target and downstream beam counter $B2$ (5 ± 1)%, muon-beam contamination (1 ± 1)%, and electron contamination [(5 ± 2)% of pions in the beam]. The total normalization correction to the data was 18% with an error of ±8%.

Figure 2 displays $\pi^{\pm}p$ angular distributions from this experiment. The errors shown represent the statistical uncertainty in the number of elastic and background events and an estimated systematic uncertainty due to the background subtraction. Other experiments[9] in the same energy range are also shown for comparison. Agreement with previous measurements done at both forward and backward angles is good. The previously observed dip near $t=-3.0$ (GeV/c)^2 is present at all momenta for both π^- and π^+ and is observed to be very deep. As has been observed before,[10] the dip near $t=-0.9$ (GeV/c)^2 is more pronounced for π^- than for π^+.

The exchange of a single Regge pole in the t channel gives rise to an energy dependence in the differential cross section of the form[11]

$$d\sigma/dt=F(t)s^{2\alpha(t)-2}.$$

Several poles and cuts can participate in πp elastic scattering, but if one trajectory dominates in a particular region of t, this should become clear in a plot of $\alpha_{\rm eff}(t)$ vs t, where $\alpha_{\rm eff}(t)$ is an experimentally determined function of t defined by the above equation. Figures 3(a) and 3(b) show the behavior of $\alpha_{\rm eff}{}^{\pm}(t)$ for elastic $\pi^{\pm}p$ scattering cross sections[12] in the region of negative t. No data for $\theta_{\rm c.m.}>120°$ have been used. For comparison, we have drawn the trajectory $\alpha(t)=0.5+t$.

One of the most notable features of the effective trajectories is that as t becomes more negative, both $\alpha_{\rm eff}{}^{\pm}(t)$ continue to fall off quite sharply with t. This is somewhat surprising since one might expect an effective trajectory to flatten out as cuts become more important. The small-$|t|$ behavior seems consistent with the dominance of a quite flat Pomeranchuk trajectory. However, for $1.0\le|t|\le2.5$ (GeV/c)^2, $\alpha_{\rm eff}{}^{\pm}(t)$ come close to the extrapolated straight line P', ρ trajectories, indicating the importance of one or both of these trajectories in this t range. Beyond $|t|=2.5$ (GeV/c)^2, the $\alpha_{\rm eff}{}^{\pm}(t)$ trajectories begin to diverge: $\alpha_{\rm eff}{}^{-}(t)$ appears to level off while $\alpha_{\rm eff}{}^{+}(t)$ drops below the linear trajectory shown. It is possible[13] to account for this unusual structure by means of the interference between two Regge contributions, one of which dominates over the other for an intermediate range

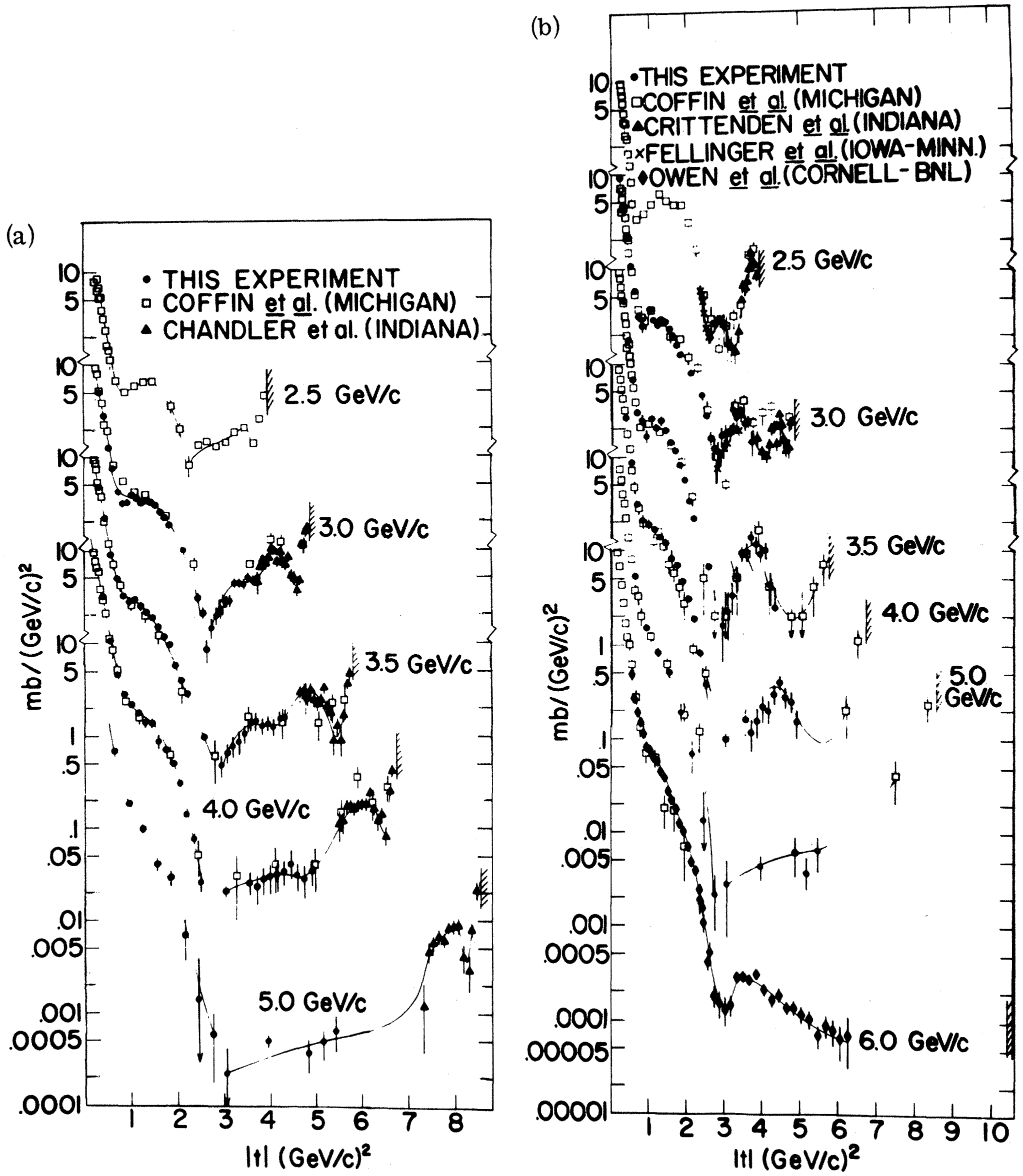

FIG. 2. Elastic differential cross sections for (a) π^+p and (b) π^-p. Sources of data plotted which are not from this experiment are listed in Ref. 9. $\theta_{\text{c.m.}} \geq 180°$ is marked by a hatched area. Hand-drawn curves connect data points at the same momentum.

of energies. Schematically writing

$$d\sigma(\pi^\pm p)/dt \sim s^{-2}|s^{\alpha_1(t)} \pm \epsilon_\pm(t)s^{\alpha_2(t)}|^2,$$

with $\epsilon_\pm$ small, $\epsilon_\pm$ being the relative strength of the two contributions, one can obtain splitting of the $\alpha_{\text{eff}}^\pm(t)$ in a simple way if $\alpha_2(t)$ lies above $\alpha_1(t)$. For the present situation, this can be accomplished by having a dominant P' (ρ) pole interfere with a higher lying ρ (P') type pole or cut. Such a model would imply that at energies above this intermediate region of $P_{\text{lab}} = 2.5$ to 8 GeV/c, $\alpha_{\text{eff}}^+(t)$ and $\alpha_{\text{eff}}^-(t)$ would again become equal in this t range, coinciding with the higher trajectory or cut. The effective trajectories, $\alpha_{\text{eff}}^\pm(t)$, were also evaluated using the expression

$$d\sigma/dt = P_l^{-2}f(t)\nu^{2\alpha(t)},$$

where $\nu = \frac{1}{2}(s-u)$ and P_l is the pion laboratory momentum, as suggested in Ref. 4. These trajectories show the same characteristics as those in Fig. 3.

Presently, a direct-channel resonance model[14] is being used in an attempt to describe existing

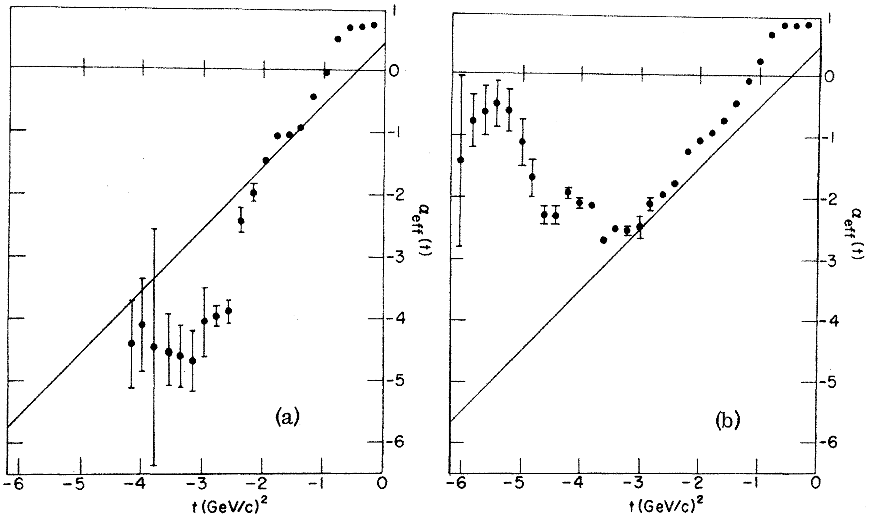

FIG. 3. $\alpha_{\rm eff}(t)$ vs t for (a) π^+p and (b) π^-p. The solid line represents a trajectory of slope $=1/({\rm GeV}/c)^2$ with the ρ-trajectory intercept. Data used in the plots for π^+p are: 2.5 GeV/c, Coffin *et al.*; 3.0, 3.5, 4.0, and 5.0 GeV/c, this experiment; 8.5 and 12.4 GeV/c, Harting *et al.*; and 8.8, 10.8, 12.8, 14.8, and 16.7 GeV/c, Foley *et al.* Data used in the plots for π^-p are: 3.0, 3.5, 4.0, and 5.0 GeV/c, this experiment; 5.8, 5.9, 7.88, 9.71, and 9.84 GeV/c, Owen *et al.*; 6.0 GeV/c, Coffin *et al.*; 10.8, 13.0, 15.0, 17.0, 18.9, 19.75, 23.18, and 25.34 GeV/c, Foley *et al.*; and 12.4 GeV/c, Harting *et al.*

$\pi^\pm$ scattering and polarization data beyond $\theta_{\rm c.m.}=90°$, including data from this experiment. In addition, preliminary optical-model calculations are being carried out in an attempt to describe the entire angular distribution at several energies.

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SCALING BEHAVIOR IN $pp \to \pi +$ ANYTHING AT HIGH ENERGY*

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We present evidence that the inclusive reaction $p+p\to\pi+$ anything approaches a scaling limit at accelerator energies.

Although the bulk of high-energy reactions is inelastic, the variety and complexity of final states encountered have made it difficult for the experimentalist to decide what to measure and have prevented detailed theoretical analysis of these processes. Of all inelastic processes, perhaps the most amenable to experimental study and to theoretical treatment are inclusive reactions—reactions in which some final particle or property of the final state is studied irrespectively of whatever else is happening. As an example of such a reaction, we discuss here the process $p+p\to\pi+$anything.

The differential cross section for this process depends in general on three variables. We can conveniently take these as the square of the center-of-mass energy s, and the longitudinal and transverse components of the momentum of the outgoing pion in the c.m. system,

$$d\sigma_\pi = \frac{d^3p}{p^0} f(p_\perp, p_\parallel, s) = \frac{\pi\, dp_\perp{}^2\, dp_\parallel}{(p_\perp{}^2 + p_\parallel{}^2 + \mu^2)^{1/2}} f(p_\perp, p_\parallel, s). \qquad (1)$$

On the basis of heuristic arguments, Feynman[1] has suggested that the structure function $f(p_\perp, p_\parallel, s)$ scales at high energy. That is, as $s\to\infty$ it becomes only a function of $p_\perp$ and the ratio $x = 2p_\parallel/s^{1/2}$. Thus,

$$d\sigma_\pi \xrightarrow[s\to\infty]{} \frac{\pi\, dp_\perp{}^2\, dx}{[x^2 + 4(p_\perp{}^2 + \mu^2)/s]^{1/2}} f(x, p_\perp). \qquad (2)$$

Analogous expressions have been suggested by Amati, Stanghellini, and Fubini,[2] and Wilson,[3] on the basis of the multiperipheral model.

The integrated inclusive cross section counts the production of n pions n times over. Hence $\Sigma_\pi \equiv \int d\sigma_\pi = \Sigma n\sigma_n$, where σ_n is the cross section for the production of n pions. It is related to the average number of π's produced in inelastic collisions

$$\bar{n}_\pi = \frac{\Sigma n\sigma_n}{\Sigma\sigma_n} = \frac{\Sigma_\pi}{\sigma_{\text{tot inel}}}. \qquad (3)$$

If the total inelastic cross section for pp scattering approaches a constant as s goes to infinity, which we shall assume, and if Feynman's scaling holds true with $f(0, p_\perp) \neq 0$, then it follows that the average number of pions produced in pp collisions increases logarithmically with s,[4]

$$\bar{n}_{\pi^\pm} = \left[\frac{\pi}{\sigma_{\text{tot inel}}} \int_0^\infty dp_\perp{}^2 f^\pm(0, p_\perp)\right] \ln(s) + \text{const}^\pm = c^\pm \ln(E/m) + d^\pm, \qquad (4)$$

where E is the incident energy in the laboratory frame. Recent cosmic-ray experiments[5] with a hydrogen target, including energies up to 800 GeV, clearly show such a logarithmic growth of the average multiplicity.

In this note we investigate the extent to which existing accelerator data for the process $p+p\to\pi+$anything have approached the Feynman scaling limit. In particular, if these data are already asymptotic, then the coefficient of $\ln(s)$ in Eq. (4) calculated from the accelerator data must agree with the coefficient of $\ln(s)$ determined experimentally from the presumably asymptotic cosmic-ray data.

The existing accelerator data[6-8] cannot be used directly to compute the integral indicated in Eq. (4) since no data for $x=0$ and varying $p_\perp$ exist. We can, however, fit the existing data with a factorized form

$$f^\pm(x, p_\perp) = F(p_\perp)G^\pm(x), \qquad (5)$$