TRINUCLEON FORM FACTORS CALCULATED FROM REALISTIC LOCAL POTENTIALS

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Charge and magnetic form factors are calculated using solutions of the three-nucleon Faddeev equations with the local two-nucleon potentials of Reid and of Malfliet and Tjon. The results give reasonable agreement with the data up to 9-fm^{-2} transfer, but do not reproduce the minimum at 11.8 fm^{-2} in the ^{3}He charge form factor.

In a recent electron elastic scattering experiment[1] knowledge of the charge and magnetic form factors of ^{3}He has been extended up to four-momentum transfers of $q^2 = 20\ \text{fm}^{-2}$. It was found that the absolute value of the charge form factor has a minimum at approximately $q^2 = 11.8\ \text{fm}^{-2}$. In view of the recent progress in solving the nuclear three-body problem through use of the Faddeev equations, it is natural to raise the question of whether a local potential can simultaneously fit the nucleon-nucleon phase shifts and the ^{3}He, ^{3}H form factors. In this note we present some results of trinucleon form-factor calculations in which we use S-state wave functions constructed by the method of Malfliet and Tjon[2] from the nucleon-nucleon potentials of Reid[3] and of Ref. 2, and we compare these and previous results for varational calculations with hard-core potentials with the experimental data.

The trinucleon form factors (restricted to S states and neglecting the exchange moments) are in the notation of Schiff[4]:

$$\begin{aligned}
2F_{\text{ch}}(^3\text{He}) &= (2F_{\text{ch}}^{\,p} + F_{\text{ch}}^{\,n})F_1 - \tfrac{2}{3}(F_{\text{ch}}^{\,p} - F_{\text{ch}}^{\,n})F_2,\\
F_{\text{ch}}(^3\text{H}) &= 2F_{\text{ch}}^{\,n} + F_{\text{ch}}^{\,p})F_1 + \tfrac{2}{3}(F_{\text{ch}}^{\,p} - F_{\text{ch}}^{\,n})F_2,\\
\mu(^3\text{He})F_{\text{mag}}(^3\text{He}) &= \mu_n F_{\text{mag}}^{\,n} F_1 + \tfrac{2}{3}(\mu_p F_{\text{mag}}^{\,p} + \mu_n F_{\text{mag}}^{\,n})F_2,\\
\mu(^3\text{H})F_{\text{mag}}(^3\text{H}) &= \mu_p F_{\text{mag}}^{\,p} F_1 + \tfrac{2}{3}(\mu_p F_{\text{mag}}^{\,p} + \mu_n F_{\text{mag}}^{\,n})F_2, \qquad (1)
\end{aligned}$$

where the body form factors F_1 and F_2 are functions of the three-momentum $\vec{q}$. They are related to the totally symmetric $U(P, Q)$ and the mixed-symmetric $V_1(P, Q)$ S-state wave functions by the expressions

$$\begin{aligned}
F_1(q) &= \int d^3P\, d^3Q\, U(P, Q) U(P, |\vec{Q} - \vec{q}/(3M)^{1/2}|)\\
F_2(q) &= -3\int d^3P\, d^3Q\, U(P, Q) V_1(P, |\vec{Q} - \vec{q}/(3M)^{1/2}|), \qquad (2)
\end{aligned}$$

where M is the nucleon mass. The integration variables $\vec{P}$ and $\vec{Q}$ of Eq. (2) are the relative momenta defined in terms of the particle momenta k_i by

$$\begin{aligned}
\vec{P} &= \tfrac{1}{2}(M)^{-1/2}(\vec{k}_2 - \vec{k}_3),\\
\vec{Q} &= \tfrac{1}{2}(3M)^{-1/2}(\vec{k}_2 + \vec{k}_3 - 2\vec{k}_1). \qquad (3)
\end{aligned}$$

The functions U and V_1 are determined by solving the Faddeev equations for a given two-nucleon interaction. For a discussion of the method used see Ref. 2.

We have treated three distinct sets of two-body potentials in this study. The first set (I-III) was that employed in Ref. 2; it consists of $^3S_1$ and $^1S_0$ potentials of a local, central Yukawa-type having short-range repulsion and long-range attraction. The strengths and ranges were determined from a fit to both the low-energy two-nucleon parameters and the s-wave phase-shift data up to 300-MeV lab energy. The second set studied consisted of the soft-core potentials of Reid (set S. C. of Ref. 3), which is a more elaborate set of repulsive and attractive Yukawa forms. The Reid potentials also possess tensor and spin-orbit components in the triplet channel. However, since we are interested in form factors restricted to S-state contributions only, the three-body equations were solved with the following simplification: The d-wave elements of the two-body T matrix in the triplet channel of the Faddeev equations were dropped. This resulted in a triton binding energy of 6.8 MeV as compared with the 6.5 MeV obtained in the case in which the d-wave matrix elements were included.[5]

The ^{3}He and ^{3}H charge and magnetic form factors associated with these two sets of potentials are shown in Fig. 1 along with the data of McCarthy et al.[1] and Collard et al.[6] In this calculation we have used the analytic forms given by Janssens et al.[7] for the nucleon electromagnetic form factors. The ^{3}H binding energy and the trinucle-

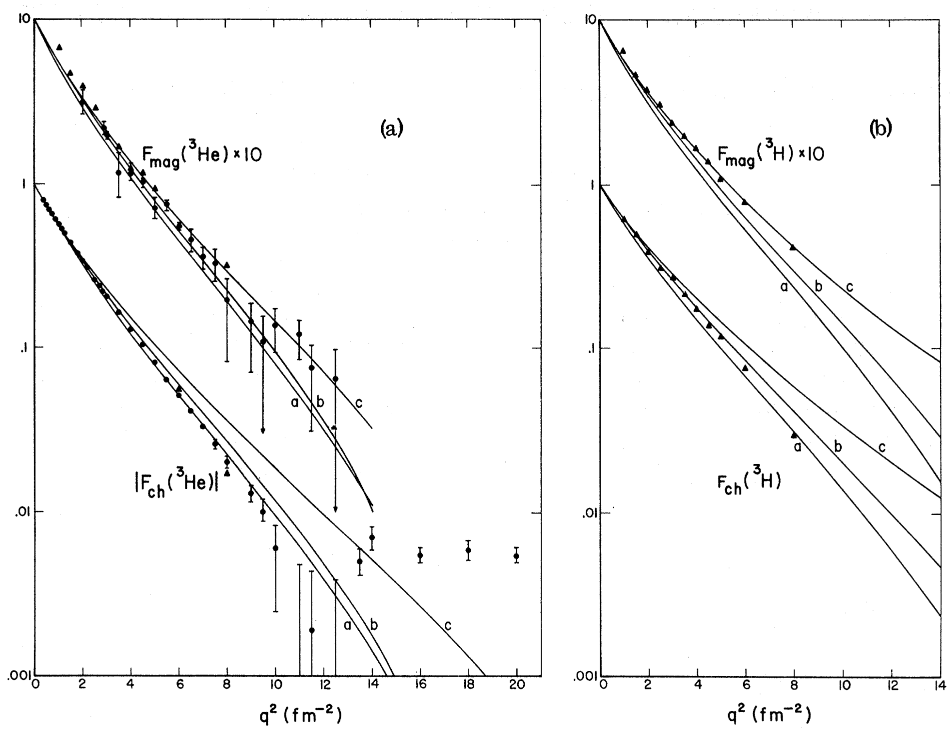

FIG. 1. Charge and magnetic form factors of ^{3}He and ^{3}H. Circles are the data of Ref. 1; triangles the data of Ref. 6. The theoretical curves are constructed from the two-nucleon potentials of (a) the Reid potential with soft cores, (b) the Malfliet-Tjon potential set (I-III) with soft cores, and (c) the Malfliet-Tjon potential set (I-IV) with a soft core in the singlet only.

on electromagnetic radii are listed in Table I. We note that the soft-core potentials (curves a and b) produce reasonable fits to the recent data up to $q^2 = 9$ fm^{-2}; however, the minimum in the calculated $|F_{ch}(^3He)|$ at 17 fm^{-2} occurs at much too high a q^2 and does not reproduce the large cross section at high momentum transfer observed in the experimental data. Since we may expect that this minimum in the form factor is directly related to the short-range correlations

Table I. Binding energy and radii of trinucleon ground states constructed from different s-wave two-nucleon potentials.

Two-nucleon potential	^{3}H Binding energy (MeV)	$R_{ch}(^3He)$ (fm)	$R_{mag}(^3He)$ (fm)	$R_{ch}(^3H)$ (fm)	$R_{mag}(^3H)$ (fm)	q^2 of dip in $F_{ch}(^3He)$ (fm^{-2})
Reid (SC) soft cores	6.8	2.05	2.09	1.80	1.97	17.0
Malfliet-Tjon soft cores (I-III)	8.3	1.93	1.96	1.70	1.87	17.0
Malfliet-Tjon soft core in singlet only (I-IV)	8.4	1.90	1.94	1.66	1.83	24.0
Experiment	8.49					
McCarthy *et al.* (Ref. 1)		1.88 ± 0.05	1.95 ± 0.11			11.8 ± 0.5
Collard *et al.* (Ref. 6)		1.87 ± 0.05	1.74 ± 0.10	1.70 ± 0.05	1.70 ± 0.05	

in the wave function, it should depend on the presence of the repulsive core in the two-body potential. To show this dependence we have recalculated the form factors using a third set of nucleon-nucleon potentials (Malfliet and Tjon, I-IV) in which a purely attractive $^3S_1$ potential was adjusted to fit the low-energy triplet parameters. The triton binding energy is changed by only 0.1 MeV and the radii do not differ significantly from those of the I-III set, but the zero in $|F_{ch}(^3\text{He})|$ moves out[8] to 24 fm^{-2}.

It is worthwhile to note that the neglected ^4D states of ^{3}He are not expected to have much effect on the charge form factor, since the ^2S and ^4D states do not interfere.[9] However, the ^2S and ^4D states will interfere in the magnetic form factor[10] and are expected to fill in the minimum in the S-state form factor. It should also be pointed out that the calculated $F_{mag}(^3\text{He})$ is in much better agreement at low momentum transfer with the recent data of Ref. 1 than with the older data of Ref. 6. A similar inconsistency between the calculations with repulsion (curves a and b) and $F_{mag}(^3\text{H})$ or Ref. 6 is apparent in Fig. 1. (The charge data appear to be consistent for both experiments.) If the theoretical fit to the latest ^{3}He magnetic data is indeed correct, then the neglected exchange-moment form factor must have essentially the same q dependence as the body form factor. This would imply that the exchange-moment density has approximately the same spatial distribution as the nucleons themselves.

A comparison of these results with previous calculations shows the same qualitative misfit to the minimum in $|F_{ch}(^3\text{He})|$. Form factors based on wave functions from variational calculations with the hard-core potentials of Gammel-Brueckner, Hamada-Johnston, and the Yale group have been generated by Delves and Blatt[11] and for the Hamada-Johnston potential by Davies.[12] Although the form factors are only given out to $q^2 = 8$-10 fm^{-2}, indications are that they will not have a dip near $q^2 = 11.8$ fm^{-2}. A variational calculation by Tang and Herndon,[13] in which they use a potential with a hard core plus exponential attraction, shows that to achieve a minimum near 11.8 fm^{-2} one would need a core radius of at least 0.7 fm. In view of efforts to date in constructing hard-core potentials, it appears unlikely that a good fit to the s-wave phase-shift data can be obtained with a hard core of this size. Finally Lim,[14] using perturbation theory, obtained a minimum as low as 13 fm^{-2} with a soft-core potential; however, the potential does not give a good fit to the low-energy scattering parameters.[15]

To summarize, we have seen that wave functions generated from local potentials fitted to the low-energy parameters and to the s-wave phase shifts give reasonable predictions for the electromagnetic form factors of ^{3}He at momentum transfers below 9 fm^{-2}, but fail to give good predictions in the region of the minimum and beyond in the charge form factor. The results of this calculation and those of the variational calculations on hard-core potentials suggest that it may be impossible to simultaneously fit the high-energy nucleon-nucleon phase shifts and the trinucleon form factors at high momentum transfer with local, repulsive two-body potentials.

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