MASS OF ^{13}O AND THE ISOBARIC MULTIPLET MASS EQUATION*

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The mass excess of ^{13}O has been determined to be 23.107 ± 0.015 MeV. This yields a value for d, the coefficient of T_z^3 in the isobaric multiplet mass equation, of −0.8 ± 3.4 keV for the $A = 13$ quartet. Additional values for d, obtained from new measurements of the masses of ^{17}Ne, ^{21}Mg, and ^{25}Si, are also consistent with zero.

Although the level energies of several isobaric multiplets with $T = \frac{3}{2}$ have been determined,[1] only the $A = 9$ quartet has been measured with sufficient accuracy to observe a significant deviation from the quadratic form of the isobaric multiplet mass equation, $\Delta M = a + bT_z + cT_z^2$. If this deviation is paramatrized by an additional term dT_z^3 in the mass equation, then d has a value[2] of 9.2 ± 3.7 keV. The nonzero value for d indicates that a first-order perturbation treatment of the isospin-nonconserving operator (assuming charge-dependent forces of tensorial rank 2 or less) is inadequate to describe the $A = 9$ quartet. Several theoretical attempts to evaluate the magnitude of d have led to conflicting results. A second-order perturbation calculation,[3] using Coulomb-energy systematics to estimate the mixing of $T = \frac{1}{2}$ states with the $T = \frac{3}{2}$ states in ^{9}Be and ^{9}B, gives a d coefficient comparable with that observed. However, further considerations based on the widths of the perturbing $T = \frac{1}{2}$ states indicate that the $T = \frac{1}{2}$ admixture is small and not consistent with the large value observed for d.[4] Henley and Lacy,[5] applying a simple nonperturbative model to the $A = 4n + 1$ isospin quartets, find generally small values for d (≲1 keV) and, in particular, obtain $d = 0.06$ keV for $A = 9$.

In order to investigate systematics that might appear in the d coefficient, we have accurately remeasured the mass of ^{13}O and we report new or improved values for the masses of ^{17}Ne, ^{21}Mg, and ^{25}Si. In all cases the values obtained for d are essentially consistent with zero. In particular, using the well-known masses for the other members of the $A = 13$ multiplet,[1] no deviation from the quadratic mass equation is observed within an error comparable with the uncertainty in the $A = 9$ determination.

The experiments were performed with ^{3}He and ^{4}He beams from the University of California at Berkeley 88-in. variable-energy cyclotron. The ^{13}O mass measurement employed semiconductor counter telescopes to observe the reaction ^{16}O(^{3}He, ^{6}He)^{13}O and the calibration reactions ^{12}C(^{3}He, ^{6}He)^{9}C and ^{12}C(α, ^{6}He)^{10}C. ^{3}He ions, bombarding a CO_2 gas target, produced ^{6}He nuclei corresponding to the ^{9}C and ^{13}O ground states. Since the peaks due to ^{9}C and ^{13}O were separated by approximately 1.4 MeV, the known mass[2] for the ^{9}C ground state provided an accurate energy calibration. The slope of the energy scale was determined by observing the ground state and first-excited state ($E_x = 3.3527$ MeV)[6] in ^{10}C produced by alpha-particle bombardment of ethane.

Two similar four-counter telescopes (consisting of 114-, 90-, 300-, and 500-μm counters) and electronic systems were simultaneously employed at equal angles on opposite sides of the beam axis.[7] Two particle identifications were performed and compared using the signals from the two successive differential-energy-loss (ΔE) detectors and the third E detector. Subnanosecond pulse-shape discrimination was employed to reduce pile up due to ^{3}He's and deuterons from the same beam burst which can produce a false signal in the ^{6}He region of the particle-identifier spectrum. Pulser calibrations following each event were used to correct the slope and zero point of the energy scale for each system. A split Faraday cup and the observation of elastically scattered particles were used to monitor the beam position.

Five parameters were recorded for each event: the total energy, two ΔE losses, particle identification, and pile up. Corrections were made and limits were placed on these parameters using an off-line computer. Peak centroids were determined using Chauvenet's theorem[8] to set limits on the extent of the ^{6}He peaks systematically. A typical set of spectra is shown in Fig. 1. The resolution was 200-300 keV.

The ^{13}O mass was obtained by averaging the Q values measured in each detector system. Two independent runs at $E_{^3\mathrm{He}} = 62.6$ MeV, $\theta_{\mathrm{lab}} = 11.65°$ and at $E_{^3\mathrm{He}} = 66.3$ MeV, $\theta_{\mathrm{lab}} = 12.19°$ were combined to yield an ^{13}O mass excess of 23.107 ± 0.015 MeV based on a ^{9}C mass excess of 28.906 ± 0.004 MeV.[2] The uncertainty in the mass ex-

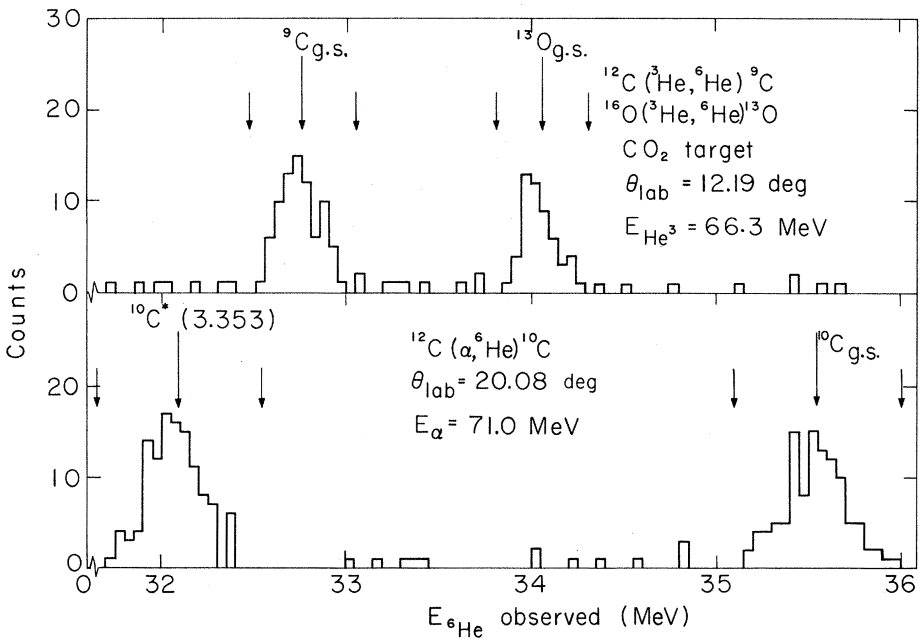

FIG. 1. Typical spectra representing approximately 25% of the total data. The short arrows indicate the region used in calculating the centroids of the peaks. The long arrows mark the positions of these centroids.

cess is primarily due to statistical errors and the uncertainty in the beam energy. Errors due to uncertainties in the scattering angle, the slope of the energy scale, counter dead layers, gas-target foil corrections, gas-pressure variations, and energy fluctuations in the beam were small because of the differential nature of the measurement. Small corrections were made for the shifts in each peak position due to the angular distribution of the reaction products across the solid angle (5×10^{-5} sr) of the detectors. The mass excess obtained is in excellent agreement with the previously measured value, 23.110 ± 0.070 MeV.[9]

The masses of ^{17}Ne, ^{21}Mg, and ^{25}Si, measured using similar techniques but with less precision, are given in Table I.[10] For the ^{17}Ne measurement, a gas target containing a mixture of 97% enriched ^{20}Ne gas and CO_2 was used. Carbon-backed ^{24}Mg (99% enriched) and natural SiO_2 targets were used for the ^{21}Mg and ^{25}Si measurements, respectively. The overall resolution for the solid targets was ≈175 keV.

The mass of ^{17}Ne (16.479 ± 0.050 MeV) is found to be in good agreement with a previous value[11] of 16.47 ± 0.25 MeV. The new mass of ^{21}Mg, 10.889 ± 0.040 MeV, supersedes the previous measurement of 10.62 ± 0.12 MeV from this laboratory.[12] For the ^{25}Si measurement the group observed could be composed of both the ground and first-excited states which are expected to lie ≈30 keV apart. If the first-excited state is populated to a significant extent, the d coefficient will be slightly more positive than that given in Table I, but it will still be consistent with zero. Values of a, b, c, and χ^2 determined by a linear–least-squares fit to the quadratic mass equation are also given in Table I. With the exception of $A=9$ all are consistent with a value of $\chi^2 \lesssim 1$. Values for d are given in the last column of Table I.

Our value for d, -0.8 ± 3.4 keV, for the $A=13$ quartet is significantly less than the value 9.2 ± 3.7 keV previously determined for $A=9$. Since additional values of d for the quartets with $A>13$ are also essentially consistent with zero, there does not appear to be a systematic increase in d with increasing average charge of the multiplet. This suggests that the nonzero d coefficient for the $A=9$ quartet is not due primarily to charge-dependent forces of tensorial rank 3 or higher.

It is of particular interest to understand the variation in magnitude of the d coefficient in the more accurately measured $A=9$ and $A=13$ quartets. In the absence of higher order, charge-dependent forces, a nonzero value for d might arise from level shifts due to nearby thresholds for isospin-allowed decays[13] or, in a perturbation treatment (where d depends on the product of off-diagonal vector and tensor Coulomb matrix elements[1,3]), it might be due to mixing of the

Table I. Completed $4n+1$, $T=-\frac{3}{2}$ multiplets. Values of a, b, and c are obtained from a least-squares fit to $\Delta M = a+bT_z+cT_z^2$. d is the coefficient of an additional T_z^3 term in ΔM. Errors, in parentheses, are in keV.

A	$T_z = -3/2$ Mass Excess (MeV)	IMME Prediction[c] (MeV)	a (MeV)	b (MeV)	c (MeV)	χ^2	d (keV)
9	28.906 (4)[a]	28.961 (22)	26.343 (4.0)	-1.315 (2.0)	0.263 (2.4)	6.16	9.2 (3.7)
13	23.107 (15)[b]	23.102 (14)[d]	19.257 (2.2)	-2.181 (3.4)	0.256 (2.7)	0.06	-0.8 (3.4)
17	16.479 (50)[b]	16.508 (23)[e]	11.651 (3.7)	-2.878 (5.4)	0.238 (8.1)	0.28	4.8 (9.2)
21	10.889 (40)[b]	10.940 (24)	4.897 (4.8)	-3.658 (6.8)	0.241 (6.4)	1.18	8.5 (7.8)
25	3.817 (50)[b]	3.796 (42)	-3.262 (8.1)	-4.387 (10.6)	0.216 (10.8)	0.10	-3.5 (10.9)
37	-13.230 (50)[c]	-13.198 (91)	-22.886 (11.6)	-6.181 (14.6)	0.174 (14.6)	0.10	5.3 (17.3)

[a] Ref. 2.

[b] Present work. For the ^{25}Si measurement we have assumed that only the ground state is populated.

[c] Ref. 1.

[d] An improved value for the ^{13}N state [M. J. Levine and P. D. Parker, Phys. Rev. 186, 1021 (1969)] and for the ^{13}C state [K. A. Snover, E. G. Adelberger, and F. Riess, Bull. Amer. Phys. Soc. 13, 1662 (1968)] has been included in the weighted average.

[e] A new value for the ^{17}O state [C. Detraz and H. H. Duhm, Phys. Lett. B 29, 29 (1969)] has been included in the weighted average.

$T=\frac{3}{2}$, $T_z=\pm\frac{1}{2}$ levels with order $J^\pi=\frac{3}{2}^-$ levels with either $T=\frac{3}{2}$ or $T=\frac{1}{2}$. A reliable test of the perturbation approach requires a knowledge of all of the parameters of the appropriate $T=\frac{3}{2}$ and $T=\frac{1}{2}$ levels in the $T_z=\pm\frac{1}{2}$ nuclei. In both the $A=9$ and $A=13$ systems it is therefore necessary to identify and characterize the $\frac{3}{2}^-$, $T=\frac{1}{2}$ states predicted[14] to lie in the energy range close to the analog levels. In addition, for the $A=9$ system a more accurate determination of the width of the ^{9}B analog state is needed.

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CLOSE SIMILARITIES IN THE EXCITATION FUNCTIONS FROM THE ELASTIC ^{16}O SCATTERING FROM NUCLEI WITH $A \approx 16$†

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Close similarities are found in the gross structures of the excitation functions from the elastic scattering of ^{16}O from ^{14}N, ^{15}N, and ^{18}O. The picture emerging from the optical-model analysis of these data is that of an interaction potential which is totally absorbing for the low-l partial waves and which is very transparent for the surface partial waves.

We wish to report on a study of the elastic scattering of ^{14}N and ^{15}N from ^{16}O and of ^{16}O from ^{18}O, in which we find strikingly close similarities among the gross structures of the excitation functions from these various systems. This investigation was stimulated by the observation[1] of unexpected, and very pronounced, structure in the excitation functions in the $^{16}O + ^{16}O$ scattering and the subsequent renewed interest[2-6] in the heavy-ion–nucleus interaction.

In view of the $^{16}O + ^{16}O$ results the scattering of systems in which either the target or the projectile ^{16}O is replaced by a nucleus with $A \approx 16$ (such as ^{14}N, ^{15}N, or ^{18}O) is of special interest. Although in such cases one is no longer dealing with the scattering of identical spin-zero nuclei to which only the even-l partial waves contribute, it is worth noting that optical-model calculations predict very nearly the same structure in the excitation functions for angles $\leq 90°$ for the scattering of identical and nonidentical particles.

Within the framework of extensive optical-model calculations we explore the properties of the heavy-ion–nucleus potential and its physical significance. Even though the optical model primarily provides merely a convenient parametrization of the elastic-scattering data, it is hoped that physical insight into the interaction of complex nuclei can be obtained from an extensive optical-model analysis of several sets of related data. We therefore pay particular attention to a determination of those nuclear properties that appear to be independent of the parametrization.

Beams of ^{14}N, ^{15}N, and ^{16}O ions have been accelerated[7,8] with the Argonne National Laboratory's High Voltage Engineering Corporation model FN tandem Van de Graaff and scattered off approximately 100-μg thick targets of SiO_2 and Al_2 $^{18}O_3$. Particles were identified by the associated-particle method, in which kinematic coincidence is required between the scattered ion and the recoiling target nucleus.[9,10] Absolute cross sections accurate to approximately 15% have been obtained by normalizing the ^{15}N and ^{18}O data to Coulomb scattering and the ^{14}N data to the measurements of Jacobson.[11] Corrections for energy loss in the target and for the varying charge state have been applied. For all these systems we have measured angular distributions at not less than four energies and excitation functions at several angles in the approximate c.m. energy range, 10-30 MeV. The ^{14}N data for c.m. energies below 16 MeV are those of Jacobson.[11]

The excitation-function data are summarized in Figs. 1 and 2. In Fig. 1 the data of all three systems studied in this work are compared with each other and with the earlier $^{16}O + ^{16}O$ data,[1,2] and in Fig. 2 the experimental excitation functions are shown together with optical-model calculations. The abscissa in Fig. 1 is qR, where $q = 2k \sin\frac{1}{2}\theta$ is the momentum transfer and $R = 1.0(A_1^{1/3} + A_2^{1/3})$ is the radius. In the left half of Fig. 1 all the $^{16}O + ^{18}O$ data have been shifted in qR by the same amount to compensate for the difference in the Coulomb-barrier heights. From Figs. 1 and 2 it is seen that diffractionlike gross