MEASUREMENT OF $T_z = -\frac{3}{2}$ MASSES AND THE ISOBARIC MULTIPLET MASS EQUATION.*

G. F. Trentelman, B M. Preedom,† and E. Kashy
Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48823
(Received 18 June 1970)

Precise measurements of the masses of ^{9}C, ^{13}O, and ^{21}Mg have been made using a magnetic spectrograph. The results are used to test the isobaric multiplet mass equation.

One of few simply calculable model-independent expressions in nuclear physics is the isobaric multiplet mass equation,

$$M(\beta, T, T_z) = a(\beta, T) + b(\beta, T)T_z + c(\beta, T)T_z^2,$$

where β represents all necessary quantum numbers in addition to the isospin. The derivation of this equation treats charge-dependent forces as a first-order perturbation to a charge-independent nuclear Hamiltonian.[1,2]

Testing the validity of this equation requires knowing the masses of all members of at least isospin $T = \frac{3}{2}$ quartets. At present the members of eight such multiplets ($A = 7$ through 37) have been measured and reported,[2,3] but until recently the experimental uncertainties associated with the $T_z = -\frac{3}{2}$ masses have been substantially larger than those of the other multiplet members.

Two recent measurements[4,5] of the ^{9}C mass excess to a precision ±4 keV have been reported, and this uncertainty is commensurate with that of other members of the $A = 9$, $T = \frac{3}{2}$ quartet ±5 keV. Application of the mass equation to this quartet indicates that a term in T_z^3 with a coefficient of 9.2 ± 3.7 keV may be added to the mass equation, at least for $A = 9$. It is of importance then to measure other of the $T_z = -\frac{3}{2}$ masses with minimal experimental uncertainty not only to check the general precision of the quadratic mass equation but also to see if the occurrence of cubic terms persists in the other multiplets.

This Letter reports the measurement of the masses of ^{9}C, ^{13}O, and ^{21}Mg using (^{3}He, ^{6}He) reactions on ^{12}C, ^{16}O, and ^{24}Mg, respectively. The ^{3}He beam from the sector-focused Michigan State University cyclotron was used at various energies between 68 and 70 MeV and the ^{6}He reaction products were detected in an Enge split-

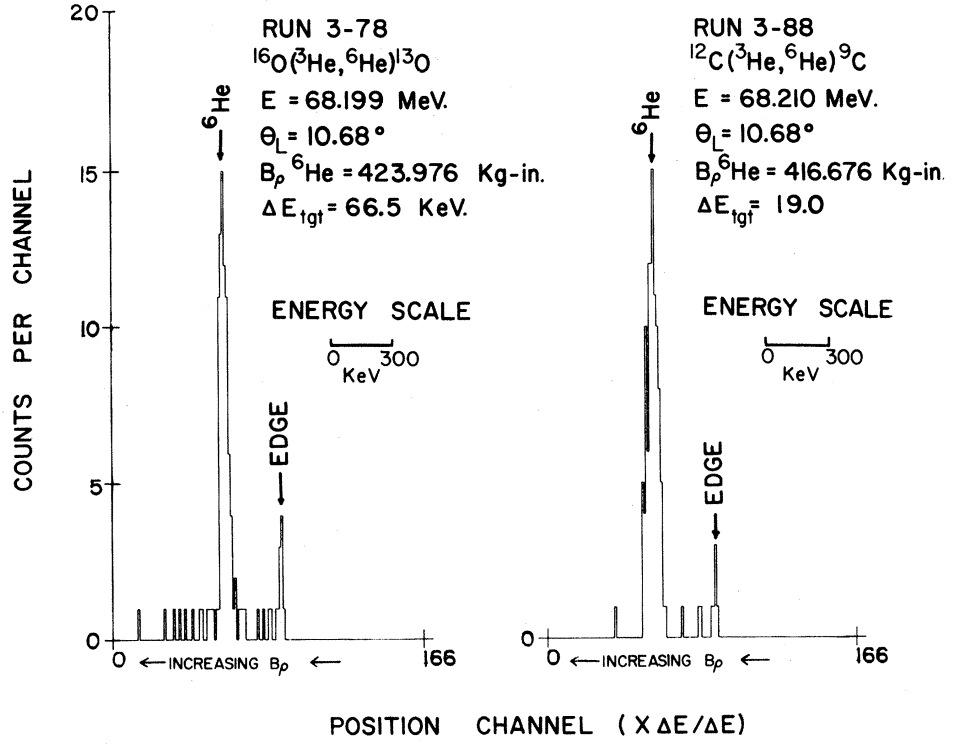

FIG. 1. Typical position spectra for $^{16}O(^{3}He, ^{6}He)^{13}O$ and $^{12}C(^{3}He, ^{6}He)^{9}C$. The ΔE_{tgt} represents energy correction to the Q value due to target effects. The peak marked "EDGE" indicates the end of the counter.

Table I. Reaction Q values and mass excesses for the $T_z=-\frac{3}{2}$ nuclei ^{9}C, ^{13}O, and ^{21}Mg.

Element	Reaction	Q Value (MeV)	Mass excess (MeV)	Mass excess previously reported (MeV)
^{9}C	^{12}C(^{3}He, ^{6}He)^{9}C	−31.578 ± 0.008	28.911 ± 0.009	28.906 ± 0.004[a]
^{13}O	^{16}O(^{3}He, ^{6}He)^{13}O	−30.506 ± 0.013	23.103 ± 0.014	23.11 ± 0.070[b]
^{21}Mg	^{24}Mg(^{3}He, ^{6}He)^{21}Mg	−27.512 ± 0.018	10.912 ± 0.018	10.95 ± 0.120[c]

[a]See Ref. 4.
[b]See Ref. 3.
[c]See Ref. 2.

pole magnetic spectrograph. The spectrograph compensates for the energy spread due to the reaction kinematics and thus allows a large-solid-angle (1.2-msr) entrance aperture to be used. This is a definite advantage as the cross sections for the (^{3}He, ^{6}He) reactions are small (~1 μb), and they also decrease with increasing target mass.

The magnetic rigidity of the particles was precisely calibrated for a particular orbital path in the spectrograph from 280 to 470 kG in. with a momentum-matching technique developed in this laboratory.[6] This technique requires, for example, the simultaneous detection at the same position on the focal plane of both protons and deuterons from a pair of reactions such as ^{12}C(p, p)^{12}C and ^{12}C(p, d)^{11}C. This condition is met at a given laboratory scattering angle only for a unique beam energy such that the magnetic rigidities of the outgoing protons and deuterons are equal. For these reactions at a laboratory scattering angle of 15.0°, the momentum matching yields E_p (beam) = 33.691 ± 0.0022 MeV and $B\rho$ (spectrograph) = 332.256 kG in. The 2.2-keV uncertainty represents the limit of the accuracy in determining the beam energy by this method and reflects the uncertainty in the mass of ^{11}C. This uncertainty, however, contributes only a small amount of the error in the determination of the absolute value of the masses presented below.

The energy of the ^{3}He beam for each run was determined by elastic scattering from ^{12}C and ^{16}O in the calibrated spectrograph described above. The laboratory scattering angle for the ^{3}He-induced reactions was sensitively measured with ^{1}H(^{3}He, ^{3}He)^{1}H scattering. Particle detection and identification for all reactions was made with a 300-μm position-sensitive silicon surface-barrier detector on line with an XDS Sigma-7 computer. Figure 1 shows typical position spectra and experimental parameters for the reactions ^{12}C(^{3}He, ^{6}He)^{9}C and ^{16}O(^{3}He, ^{6}He)^{13}O. Eight such spectra were obtained for ^{9}C, nine for ^{13}O, and four for ^{21}Mg. A more detailed account of the experimental procedures and their intrinsic uncertainties will be reported later.

Table I lists the reactions, their measured Q values and resulting mass excesses, as well as the latest published values of these masses for comparison. A principal source of error in the data shown in this table is the uncertainty in the energy loss in the targets. Other numerous contributions to the uncertainties arise from sources such as calibration of the spectrograph, statistical uncertainty in the centroids, scattering-angle determination, etc. A detailed analysis of these errors will be presented in a later paper.

Table II displays the coefficients $a(\beta, T)$, $b(\beta, T)$, and $c(\beta, T)$ of the quadratic mass equation obtained from a weighted least-squares fit of the

Table II. Empirically determined coefficients for the mass equation $M=a+bT_z+cT_z^2$ using the latest $T_z=-\frac{3}{2}$ mass excess values.[a] The last column indicates the coefficient of a T_z^3 term assuming the equation to have the form $M=a+bT_z+cT_z^2+dT_z^3$. The coefficients were determined from a weighted least-squares fit, and the χ^2 for the fit with the quadratic equation is indicated.

Mass	$a(\beta, T)$ (MeV)	$b(\beta, T)$ (MeV)	$c(\beta, T)$ (MeV)	χ^2	$d(\beta, T)$ (MeV)
9	26.343 ± 0.004	−1.3185 ± 0.003	0.266 ± 0.003	4.0	0.0083 ± 0.0039
13	19.257 ± 0.0027	−2.1802 ± 0.0035	0.256 ± 0.003	0.002	−0.0002 ± 0.0035
21	4.8987 ± 0.0046	−3.6573 ± 0.005	0.240 ± 0.0048	1.28	0.0057 ± 0.0051

[a]For mass excesses of $T_z=\frac{3}{2}, -\frac{1}{2}$; for $T=\frac{3}{2}$ multiplet members, see Table I in Ref. 2.

mass-9, -13, and -12 quartets using the $T_Z = -\frac{3}{2}$ masses measured here and the masses taken from Ref. 2 for the $T_Z = \frac{3}{2}$, $\frac{1}{2}$, and $-\frac{1}{2}$ members. The value of χ^2 was calculated from the equation

$$\chi^2 = \sum_i \left[\frac{M(\text{calc}) - M(\text{expt})}{\sigma_M(\text{expt})} \right]^2 .$$

The deviation of the data from the quadratic fit is shown in Fig. 2. It is seen that the quadratic mass dependence predicts the mass values within their measured errors for $A = 13$ and 21 and slightly misses two of the values for $A = 9$.

As has been pointed out, one may expect cubic terms in the isobaric multiplet mass equation to arise from a more exact treatment of charge-dependent forces.[3] Adding a term $d(\beta T) T_Z{}^3$ to the quadratic equation above and fitting to the data as before, one obtained the values of d presented in Table II. The uncertainty shown for d is determined as much by the errors in the $T_Z = \frac{1}{2}$ members of the quartet as by the present values of the $T_Z = -\frac{3}{2}$ members although the latter actually have larger errors. Only the $A = 9$ quartet shows some justification for a cubic term. If such a cubic term does indeed exist, its magnitude has been estimated to be of the order of $c\alpha Z_{\text{av}}$, where c is the coefficient of the quadratic term, Z_{av} is the average charge of the isobaric multiplet, and α is the fine-structure constant.[2] For $A = 9$ this value is approximately 9 keV and the value of $d = 8.3 \pm 3.9$ keV obtained here is not inconsistent with that estimate. Pursuing this topic further, measurements of the masses from ^{25}Si to ^{37}Ca are in progress using the same experimental techniques presented here.

It is clear that measurements of improved accuracy are required for all members of the isobaric multiplets if one hopes to use the mass equation to obtain more definite conclusions concerning the role of charge-dependent forces in nuclei. In the meantime, in view of the present results, one can use the quadratic isobaric multiplet mass equation to extrapolate to unknown masses with a high degree of confidence.

This work would not have been possible without the continuing effort of the Michigan State University cyclotron staff. We are also indebted to Professor R. Sherr for many stimulating discussions on the importance of testing the mass equation.

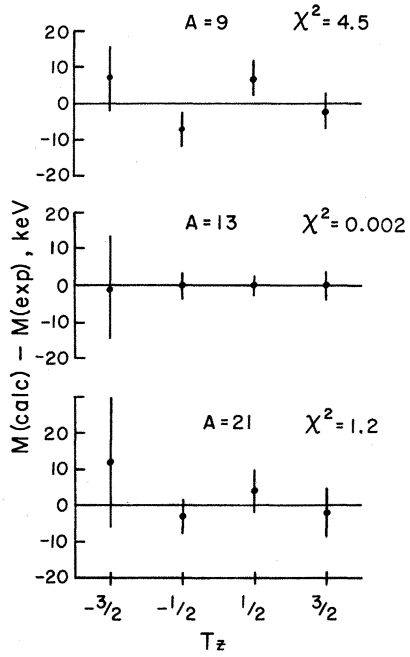

FIG. 2. Deviation of experimental $T = \frac{3}{2}$ multiplet members from quadratic mass equation using the coefficients of Table II.

*Work supported by the National Science Foundation.

†Present address: Physics Department, University of South Carolina, Columbia, S. C.

[1]D. H. Wilkinson, Phys. Rev. Lett. 13, 571 (1964), and Phys. Lett. 11, 243 (1964).

[2]G. T. Garvey, in *Proceedings of the Second Conference on Nuclear Isospin, Asilomar-Pacific Grove, California, 13-15 March 1969*, edited by J. D. Anderson, S. D. Bloom, J. Cerny, and W. W. True (Academic, New York, 1969), p. 703.

[3]J. Cerny, Annu. Rev. Nucl. Sci. 18, 27 (1968).

[4]C. A. Barnes, E. G. Adelberger, D. C. Hensley, and A. B. McDonald, in *Nuclear Physics: An International Conference*, edited by R. L. Becker, C. D. Goodman, P. H. Stelson, and A. Zucker (Academic, New York, 1967), p. 261.

[5]J. M. Mosher, R. W. Kavanagh, and T. A. Tombrello, Bull. Amer. Phys. Soc. 14, 1167 (1969).

[6]G. F. Trentelman and E. Kashy, to be published.