We have studied the equations of motion for the spin-boson problem and other approximations and equivalences. A more detailed account of this work will be published shortly.

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ISOSPIN FORBIDDEN RESONANCES IN THE REACTION ^{93}Nb(p, α)^{90}Zr *

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Isospin forbidden resonances ($\Delta T = 1$) have been observed in the reaction ^{93}Nb(p,α)^{90}Zr. Angular distributions to the 0^+ ground state measured at the 4^+ (59-keV) and 2^+ (334-keV) analog resonances indicate that the 2^+ isospin analog state is formed via a pure $d_{5/2}$ capture whereas the 4^+ isospin analog state is formed by a coherent admixture of $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ components.

Isospin-forbidden resonances in heavy nuclei such as those previously observed[1,2] in (p,n) reactions are thought to arise from an enhancement in the compound-nuclear (CN) cross section due to Coulomb mixing of the isospin analog state (IAS) into the many underlying states of one unit less isospin ($T_<$ states).[1,3] Such reactions are commonly described[1,3] in the framework of the Hauser-Feshbach (HF) theory where the presence of the IAS is manifested by an enhancement in the proton transmission function. Although the basic assumption of the HF theory is the independence of formation and decay, a further approximation is usually made that the reaction amplitudes corresponding to the various $j = lts$ channels add incoherently in the angular distribution of the reaction products. In other words, the matrix of the transmission functions[4] $\tilde{T}$ is assumed to be diagonal. Such an approximation has been used[2] in the analysis of previous (p,n) data. In contrast, the analysis of a resonance in the reaction ^{93}Nb(p,α)^{90}Zr studied here required the use of a nondiagonal $\tilde{T}$ matrix.[5] This allowed us to measure small admixtures in the IAS wave function.

The reaction ^{93}Nb(p,α)^{90}Zr was studied between $E_p = 4.7$ and 6.2 MeV using the proton beam from the Stony Brook High Voltage Engineering Corporation model FN tandem accelerator and a self-supporting Nb foil ($\Delta E \sim 10$ keV for 5-MeV protons). The α particles were detected with two pairs of counter telescopes operated in a pass-through reject mode. Excitation functions for the transition to the ground state of ^{90}Zr obtained at $\theta_L = 90°$ and 155° are shown in Fig. 1. The lowest seven IAS have previously been observed in the (p,n) reaction.[6] We see strong resonances in the (p,α) reaction[7] and attribute them to the formation of low-lying IAS with natural parity. As a consequence of angular-momentum and parity conservation, the unnatural-parity states cannot decay by α emission to a 0^+ state and thus are not observed in the ground-state transition. The strong resonances at $E_p = 4.885$ and 5.175 MeV correspond, respectively, to the analogs of the 4^+ (59-keV) and 2^+ (334-keV) states[8] of ^{94}Nb. The first five states and the lowest 2^+ state in ^{94}Nb have been suggested[8] to have pure $\nu(d_{5/2}{}^3)_{J'}$ neutron configuration with a predominant $J' = \frac{5}{2}^+$ component coupled to a $\frac{9}{2}^+$ proton configuration.

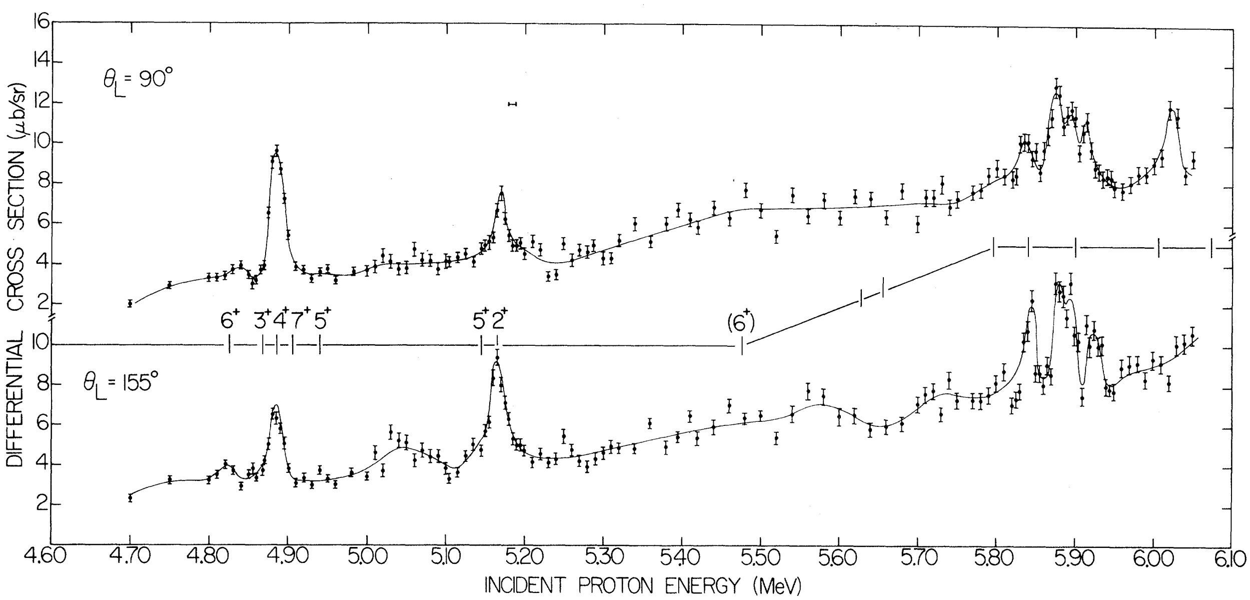

FIG. 1. Excitation functions for the reaction $^{93}Nb(p,\alpha)^{90}Zr(0^+,g.s.)$ obtained at $\theta_L = 90°$ and 155°. The incident energies refer to the lab frame. The lines drawn through the data points are intended to be a visual aid. The vertical markers indicate the expected positions of the analogs of low-lying ^{94}Nb states seen in the (d,p) study of Ref. 8. Only statistical uncertainties are shown.

Since the target has a $\nu(d_{5/2}{}^2)_{0^+}$ configuration, the IAS are expected to be formed via $j = d_{5/2}$ capture.

As evident in Fig. 1, the 6^+(g.s.) IAS is barely seen, if present at all. However, the same state is seen as a strong resonance in the transition to the 5^- (2.315-MeV) state in Zr^{90} where the 6^+ IAS can decay via $l_\alpha = 1$. The small cross section to the 0^+ ground state can thus be attributed to the small barrier penetrability for $l_\alpha = 6$ decay. In general, analog resonances (including those with unnatural parity) are seen in the transitions to the first five states of ^{90}Zr.

Angular distributions for the ground-state transition, measured at energies on and near the lowest 4^+ and 2^+ resonances, are displayed in Fig. 2. The 2^+ angular distribution is seen to be in accord with the predicted shape (dashed line) for a single $j = d_{5/2}$ capture while the 4^+ resonance data are not. One possible explanation might be that fluctuations[9] such as those observed off the analog resonances (see Fig. 1) are modulating the 4^+angular distribution. We view this as unlikely because the 4^+ resonance can be fitted well by a function consisting of a Breit-Wigner resonance plus a constant background with the width of the resonance equal to that observed in the (p,n) reaction.[6] We are thus led to consider possible admixtures of configurations in the 4^+ IAS. In particular, the next two single-particle orbits above the $2d_{5/2}$ are the $3s_{1/2}$ and $2d_{3/2}$. These can admix into the 4^+ state but not the 2^+ state because of the $\frac{9}{2}^+$ proton configuration. The (p,α) angular distribution on the 4^+ resonance should then reflect these admixtures and in fact should be particularly sensitive to an $s_{1/2}$ admixture because of the relatively large s-wave penetrability.

In order to demonstrate that these admixtures add coherently (nondiagonal $\tilde{T}$) rather than incoherently (diagonal $\tilde{T}$), the 4^+ angular distribution (with the off-resonance contribution subtracted) was fitted twice using a minimum-χ^2 criterion; one fit included interference terms while the other did not. More explicitly, the expression for the angular distribution can be written as

$$W(\theta) = \sum_{jj'} C_{jj'} f_{jj'}(\theta), \tag{1}$$

where the values of j and j' range from $\frac{1}{2}$ to $\frac{5}{2}$, the $C_{jj'}$ are the parameters of the fits, and the $f_{jj'}(\theta)$ are explicit functions of the Legendre polynomials.[10] An incoherent sum of j admixtures implies the neglect of terms with $j \neq j'$.

Within the framework of the HF theory, the parameters $C_{jj'}$ are related to the α and proton $\tilde{T}$-matrix elements. For the case studied here where there are many open channels (p, n, and α), a very good approximation to this relation is

$$C_{jj'} \propto \tilde{T}_{\alpha\alpha} \sum_{jj'} (\tilde{T}_{jj'} - \tilde{T}_{jj'}{}^0), \tag{2}$$

where the $\tilde{T}_{jj'}{}^0$ are the proton matrix elements in the absence of the analog resonance. The matrix $\tilde{T}$ is defined by the relation $\tilde{T} = I - \langle S\rangle^\dagger\langle S\rangle$,

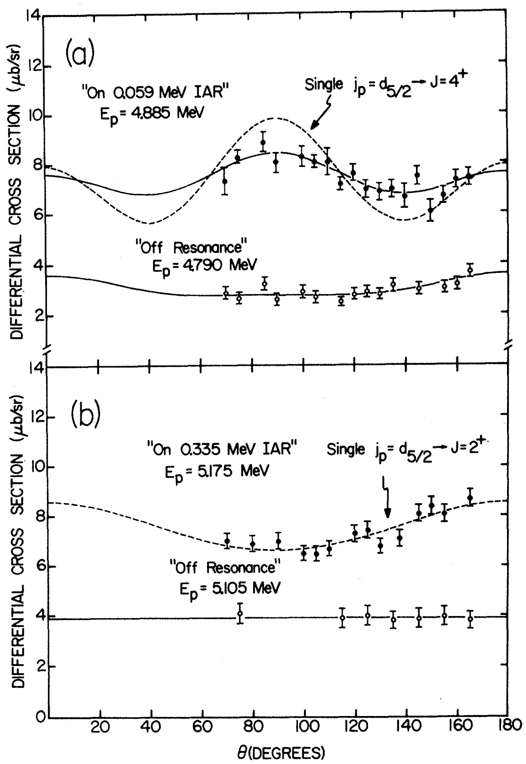

FIG. 2. Angular distributions $W(\theta)$ for the reaction ^{93}Nb$(p,\alpha)^{90}$Zr$(0^+$, g.s.). The solid lines represent the results of a least-squares fit with a sum of even Legendre polynomials up to fourth order. The dashed lines represent the expected $W(\theta)$ for a single $j = d_{5/2}$ capture. These were obtained by adding the theoretical $W(\theta)$ to the off-resonance data and requiring the resulting angle-integrated cross section to be the same as that of the on-resonance data. Only statistical uncertainties are shown.

where I is the unit matrix and $\langle S \rangle$ is the energy-averaged S matrix whose diagonal and off-diagonal elements for the proton channels include a resonance term.[11] This resonance term differs from the Breit-Wigner amplitudes by the presence of the so-called background mixing phases.[3] If we assume that the mixing phases and the absorption in the proton channels are unimportant, the resulting T-matrix elements yield an expression for the angular distribution which can be written in a form where the parameters are similar to those obtained if we have considered a coherent sum of Breit-Wigner amplitudes. The neglect of the mixing phases is justified by the apparent symmetry in the resonance shape. The proton absorptions, calculated with an optical model code,[12] were found to be small and, therefore, may be ignored. Thus, the fit to the angular distribution involves only three parameters, even for the case with interferences, because the relative phases of the formation channels differ from the corresponding relative scattering phases of the optical model only by an angle π.

A good fit to the data, comparable with the solid line in Fig. 2(a), was obtained for both the cases with and without interferences (normalized χ^2 value of 1.0 which corresponds to a 50% confidence level). Fortunately, the resulting parameters of the fits for the two cases differ significantly. In particular, the required amount of $s_{1/2}$ admixture, expressed as a ratio of proton partial widths, $\Gamma_{1/2}/\Gamma_{5/2}$, is 0.09 ± 0.02 for the case with interferences; the corresponding value without interference effects is 1.0 ± 0.2. Thus the ambiguity can be resolved by requiring the admixtures to be consistent with the structure of the parent 4^+ state in ^{94}Nb.

We have, therefore, recorded forward-angle spectra for the reaction ^{93}Nb$(d,p)^{94}$Nb at $E_d = 9.0$ MeV using the University of Pennsylvania multigap spectrograph. The extreme forward-angle cross sections should be sensitive to the $s_{1/2}$ admixture even though the l_n amplitudes add incoherently because the calculated single-particle stripping cross section[13] for $l_n = 0$ is much larger than that for $l_n = 2$. The measured differential cross sections leading to the 59-keV state (4^+) of ^{94}Nb yield no evidence for an appreciable $l_n = 0$ component.[14] In particular, the extracted upper limit of the $l_n = 0$ contribution to the measured (d,p) cross section allows us to set an upper limit on the $s_{1/2}$ partial width for the 4^+ IAS to be $\Gamma_{1/2}/\Gamma_{5/2} \lesssim 0.1$.

In view of the evidence presented by the (d,p) data that the 4^+ parent state contains very little $s_{1/2}$ admixture, we conclude that the various components in the 4^+ IAS formation contribute coherently in the angular distribution and that the wave function of the 4^+ parent state contains admixtures which can be written as

$$|4^+\rangle \approx a\{[C \times d_{5/2}] - (0.02 \pm 0.01)^{1/2}[C \times s_{1/2}] + (0.06 \pm 0.02)^{1/2}[C \times d_{3/2}] + \cdots\}, \quad (3)$$

where a is a normalization constant and C is the ^{93}Nb ground state.

In summary, the present study of isospin-forbidden resonances in the (p,α) reaction on a heavy nucleus is seen to yield useful spectroscopic information on the IAS. The fact that resonances are observed in both the α channels and the neutron channels,[6] where the isospin changes by one unit, indicates the consistency of the HF approach for the interpretation of these resonances. The analysis of the 4^+ angular distribution indicated that the channel correlations should not be ignored in the enhanced compound-nuclear cross section. The presence of this correlation further suggests a sensitive technique for studying small admixtures in the wave function of the analog state.

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