

<sup>8</sup>J. R. Sanmartin, to be published.

<sup>9</sup>J. M. Dawson and C. Oberman, *Phys. Fluids* **5**, 517 (1962), and *ibid.* **6**, 394 (1963).

<sup>10</sup>W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. Oberman, *Phys. Rev. Lett.* **24**, 987 (1970). Recent computer experiments have shown that  $\nu_a$  may have a weak dependence on the electron-ion mass ratio. The simple estimate of  $\nu_a$  given in Eq. (5) does not show this behavior.

<sup>11</sup>J. M. Dawson, P. K. Kaw, and B. Green, *Phys. Fluids* **12**, 875 (1969).

<sup>12</sup>This can be verified from Nishikawa's Eq. (I. 54), if one replaces the ion wave damping coefficient  $\Gamma_1$  by

the ion acoustic frequency  $\omega_1$  (for  $T_e \approx T_i$ ).

<sup>13</sup>V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas* (Addison-Wesley, Reading, Mass., 1964), pp. 364-6.

<sup>14</sup>The finite size of the waveguide the detailed form of the wave fields, uncertainties in density, temperature, etc. will also contribute to  $\zeta$ .

<sup>15</sup>Notice that the critical field for the reflection coefficient falloff is higher than the threshold field required for the excitation of the instability. This is because the anomalous collision frequency has to come up to a sufficiently high level before one sees the effect on the reflection coefficient.

### SPECTROSCOPIC MEASUREMENT OF THE FREQUENCY, INTENSITY, AND DIRECTION OF ELECTRIC FIELDS IN A BEAM-PLASMA INTERACTION BY THE HIGH-FREQUENCY STARK-ZEEMAN EFFECT\*

William S. Cooper, III, and Roger A. Hess

Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

(Received 3 June 1970)

We show how the Zeeman pattern of the "satellites" of spectral lines produced by the high-frequency Stark effect can be used to determine the direction of an oscillating electric field relative to the magnetic field. We illustrate the use of this effect as a diagnostic technique in plasma physics by observations of the Zeeman pattern of the "plasma satellites" of the 4922-Å He-I line produced by high-frequency electric fields in a beam-plasma interaction.

The high-frequency Stark effect is a powerful spectroscopic technique for studying strong oscillating electric fields in plasmas.<sup>1</sup> Oscillating electric fields induce multiple quantum transitions which produce new spectral lines, "satellites" of normally forbidden or allowed lines. It is possible to determine the frequency of the perturbing electric field from the spacing of these satellites, and the strength of the field from measurements of the intensities of the satellites relative to the intensity of a nearby allowed line, or from the observed Stark shifts of the lines.<sup>2</sup>

In most cases in which one would like to use this method to study oscillating electric fields in a plasma, the plasma is permeated by a magnetic field. The presence of the magnetic field complicates the theoretical calculations, but it does introduce one valuable simplification in the data analysis. In the absence of a magnetic field, one can only determine the direction of the electric field from measurements of the slight polarization of the satellites.<sup>2</sup> In the presence of a magnetic field, however, one can determine the direction of the electric field relative to the magnetic field very simply by inspection of the Zeeman pattern of the satellites. In this Letter we

report the application of this new diagnostic technique to a beam-plasma interaction.

To calculate the high-frequency Stark effect in the presence of a constant magnetic field, one must solve the time-dependent Schrödinger equation with a Hamiltonian containing terms describing the interaction of an oscillating electric field and a static magnetic field with the excited atom. This can be done by including a magnetic interaction term within the framework of the calculations described by Cooper and Hicks.<sup>3</sup> We have done this and will report on it in a later publication.

If the electric fields are not too strong, such detailed calculations are not necessary since only two satellites are observed in this case. As discussed in Ref. 2 and in earlier publications cited therein, these satellites may be considered to be produced by the following two-quantum process: An excited atom in excited state  $i$  either absorbs one quantum from the electric field or emits one quantum to the field by an electric dipole transition and goes to an intermediate virtual state. This virtual state then decays to the final state  $k$  by a second electric dipole transition, which results in the emission of an optical photon. A direct electric dipole transition

from state  $i$  to state  $k$  is forbidden.

The probability of a transition from state  $i$  to state  $k$  is proportional to the product of the squares of two matrix elements, each one corresponding to one step of the two-step process described above; the pertinent matrix elements are given by Eqs. (2) and (3) of Ref. 2. Once a direction of observation is specified (the direction of emission of the optical photon relative to the magnetic field), the selection rules for the change in the magnetic quantum number ( $\Delta m$ ) follow, in the usual fashion, from the condition that these matrix elements not vanish.

The selection rules for  $\Delta m$  in the first step in the two-quantum process, in which the change in  $m$  is  $\Delta m_1$ , may be summarized as follows: If  $\vec{E}$  is parallel to  $\vec{B}$ ,  $\Delta m_1 = 0$ . If  $\vec{E}$  is perpendicular to  $\vec{B}$ ,  $\Delta m_1 = \pm 1$ . If  $\vec{E}$  is right-hand circularly polarized and a field quantum is emitted, or if  $\vec{E}$  is left-hand circularly polarized and a field quantum is absorbed,  $\Delta m_1 = +1$ . If  $\vec{E}$  is right-hand circularly polarized and a field quantum is absorbed, or if  $\vec{E}$  is left-hand circularly polarized and a field quantum is emitted,  $\Delta m_1 = -1$ . If  $\vec{E}$  is neither parallel nor perpendicular to  $\vec{B}$ ,  $\Delta m_1 = 0$  or  $\pm 1$ . We have followed the convention in plasma physics in defining right- and left-hand waves; in a left-hand wave the electric field vector rotates in the same sense as a positive ion.

In the second step of the two-quantum transition, in which the optical photon is emitted, the usual selection rules for  $\Delta m_2$  (the change in  $m$  in this step) for an electric dipole transition apply: For observation perpendicular to the magnetic field, if  $\Delta m_2 = 0$ , the photon is polarized parallel to  $\vec{B}$  ("P" polarization), and if  $\Delta m_2 = \pm 1$ , the photon is polarized perpendicular to  $\vec{B}$  ("S" polarization).

The Zeeman pattern of a satellite can now be calculated by evaluating the pertinent matrix elements, provided the magnetic splitting of the various levels is known. In Fig. 1 we show the results of calculations of the "normal" Zeeman patterns of satellites, which would be observed in transitions between singlet states, or if the magnetic field were large enough for the complete Paschen-Back effect to exist. The patterns are shown for observation normal to  $\vec{B}$ , for several possible electric field configurations. As drawn, the patterns apply to transitions of the type  $(n, l) \rightarrow (n, l-2)$ ; for example, the satellites of the 4922-Å ( $4^1D \rightarrow 2^1P^0$ ) line of He I. The relative intensities of the components are approx-

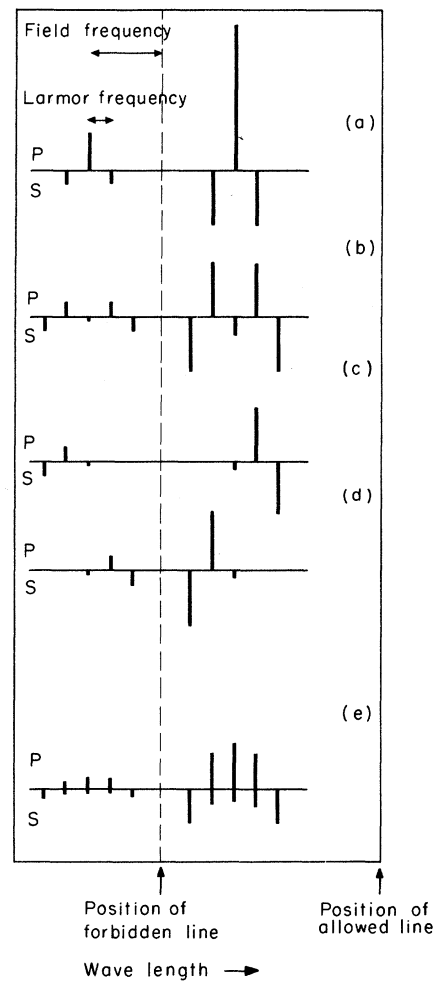


FIG. 1. Normal Zeeman patterns of the satellites of a forbidden line, showing both P and S polarization for various configurations of the electric field, all for direction of observation perpendicular to  $\vec{B}$ . Shown are: (a)  $\vec{E}$  parallel to  $\vec{B}$ , (b)  $\vec{E}$  perpendicular to  $\vec{B}$  and randomly distributed to azimuth, (c)  $\vec{E}$  perpendicular to  $\vec{B}$  and left-hand circularly polarized, (d)  $\vec{E}$  perpendicular to  $\vec{B}$  and right-hand circularly polarized, and (e)  $\vec{E}$  random in direction.

imately correct as shown. We have assumed that all initial states are equally populated.

It is obvious from Fig. 1 that by simple inspection of the Zeeman pattern of the satellites one can determine if  $\vec{E}$  is parallel or perpendicular to  $\vec{B}$ , and in the latter case, whether or not the electric field is circularly polarized. If it is, one can also determine the sense of the polarization. If the electric field has components both parallel and perpendicular to  $\vec{B}$ , some mixture of the patterns (a) through (d) is obtained; an example, pattern (e), is shown for the electric field random in direction.

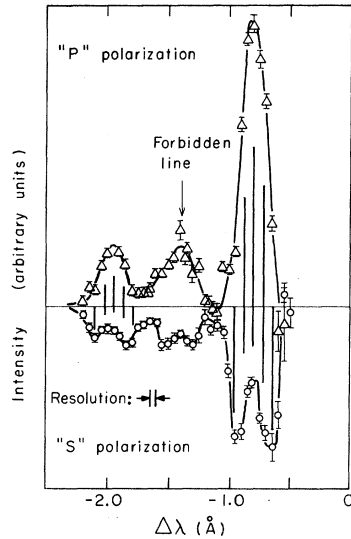


FIG. 2. Measured (triangles and circles) and calculated (vertical solid lines) Zeeman pattern of the satellites produced by high-frequency electric fields generated in a beam-plasma interaction, observed normal to  $\vec{B}$ . The horizontal line represents zero intensity; "P" components are shown above this line and "S" components below it.  $\Delta\lambda$  is the separation from the center of the allowed 4922-Å He-I line.

As an example of the application of this technique to plasma diagnostics, we show in Fig. 2 an experimentally measured Zeeman pattern of the satellites of the 4922-Å line of He I. The satellites were produced by high-frequency electric fields generated by a strong plasma instability. We produced the plasma by shooting a 4-keV, 24-mA, 1-mm-diam electron beam along a 7-kG magnetic field into a chamber containing helium at a pressure of 0.2 Torr. The beam initially creates a plasma by collisional ionization, then interacts with this plasma and maintains it by a beam-plasma interaction that we believe is similar to one observed by Seidl and Šunka.<sup>4</sup>

Light radiated perpendicular to the magnetic field was spectrally resolved with a resolution of about 0.05 Å by a Fabry-Perot interferometer and a grating monochromator in tandem. At the point of observation the beam had penetrated 2.8 cm into the chamber; this was the point of maximum light emission. The electron beam, and therefore also the instability, was switched on and off at a rate of 100 kHz. By using gated scalars to count the pulses from the photomultiplier that detected the light, we were able to make time-resolved (1- $\mu$ sec gate width) measurements of the 4922-Å He-I line profile just

before the beam was turned off (instability present) and just after the beam was turned off (instability absent). By subtraction of the "beam-off" signal from the "beam-on" signal we were then able to discriminate against the interfering wing of the allowed line and to make a differential measurement of the intensities of the satellites. This difference in the two normalized counting rates is what is shown, as a function of  $\Delta\lambda$  (the separation from the center of the allowed line), as the measured data in Fig. 2. The error bars shown represent statistical counting errors.

A comparison of the measured Zeeman pattern of the satellites with the patterns shown in Fig. 1 indicated that the electric fields associated with this beam-plasma interaction have components both parallel to and perpendicular to the magnetic field. The time-averaged direction of the electric field appears to be random. Once the Zeeman pattern was identified, the frequency of the electric field could be determined; in this case it was about 71 GHz, which was 3.6 times the electron gyrofrequency, and was comparable with the electron plasma frequency in the region of the beam. We estimated the electron density from the radial dependence of the shape of the 4922-Å line profile to be between  $4 \times 10^{13}$  and  $10^{14}$  cm<sup>-3</sup> on axis; the corresponding electron plasma frequencies are 57 and 90 GHz, which bracket the measured frequency.

We also made measurements at several wavelengths of the intensity variation across the beam; these data, after "Abel inversion," yielded the intensities of the stronger satellite and of the allowed line as functions of radius. The radial dependence of the strength of the high-frequency electric field associated with the instability then followed from the perturbation calculations outlined in Ref. 2. These measurements indicated that the high-frequency electric fields were confined in radius to the immediate vicinity of the beam and reached an rms value of about 2 kV/cm.

The vertical solid lines in Fig. 2 show the results of machine calculations of the satellite pattern expected for the parameters quoted above:  $E_{rms} = 2$  kV/cm,  $|\vec{B}| = 7$  kG, the field frequency = 70.5 GHz, the direction of  $\vec{E}$  random, and the direction of observation normal to  $\vec{B}$ . We used a computer to solve the time-dependent Schrödinger equation by a method outlined in Ref. 3. The calculated intensities have been suitably scaled for comparison with the measurements.

The measured data shown in Fig. 2 also clearly indicate a low-frequency component of the electric field, which produced the signal labeled "forbidden line" (the frequency is so low that the two satellite patterns have merged and are not resolved). This field, which is also random in direction, is probably the quasistatic field of the ions, observable in this differential measurement because the ion density is higher when the beam is on than when the beam is off. The presence of this signal illustrates the unique ability of this diagnostic method to determine the frequency spectrum of electric fields in a plasma.

Several precautions in the use of this diagnostic technique must be observed. First, it is clear from Fig. 1 that some patterns are displaced from the positions the satellites would occupy if there were no magnetic field. If the field frequency and the Larmor frequency are comparable, serious errors in using the high-frequency Stark effect to determine the frequency of the electric field can result unless the direction and polarization of the electric field are first determined from the Zeeman pattern. For instance, the field frequency as deduced from a simple measurement of the apparent separation of the satellites observed in unpolarized light would be in error by about 1.5 times the Larmor frequency if the electric field were circularly polarized. Second, if the magnetic splitting is comparable with the separation of the interacting upper levels, the intensities and positions (Stark shifts) of the satellites depend on the magnetic field strength, and any theory used to derive mea-

surements of the electric field strength must correctly include the effect of the magnetic field. Finally, since few plasmas are near thermal equilibrium, the assumption that all initial states of a transition are equally probable may break down. This would produce changes in the intensity of components of a satellite Zeeman pattern, but not in their positions or polarization.

In related but different beam-plasma interactions observed under different conditions in the same experiment, we have observed satellite Zeeman patterns resembling Figs. 1(a) and 1(d); we have also learned that patterns similar to Fig. 1(b), indicating that the electric and magnetic fields are perpendicular, have been observed under conditions of strong ion heating in the "Burnout V" experiment at Oak Ridge National Laboratory.<sup>6</sup>

We thank Mr. E. B. Hewitt for his invaluable assistance in this experiment.

---

\*Work done under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup>M. Baranger and B. Mozer, *Phys. Rev.* **123**, 25 (1961).

<sup>2</sup>W. S. Cooper, III, and H. Ringler, *Phys. Rev.* **179**, 226 (1969).

<sup>3</sup>W. S. Cooper, III, and W. W. Hicks, University of California, Lawrence Radiation Laboratory Report No. UCRL-19728, 1970 (unpublished).

<sup>4</sup>M. Seidl and P. Šunka, *Nucl. Fusion* **7**, 237 (1967).

<sup>5</sup>We thank Dr. John G. Conway for lending us this instrument.

<sup>6</sup>R. V. Neidigh and F. R. Scott, private communication.