Notice that the time scale (29) is comparable with the rapid skin-penetration time of the B_θ field. Our analysis therefore does not really depend on having reached the assumed steady state, but should be valid any time after the B_θ penetration has occurred.

*Work supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-3927.

[1]T. E. Stringer, Phys. Rev. Lett. 22, 770 (1969).

[2]M. N. Rosenbluth and A. J. Taylor, Phys. Rev. Lett. 23, 367 (1969).

[3]N. K. Winsor, J. L. Johnson, and J. M. Dawson, "A Numerical Model for Toroidal Plasma Containment with Flow" (to be published); J. M. Greene, J. L. Johnson, K. E. Weimer, and N. K. Winsor, to be published.

[4]R. D. Hazeltine, E. P. Lee, and M. N. Rosenbluth, "Resistive Plasma Rotation and Shock Formation in Toroidal Geometry" (to be published).

[5]T. E. Stringer, private communication.

[6]A. A. Galeev, private communication.

[7]H. Grad and J. Hogan, Phys. Rev. Lett. 24, 1337 (1970). The same solution was found in 1965 by F. Maschke and published in an internal report of Commissariat à l'Energie Atomique, Fontenay-aux-Roses, France.

[8]J. M. Greene, J. L. Johnson, and K. E. Weimer, unpublished.

INTERPRETATION OF AN EXPERIMENT ON THE ANOMALOUS ABSORPTION OF AN ELECTROMAGNETIC WAVE IN A PLASMA*

P. Kaw, E. Valeo, and J. M. Dawson
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 26 June 1970)

In this Letter we propose an interpretation of a recent experiment on the anomalous absorption of a powerful electromagnetic wave in a collisionless plasma. The electromagnetic wave excites an instability, driving up the ion-density fluctuations. The ion waves scatter energy efficiently from the electromagnetic wave into longitudinal plasma modes which are then absorbed in the plasma.

Recently, Gekker and Sizukhin[1] carried out an experiment demonstrating anomalous absorption of a powerful electromagnetic wave in a collisionless plasma. A schematic of their experimental arrangement is shown in Fig. 1(a). Powerful traveling electromagnetic waves in the H_{11} mode were set up in a cylindrical waveguide, AB; the waves were launched at A and absorbed at B by a matched impedance. A plasma stream was injected at B inward along the axis.[2] The leading edge of the plasma had a density profile, presumably of a type similar to that shown in Fig. 1(b). The experiment consisted in measuring the reflection coefficient as a function of the intensity of the incident electromagnetic wave. When the intensity exceeded a certain critical value, the reflection coefficient dropped from 100% to almost zero. Since the incident frequency was less than the peak plasma frequency, the transmission coefficient was negligible[1]; thus, most of the incident energy was absorbed in the plasma. Gekker and Sizukhin[1] suggest that "this phenomenon is apparently connected with the excitation of an instability that leads to plasma heating"; however, they did not even speculate on the nature of the instability. In this Letter we offer a possible interpretation of this experiment.

Our interpretation of the experiment is as follows: The incident electromagnetic wave travels almost absorption-free until it reaches a region where the incident frequency ω is close to the local plasma frequency. When the incident wave is weak, there is a perfect reflection of the wave from this region. However, if the electric field strength of the incident wave is sufficiently large, it excites instabilities in a thin layer in this re-

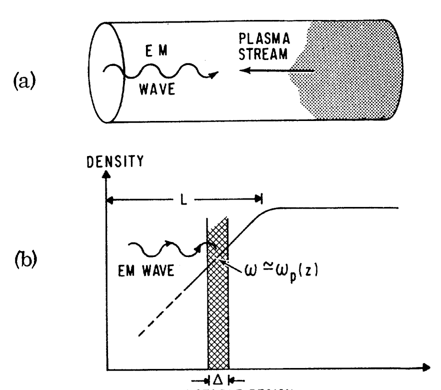

FIG. 1. (a) Schematic of the experimental arrangement used by Gekker and Sizukhin. (b) Density profile of the leading edge of the plasma in the experiment; $L \simeq 10^2$ cm, $\Delta \simeq 10$ cm.

gion [see Fig. 1(b)].[3] The most likely instabilities are the two parametric instabilities of the hydrodynamic type, discussed in detail recently by several people.[4-8] These instabilities lead to strong ion-density fluctuations and a considerable enhancement of the high-frequency resistivity around the plasma frequency.[7-9] (A demonstration of this effect, due to one of the two instabilities, together with the resultant enhanced heating of a plasma—in a computer simulation experiment—was recently reported.[10]) The result of the instability, therefore, is that there exists a strong anomalous absorption of the electromagnetic wave in the unstable region. Thus, if the electric field amplitude is sufficiently large, the reflection coefficient drops to zero. It is of interest to note that this method of anomalous absorption is particularly efficient because the wave is spending a lot of time in the unstable region (its group velocity is small when $\omega \approx \omega_p$).

The reflection coefficient of a plane electromagnetic wave normally incident on an inhomogenous overdense plasma can be readily worked out, and in the approximation of geometric optics is given by[11]

$$|R|^2 = \exp(-K_T), \tag{1}$$

where

$$K_T = 2\int_\Delta \frac{\nu_a(z)\omega_p^2(z)dz}{c\omega[\omega^2-\omega_p^2(z)]^{1/2}}; \tag{2}$$

ν_a is the effective collision frequency, the integration is carried out over the width Δ of the unstable region, and the rest of the symbols have obvious meanings. Assuming that the density variation in the unstable region is linear—i.e.,

$$\omega_p^2(z) = \omega^2(z/L),$$

where L is the typical scale length of density variation—one can carry out the integration in Eq. (2). We will show later that for the experiment, $\Delta/L \ll 1$; in this case, we find

$$K_T \simeq 4(\nu_{a0}/c)(L\Delta)^{1/2}. \tag{3}$$

Let us now make an estimate of the anomalous collision frequency due to the two parametric instabilities. The oscillating two-streaming instability (in which a purely growing ion mode and a high-frequency electron plasma mode both grow) is excited when the incident electric field strength exceeds a threshold value given by[6-8]

$$\frac{eE_0}{m\omega_p V_T} \simeq \sqrt{8}\left(1+\frac{T_i}{T_e}\right)^{1/2}\left(\frac{\nu}{\omega_p}\right)^{1/2}, \tag{4}$$

where V_T is the electron thermal velocity, ν is the electron collision frequency, and the rest of the symbols have their usual meanings. For the parametric ion-acoustic instability (in which an ion-acoustic wave and an electron plasma wave both grow), the thresholds are lower when $T_e \gg T_i$. However, if $T_e \approx T_i$ (which is quite likely for the type of spark plasma used in the experiment), the threshold is again given by Eq. (4) within factors of the order of unity.[12] Since the normal electron collision frequency in the experiment is quite small, the threshold field given by Eq. (4) is readily exceeded and the plasma becomes unstable. As the ion modes grow, the ions become strongly correlated and begin very efficiently to scatter energy from the external electromagnetic wave into longitudinal plasma modes, which then are absorbed in the plasma (mainly due to Landau damping by electrons). This enhanced energy absorption can be interpreted as an enhanced collision frequency, ν_a. Thus, one might expect the instability to keep on growing until the anomalous collision frequency ν_a becomes so large that E_0 no longer exceeds the modified threshold field. At this point the instability will shut itself off; one may say, therefore, that the anomalous collision frequency is given by

$$\nu_a \simeq \omega_p(eE_0/4m\omega_p V_T)^2, \tag{5}$$

where we have taken $T_i \simeq T_e$. This equation cannot be trusted for values of ν_a/ω_p greater than a few tenths since one would not expect ν_a to increase indefinitely (this was also noted in the computer simulation experiment[10]).

Combining Eqs. (1), (3), and (5), one obtains

$$|R|^2 = \exp\left[-\zeta\frac{\omega_p}{c}(L\Delta)^{1/2}\left(\frac{eE_0}{2m\omega_p V_T}\right)^2\right], \tag{6}$$

where we have introduced a parameter ζ of order unity to take care of the fact that all the expressions we have derived are only order-of-magnitude expressions.

We will now make estimates of Δ, the width of the unstable region, and E_0, the internal field in the plasma. It is quite well known that when an electromagnetic wave is incident on a linear layer, the superposition of the incident and reflected waves leads to a standing-wave pattern with a large maximum near the point $\omega \approx \omega_p$. The width of this maximum gives a rough estimate of the width of the unstable region, and is given by[13]

$$\Delta \simeq (c^2L/\omega^2)^{1/3}. \tag{7}$$

A better estimate may be made by letting the width of the resonance around ω_p determine the width of the unstable region; further, Δ in all probability will be a function of E_0, increasing with greater field strengths. We will put all of these factors, which are typically of order unity, into the parameter ζ introduced in Eq. (6). The internal field at the first maximum is greater than the vacuum field by an amount[13]

$$E_0/E_V \simeq (\omega L/c)^{1/6}. \tag{8}$$

Using (7) and (8), Eq. (6) takes the form

$$|R|^2 = \exp\left[-\zeta\left(\frac{\omega L}{c}\right)\left(\frac{eE_V}{2m\omega V_T}\right)^2\right]. \tag{9}$$

Equation (9) is our final expression. The only unknown is the parameter ζ, which is of order unity, and is uncertain because of the simplified analysis used here.[14] From the experiment of Gekker and Sizukhin,[1] we use the following data: $\omega \simeq 2\times10^{10}$ sec^{-1}, $V_T \simeq 1.2\times10^8$ cm/sec, $c/\omega \simeq 10/2\pi$ cm, and $L \simeq 10^2$ cm.

Taking $\zeta \simeq \frac{1}{4}$ (to fit the experimental point where the reflection coefficient drops to e^{-1} of its original value), one obtains the theoretical curve shown in Fig. 2. The curve has not been extended to higher field strengths since ν_a/ω_p starts exceeding a few tenths and hence our Eq. (5) is suspect. The agreement in the moderately strong field regions is reasonably good. For very strong fields, our qualitative arguments about the saturation of ν_a suggest a saturation in the absorption; the experiment, however, indicates that the absorption keeps on increasing. This may be due to the fact that the width of the unstable region also keeps increasing.

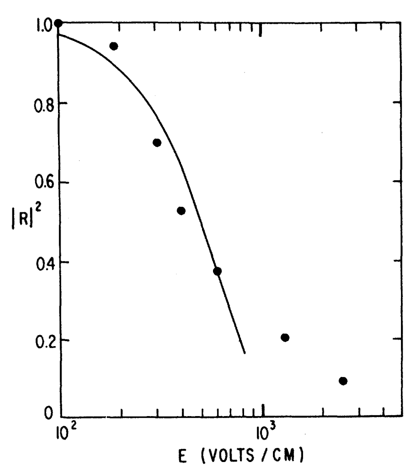

FIG. 2. Variation of the reflection coefficient with the intensity of incident electric field strength. Solid curve, theory; closed circles, experimental points.

One might question the use of the infinite homogeneous plasma theory of the instability for our inhomogeneous situation. It should be pointed out, however, that the experiment satisfies the following inequalities: $L(\sim10^2\text{ cm}) \gg \lambda_{fs}$, $\Delta(\sim10\text{ cm}) \gg \lambda_D(\sim10^{-3}\text{ cm})$, where λ_{fs} is the free-space wavelength and λ_D the Debye length. Thus one can excite ion waves with $\lambda \gg \lambda_D$ but still short compared with λ_{fs}, L, or Δ; this also justifies the use of the dipole approximation for the instability calculation. It is also of interest to note that the maximum growth rate of the instability is $\gamma \sim (m/M)^{1/3}\omega_p \sim 10^8$ sec^{-1}; this shows that the waves have sufficient time to grow quite a bit before they convect out of the unstable region ($t_c \sim \Delta/V_{thi} \sim 10^{-6}$) or before the incident electromagnetic wave is switched off ($\sim10^{-6}$ sec).

Considering the uncertainties in both theory and experiment, the agreement obtained is remarkably good. Given more data, it may be advantageous to plot $\ln|R|^2$ vs $|E_V|^2$ and check the linearity of this relation suggested by Eq. (6). One can also experimentally confirm the dependence of the critical electric field for the reflection coefficient falloff[15] on the frequency, temperature, and scale length. Finally, one may be able to probe the plasma and study (a) the ion-density fluctuations that are critical to the proposed anomalous absorption mechanism, and (b) the production of hot electrons in the tail of the electron velocity distribution, which is a direct consequence of the proposed mechanism.[10]

We are very grateful to Dr. J. McBride and Dr. C. Oberman for their comments.

*Work performed under the auspices of the U. S. Atomic Energy Commission, Contract No. AT(30-1)-1238.

[1]I. R. Gekker and O. V. Sizukhin, Pis'ma Zh. Eksp. Teor. Fiz. 9, 408 (1969) [JETP Lett. 9, 243 (1969)].

[2]The streaming velocity of the plasma was 10^7 cm/sec. The Doppler shift in the frequency that the plasma sees is extremely small and will be ignored.

[3]The unstable layer is thin only when $V_E = eE_0/m\omega < V_T$; when the field strength of the incident wave is so large that $V_E \gtrsim V_T$, the entire region before the point $\omega \approx \omega_p$ may become unstable.

[4]V. P. Silin, Zh. Eksp. Teor. Fiz. 48, 1679 (1965) [Sov. Phys. JETP 21, 1127 (1965)].

[5]D. F. Dubois and M. V. Goldman, Phys. Rev. Lett. 14, 544 (1965); Phys. Rev. 164, 207 (1967).

[6]K. Nishikawa, J. Phys. Soc. Jap. 24, 916, 1152 (1968).

[7]P. K. Kaw and J. M. Dawson, Phys. Fluids 12, 2586 (1969).

[8]J. R. Sanmartin, to be published.

[9]J. M. Dawson and C. Oberman, Phys. Fluids 5, 517 (1962), and *ibid.* 6, 394 (1963).

[10]W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. Oberman, Phys. Rev. Lett. 24, 987 (1970). Recent computer experiments have shown that ν_a may have a weak dependence on the electron-ion mass ratio. The simple estimate of ν_a given in Eq. (5) does not show this behavior.

[11]J. M. Dawson, P. K. Kaw, and B. Green, Phys. Fluids 12, 875 (1969).

[12]This can be verified from Nishikawa's Eq. (I. 54), if one replaces the ion wave damping coefficient Γ_1 by the ion acoustic frequency ω_1 (for $T_e \simeq T_i$).

[13]V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas* (Addison-Wesley, Reading, Mass., 1964), pp. 364-6.

[14]The finite size of the waveguide the detailed form of the wave fields, uncertainties in density, temperature, etc. will also contribute to ζ.

[15]Notice that the critical field for the reflection coefficient falloff is higher than the threshold field required for the excitation of the instability. This is because the anomalous collision frequency has to come up to a sufficiently high level before one sees the effect on the reflection coefficient.

SPECTROSCOPIC MEASUREMENT OF THE FREQUENCY, INTENSITY, AND DIRECTION OF ELECTRIC FIELDS IN A BEAM-PLASMA INTERACTION BY THE HIGH-FREQUENCY STARK-ZEEMAN EFFECT*

William S. Cooper, III, and Roger A. Hess
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
(Received 3 June 1970)

We show how the Zeeman pattern of the "satellites" of spectral lines produced by the high-frequency Stark effect can be used to determine the direction of an oscillating electric field relative to the magnetic field. We illustrate the use of this effect as a diagnostic technique in plasma physics by observations of the Zeeman pattern of the "plasma satellites" of the 4922-Å He-I line produced by high-frequency electric fields in a beam-plasma interaction.

The high-frequency Stark effect is a powerful spectroscopic technique for studying strong oscillating electric fields in plasmas.[1] Oscillating electric fields induce multiple quantum transitions which produce new spectral lines, "satellites" of normally forbidden or allowed lines. It is possible to determine the frequency of the perturbing electric field from the spacing of these satellites, and the strength of the field from measurements of the intensities of the satellites relative to the intensity of a nearby allowed line, or from the observed Stark shifts of the lines.[2]

In most cases in which one would like to use this method to study oscillating electric fields in a plasma, the plasma is permeated by a magnetic field. The presence of the magnetic field complicates the theoretical calculations, but it does introduce one valuable simplification in the data analysis. In the absence of a magnetic field, one can only determine the direction of the electric field from measurements of the slight polarization of the satellites.[2] In the presence of a magnetic field, however, one can determine the direction of the electric field relative to the magnetic field very simply by inspection of the Zeeman pattern of the satellites. In this Letter we report the application of this new diagnostic technique to a beam-plasma interaction.

To calculate the high-frequency Stark effect in the presence of a constant magnetic field, one must solve the time-dependent Schrödinger equation with a Hamiltonian containing terms describing the interaction of an oscillating electric field and a static magnetic field with the excited atom. This can be done by including a magnetic interaction term within the framework of the calculations described by Cooper and Hicks.[3] We have done this and will report on it in a later publication.

If the electric fields are not too strong, such detailed calculations are not necessary since only two satellites are observed in this case. As discussed in Ref. 2 and in earlier publications cited therein, these satellites may be considered to be produced by the following two-quantum process: An excited atom in excited state i either absorbs one quantum from the electric field or emits one quantum to the field by an electric dipole transition and goes to an intermediate virtual state. This virtual state then decays to the final state k by a second electric dipole transition, which results in the emission of an opti cal photon. A direct electric dipole transition