TWO-FIREBALL PHENOMENON AND THE MULTIPERIPHERAL MODEL*

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In the context of the Chew-Snider version of the Amati-Bertocchi-Fubini-Stanghellini-Tonin pion-exchange model, we investigate certain multiperipheral mechanisms that could account for those phenomena that have hitherto motivated the two-fireball model. These mechanisms are (1) double diffractive dissociation, (2) the presence of a sequence of neutral particles on the multiperipheral chain, and (3) statistical fluctuations in the $\log\tan\theta$ spectrum. At cosmic-ray energies they are of equal importance.

The two-fireball model was proposed by various authors[1] in the late 1950's to explain a remarkable property of cosmic-ray events observed in photographic emulsions, namely, that the spectrum of secondary particles for some events, when presented in $\log\tan\theta_{\mathrm{lab}}$, appeared to have large gaps, separating the secondary particles into "forward" and "backward" clusters. According to the model, secondary particles were thought to arise from two well-defined centers, one moving in the projectile direction and one in the target direction in the center of mass.

In interpreting these data in the context of the multiperipheral model, we shall stress what appears to us to be the best-established empirical feature, namely, the frequent presence of substantial gaps in the $\log\tan\theta_{\mathrm{lab}}$ spectrum. We call this the "fireball effect." The decay distribution of the "fireballs," i.e., the forward and backward clusters, is empirically less well understood, and it is here that the multiperipheral model is at variance with the two-fireball model. (In our model the fireball is not a well-defined entity. Its average mass increases without bound as the overall energy increases.)

At first sight the fireball effect would seem to contradict predictions of the multiperipheral model. Consider a typical multiperipheral event described by the diagram in Fig. 1. All the two-particle subenergies[2] s_{ij} and momentum transfers t_i are assumed to be small compared with the overall energy. The final particles all have small transverse momenta $p_\perp$, and their longitudinal momenta in the laboratory frame $p_\parallel$ are arranged in an approximately sequential order, more or less uniformly spaced in the variable $\ln p_\parallel$.[3] (Assuming that all particles have the same $p_\perp$, the uniform distribution in $\ln p_\parallel$ transforms into a uniform distribution in $\log\tan\theta$.)

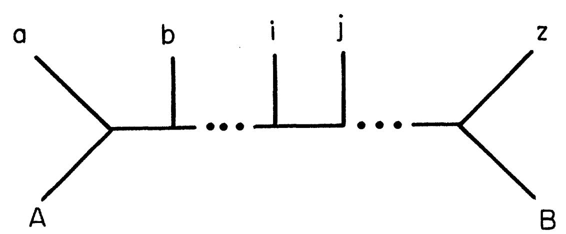

FIG. 1. Multiperipheral diagram for the process $A+B \rightarrow a+b+\cdots+z$.

However, when the model permits a broad distribution in the subenergies so that a few can greatly exceed the average, individual events can deviate markedly from the above description. A large two-particle subenergy s_{ij} will produce a "gap" in the $p_\parallel$ distribution between p_i and p_j, dividing the momenta into two groups, thereby producing a "fireball-type event." In the Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) multiperipheral model,[4] the two-particle subenergy distribution is proportional to the $\pi\pi$ elastic cross section which is dominated by resonances (chiefly the ρ), but which also has a small "high-energy tail" from Pomeranchukon exchange, extending to quite high energies. Thus there is a small probability that an abnormally large subenergy (hence a gap) will occur. This is the multiperipheral mechanism for double diffractive dissociation.[5] A model employing this effect has been studied for other purposes by Chew and Snider[6]; we will use their model as a basis for making quantitative predictions.

If one observes only charged secondaries, a sequence of neutral particles on the multiperipheral chain can also produce a "gap" in the momentum distribution. This is the second source of fireball-type events in the multiperipheral model.

Finally, for experiments in which only angles are measured and not momenta, there exists yet a third mechanism for producing gaps[7] in $\log\tan\theta$. Fluctuations in the transverse momentum $p_\perp$ can produce gaps in $\log\tan\theta$, even for a regular distribution in $\ln p_\parallel$. We will calculate the probability for this effect from the experimental $p_\perp$ distribution.

Before calculating the probability for producing "fireball-type events" from these three mechanisms, we must define our terms.

We offer two definitions of a "two-fireball event." (1) The first relies upon a knowledge of only the direction of the produced secondary, which is usually the only reliable datum in emulsion experiments. If the particle spectrum for an event in the variable $\log\tan\theta_{\rm lab}$ exhibits a gap larger than 1.3, say, then we define it to be a two-fireball event. (2) For experiments (mainly in the future) that measure the momenta of the produced particle as well, we propose a second, more specific definition of a two-fireball event. If the laboratory momenta of an event can be divided into two groups, fast and slow, so that the squared invariant mass of all pairs of particles, one chosen from the fast group and one from the slow group, is greater than some minimum, say 3 GeV^2, then we call the event a two-fireball event with a "gap in momentum space."[8] We will assume that whenever the subenergy of an adjacent pair of particles on the chain exceeds such a large value, a gap in the momentum spectrum practically always results in accordance with our two-fireball criterion. Both definitions can be generalized in an obvious way for n-fireball events.

To obtain a quantitative prediction for the probability of fireball-type events due to the first mechanism, Pomeranchukon (P) exchange, we shall consider the model of the "schizophrenic" Pomeranchukon.[6] In this model the kernel is approximated as a sum of two components, one with strength g_R^4 representing the low-subenergy resonances in the $\pi\pi$ elastic cross section, and the other with strength g_P^4 representing the "high-energy tail" of that cross section.[9] The strength of the high-subenergy part was determined in another paper[5] where the $\pi\pi$ cross section, appearing in the kernel of the ABFST integral equation, was arbitrarily split into the two contributions above the g-meson peak at $s_{ij} = 3$ GeV^2, hence the choice of 3 GeV^2 in our definition of a two-fireball event.

The probability of single P exchange per event, when it is small, is well approximated by the average number of P exchanges per event. The latter is readily calculated by differentiating the logarithm of the total cross section with respect to $\log g_P^4$ just as the average number of pion pairs is found by differentiating with respect to $\log(g_R^4+g_P^4)$. We are excluding the special class of single diffractive-dissociation and elastic-scattering events from the "two-fireball events." These events are associated with P exchange at the very ends of the chain. This exclusion can be accomplished by not differentiating with respect to the g_P^2 factor that corresponds to P exchange at the ends. After some algebra, using the parameters of Chew and Snider, we obtain the result plotted in Fig. 2 for the average number $\bar{n}_P$ of fireballs per event in a $\pi\pi$ collision. Shown also, for comparison, is the average multiplicity $\bar{n}$ of pions, i.e., the average number of pions produced per inelastic collision. It is possible to extend this result to πp and pp collisions by means of a crude scaling law. The plot of $\bar{n}_P$ vs $\bar{n}$ in Fig. 3 is independent of this scale, however, and universal to all reactions in this model. For $\bar{n} \geq 7$, the curve is fitted roughly by the expression

$$\bar{n}_P = 0.10(\bar{n}-6). \qquad (1)$$

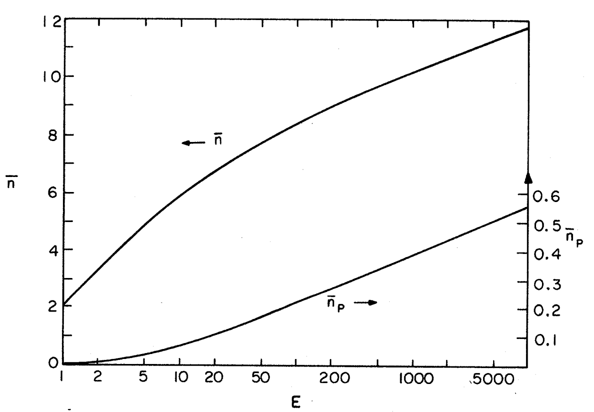

FIG. 2. Right scale, average multiplicity of Pomeranchukon exchange per inelastic event, $\bar{n}_P$; left scale, average (produced) pion multiplicity per inelastic event, $\bar{n}$ versus energy for $\pi\pi$ collisions, as predicted by the schizophrenic Pomeranchukon model (see Ref. 6).

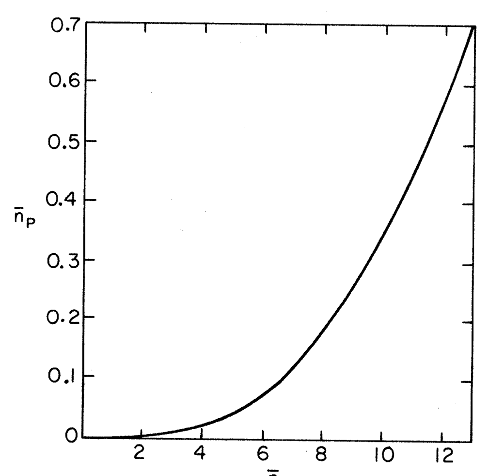

FIG. 3. Average multiplicity of Pomeranchukon exchange per inelastic event versus average (produced) pion multiplicity per inelastic event.

To estimate the frequency of neutral-particle gaps, we first find the average number of sequential neutrals required to produce an apparent "gap" with a 3-GeV^2 "subenergy." To do this, we shall use an approximate expression derived by Chew and Pignotti,[10]

$$s/s_0 \approx \prod_{\text{all energies}} (s_{ij}/c). \tag{2}$$

From the rate at which the average multiplicity of secondaries grows with energy, it is possible to estimate the average value of the ratio s_{ij}/c. Recent preliminary experimental results for secondary multiplicities in pp collisions are fitted with the expression

$$\bar{n} = a \ln s + b \tag{3}$$

for $a = 1.13$.[11] Since from Eq. (2)

$$\ln(s/s_0) \approx \bar{n} \ln(\bar{s}_{ij}/c), \tag{4}$$

we estimate that

$$\ln(\bar{s}_{ij}/c) \approx 1/a = 0.9. \tag{5}$$

Comparison of the Chan-Łoskiewicz-Allison multiperipheral model[12] with experimental data indicates that a typical value of the average subenergy is 0.5 GeV^2.[13] With this value for s_{ij}, we obtain $c \approx 0.2$ GeV^2.

Corresponding to Eq. (2), there is an expression relating the subenergy of two nonadjacent particles i and l to the intervening adjacent-particle subenergies:

$$s_{il}/c \approx (s_{ij}/c) \cdots (s_{kl}/c). \tag{6}$$

Thus when the number of intervening particles is two, the accumulated subenergy s_{il} is already 3 GeV^2 on the average. Hence only two successive neutral particles are required to produce the appearance of a large subenergy between adjacent particles. If it is further assumed that the probability of a neutral particle is one-third per particle and uncorrelated between adjacent particles, then we estimate the average multiplicity of neutral gaps to be $(\frac{1}{3})^2$ times the average multiplicity, i.e.,

$$\bar{n}_{\text{II}} = 0.1(\bar{n} - 3). \tag{7}$$

(At least four produced particles are required to make two fireballs, each containing at least two particles, in addition to two neutral particles; hence the 3.)

These two mechanisms, P exchange and neutral gaps, constitute our model for producing fireball events, according to our second definition in terms of gaps in momentum space. Their combined probability is

$$\bar{n}_{\text{P}} + \bar{n}_{\text{II}} = 0.2\bar{n} - 0.9. \tag{8}$$

When only the angles of the secondaries are measured, one must resort to our first definition in terms of gaps in $\log\tan\theta$. We show below that for particles with average transverse momenta, the two definitions are equivalent. We only need add to the above probability the probability that an average subenergy and abnormal transverse momenta produce a gap in $\log\tan\theta$.

The average spacing in $\log\tan\theta$, i.e., the average value of $\kappa \equiv \log[(p_{\parallel j}/p_{\perp j})(p_{\perp i}/p_{\parallel i})]$ is easily estimated from the expression for the average multiplicity. The total length of the $\log\tan\theta$ plot is $\log s$ + const. Therefore with $\bar{p}_{\perp j} \approx \bar{p}_{\perp i}$, $\bar{n}\bar{\kappa} \approx \log s$ + const. Hence[14]

$$\bar{\kappa} \approx \log\langle p_{\parallel j}/p_{\parallel i}\rangle \approx (\log e)/a = 0.38. \tag{9}$$

To estimate the spacing when $s_{ij} > 3$ GeV^2, we can either use the rule of thumb, derived in the discussion of neutral-particle gaps, that three average gaps equal a P gap, or we can calculate the spacing directly, using the expression for s_{ij} in terms of the momenta, when $p_{\parallel j} \gg p_{\parallel i} \gg p_{\perp i}, p_{\perp j}$,

$$s_{ij} \approx (p_{\parallel j}/p_{\parallel i}) w_i^2, \tag{10}$$

where $w_i = (p_{\perp i}^2 + m_\pi^2)^{1/2}$. Then for $p_\perp \gtrsim m_\pi$,

$$\kappa \approx \log(s_{ij}/w_i w_j). \tag{11}$$

If we use a typical experimental value $\bar{p}_\perp^2 = 0.15$, then when $s_{ij} > 3$ GeV^2, $\kappa \gtrsim 1.3$.

For an average subenergy $s_{ij} \approx 0.5$, the gap exceeds 1.3 only when $w_i w_j \lesssim 0.025$ [cf. Eq. (11)]. Assuming that the distribution in $p_\perp^2$ is approximately uncorrelated with s_{ij} and given by

$$dN/dp_\perp^2 = \exp(-p_\perp^2/0.15), \tag{12}$$

the probability of such an occurrence is about 5% per particle. However, this result is highly sensitive to the subenergy in this range. For $s_{ij} = 0.8$ (a value less likely than 0.5 by a factor of about 2^{13}) the probability is 20%. In order of magnitude the average number of gaps from this source is

$$\bar{n}_{\text{III}} \approx 0.1(\bar{n} - 1). \tag{13}$$

The net probability for gaps in $\log\tan\theta$ is then roughly $0.3\bar{n} - 1.0$ for $\bar{n} \gtrsim 5$.

Although the multiperipheral model predicts that for individual events there may be gaps in the distributions, the combined distribution of many events will not have dips. The gaps occur with equal frequency anywhere along the chain.

We are encouraged by the observation reported by Dobrotin and Slavatinsky[15] that in cosmic ray events at energies in the range 100-1000 GeV a marked asymmetry is observed in the particle distribution in the center of mass for some events. Moreover, the multiperipheral model tends to agree with their observations that some of the secondaries in the "decay" of the fireball have abnormally large energies in the fireball center of mass. These would correspond to the left- and right-most particles in the fireball group on the multiperipheral chain.

Furthermore, our results are not necessarily in disagreement with the detailed statistical analysis of Gierula, Mięsowicz, and Zieliński and of Gierula and Wojner[16] for bimodality in the $\log\tan\theta$ spectrum from emulsion experiments. A positive D test for bimodality is not a positive test for two separate peaks. A trapezoidal distribution gives a positive D test for bimodality.

Because of the attractiveness of the two-fireball idea, many experimenters have hitherto focused their attention on the structure of the fireball clusters and on the separation of the "centers" of the clusters. From the standpoint of the multiperipheral model, however, a frequency distribution of gap sizes would be a useful analytical tool.

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[1]P. Ciok, T. Coghen, J. Gierula, R. Hołyński, A. Jurak, M. Mięsowicz, T. Sanieski, and J. Pernegr, Nuovo Cimento 8, 166 (1958), and 10, 741 (1958); G. Cocconi, Phys. Rev. 111, 1699 (1958); K. Niu, Nuovo Cimento 10, 994 (1958).

[2]The two-particle subenergy is the invariant $s_{ij} = (P_i+P_j)^2$.

[3]For a discussion of this point, see C. E. DeTar, to be published.

[4]L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962).

[5]G. F. Chew, T. Rogers, and D. R. Snider, Univ. of Calif. Lawrence Radiation Laboratory Report No. UCRL-19457, 1970 (unpublished).

[6]G. F. Chew and D. R. Snider, Univ. of Calif. Lawrence Radiation Laboratory Report No. UCRL-19455, 1970 (unpublished).

[7]Czyżewski and Krzywicki have shown that an idealized multiperipheral model which treats final particles *independently* and gives a rectangular distribution in $\log\tan\theta$ can, on the average, through statistical fluctuations account for the experimental data. [See O. Czyżewski and A. Krzywicki, Nuovo Cimento 30, 603 (1963).] We are concerned here with two-particle correlations.

[8]The numbers 1.3 in $\log\tan\theta$ and 3 GeV^2 in subenergy were chosen so as to be roughly compatible when $\bar{p}_\perp \approx 0.4$ GeV/c.

[9]The multiperipheral model of the P. N. Lebedev Institute also allows for the production of fireballs. In their model, however, pion rather than P exchange divides the fireballs. Thus the gaps would presumably be quite small. [See V. N. Akimov, D. S. Chernavskiĭ, I. M. Dremin, and I. I. Roysen, Nucl. Phys. B14, 285 (1969).]

[10]G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[11]L. W. Jones *et al.*, to be published. We thank D. Lyon for supplying us with these data before publication.

[12]H.-M. Chan, J. Łoskiewicz, and W. W. M. Allison, Nuovo Cimento 57A, 93 (1968).

[13]Z. Ajduk, L. Michejda, and W. Wójcik, Acta Phys. Pol. A37, 285 (1970).

[14]In comparing this with emulsion data, it must be kept in mind that multiplicities tend to be higher in collisions with nuclei.

[15]N. A. Dobrotin and S. A. Slavatinsky, in *Proceedings of the Tenth International Conference on Cosmic Rays*, edited by J. R. Prescott *et al.* (Univ. of Calgary, Calgary, Alta., Canada, 1967), Pt. A, p. 416.

[16]J. Gierula, M. Mięsowicz, and P. Zieliński, Nuovo Cimento 18, 102 (1960); J. Gierula and E. Wojner, Institute for Nuclear Research, Warsaw, Report No. 1126/VII/PH (to be published).