RANGE OF VIRTUAL PHOTONS IN DEEP INELASTIC ep SCATTERING*

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The relative importance of the different laboratory distances of interaction of highly virtual photons is discussed in terms of the data on inelastic ep scattering in the scaling region. The definition of a characteristic distance distinguishing between short-range and long-range interactions is given. The magnitude of this quantity is extracted from the data to be about 0.65 F.

The importance of the question of the range of interactions in high-energy processes was recognized several years ago by Gribov, Ioffe, and Pomeranchuk.[1] They conjectured that the dominant distances in such processes might increase linearly with energy. Recently Ioffe[2] has argued that, as observed in inelastic ep scattering,[3] the weak q^2 dependence of the structure function $\nu W_2(q^2, \nu)$ for high ν and $|q^2| \geqslant 0.5$ GeV^2 is incompatible with a solely short-range (i.e., range $\leqslant$ proton diameter) interaction in the laboratory frame.[4] Indeed, according to Ioffe,[2] the data[5] in this region of high ν and relatively small $|q^2|$ indicate the presence of interactions with ranges of the order of several pion Compton wavelengths.

Although Ioffe's idea seems to make sense qualitatively, there are three shortcomings in his analysis that one can point to:

(1) Current conservation requires[6] the presence of light-cone singularities involving derivatives of delta functions in the commutator of two hadronic electromagnetic currents. These were ignored by Ioffe. However, in the limit $\nu \to \infty$, these singularities could introduce additional factors of ν in Ioffe's expressions by virtue of partial integration. These would affect his conclusions seriously. One would have to know the exact form of the dominant light-cone singularities of the structure functions to see whether the argument could be refined.[7]

(2) Ioffe has not made a precise statement on the strength of the long-range interaction of the virtual photon.

(3) It is not possible from his analysis to understand quantitatively the relative importance of the different laboratory distances of interaction in inelastic ep scattering.

In this note we show a way to resolve the above-mentioned difficulties by concentrating on the scaling region of the process $e+p \to e+$anything. This has been possible following the recent demonstration by Jackiw, Van Royen, and West[8] and others[9] that, taking scaling as an empirical fact, the leading light-cone behavior in configuration space that dominates the inelastic structure function $\nu W_2(q^2, \nu)$ is given under certain reasonable theoretical assumptions by a term proportional to $\theta(x^2)\epsilon(x\cdot P)$. Consequently, in the scale region, the usual four-dimensional Fourier integral for $\nu W_2(q^2, \nu)$ in terms of the electromagnetic current commutator in configuration space collapses into a one-dimensional integral on the surface of the light cone. The corresponding integration variable can be interpreted—in the lab frame—as the interaction time or equivalently as the interaction range for the asymptotic process. The unknown function in the integrand which illustrates in a quantitative manner the relative importance of the various distances of ineraction involved can be explicitly obtained in terms of the observed scaling function $F_2(\omega)$.

We start with the spin-averaged, local, and current-conserving decomposition of the electromagnetic current commutator between two identical protons in configuration space:

$$\frac{i}{2\pi}\frac{P_0}{M_p}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle = [g_{\mu\nu}\Box - \partial_\mu\partial_\nu]c_1(x^2, x\cdot P)$$
$$+[P_\mu P_\nu\Box - P\cdot\partial(\partial_\mu P_\nu + \partial_\nu P_\mu) + g_{\mu\nu}(P\cdot\partial)^2]M_\mu^{-1}c_2(x^2, x\cdot P),$$

where $c_{1,2}(x^2, x\cdot P) = -c_{1,2}(x^2, -x\cdot P)$ and $c_{1,2}(x^2, x\cdot P) = 0$ for $x^2 < 0$. The Fourier transforms

$$C_{1,2}(\omega, \nu) = -i\int d^4x\, e^{iq\cdot x} c_{1,2}(x^2, x\cdot P)$$

(with $\omega = -q^2/2M_p\nu$ and $\nu M_p = P\cdot q$) are functions in momentum space that are free of kinematic singularities. These are related to the usual structure functions $W_{1,2}(q^2, \nu)$ by the equations

$$C_1 = (4\omega^2\nu M_p^2)^{-1}(\nu W_2 - 2\omega M_p W_1), \quad C_2 = (2\omega\nu M_p^2)^{-1}W_2.$$

The general structure of the functions $c_{1,2}(x^2, x\cdot P)$ in terms of their light-cone singularities has been taken by Jackiw, Van Royen, and West[8] on the basis of locality and causality to be of the form

$$c_{1,2}(x^2, x\cdot P) = \epsilon(x\cdot P)[\theta(x^2)f_{1,2}(x^2, x\cdot P) + \sum_{n=0}^{n} \delta^{(n)}(x^2)s_{1,2}{}^{(n)}(x\cdot P)], \tag{1}$$

where $\delta^{(n)}(x^2) = [d^n/d(x^2)^n]\delta(x^2)$, and the functions f and s are even in $x\cdot P$. Furthermore, the Fourier transforms of $f_{1,2}(x^2, x\cdot P)$ and $s_{1,2}{}^{(n)}(x\cdot P)$ in the one-dimensional variable $x\cdot P$ have finite support [this is due to the mass spectrum condition $q\cdot P \geq \frac{1}{2}(-q^2)$ and can be shown using the Deser-Gilbert-Sudarshan representation]. Assuming (a) that $f_{1,2}(x^2, x\cdot P)$ and $s_{1,2}{}^{(n)}(x\cdot P)$ are sufficiently regular and uniform for various nonrigorous manipulations (e.g., interchange of limit and integration), and (b) that the functions $C_{1,2}(q^2, \nu)$ do not have singularities like $\epsilon(q\cdot P)\delta(q^2)$ (these could be interpreted as coming from massless particles which are not present in the problem), the authors of Ref. 8 have shown that nonzero limits of W_1 and νW_2, when $\nu\to\infty$ with ω fixed, can exist when the leading light-cone singularity of

$$\begin{Bmatrix} c_1(x^2, x\cdot P) \\ c_2(x^2, x\cdot P) \end{Bmatrix} \text{ is proportional to } \begin{Bmatrix} \epsilon(x\cdot P)\delta(x^2) \\ \epsilon(x\cdot P)\theta(x^2) \end{Bmatrix}.$$

In other words, the presence of singularities such as $\delta^{(n)}(x^2)$ $(n \geq 0)$ in $c_2(x^2, x\cdot P)$, and $\delta^{(m)}(x^2)$ $(m\geq 1)$ in $c_1(x^2, x\cdot P)$ would be incompatible with the observed scaling of the structure functions $W_1(q^2, \nu)$ and $\nu W_2(q^2, \nu)$, respectively. With this result, the authors of Ref. 8 have been able to write

$$F_2(\omega) = \lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} \nu W_2(q^2, \nu) = -\lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} 2i\omega\nu^2 M_p{}^2 \int d^4x\, e^{iq\cdot x}\epsilon(x\cdot P)\theta(x^2)f_2(x^2, x\cdot P), \tag{2}$$

or

$$F_2(\omega) = 4\pi i\omega \int_{-\infty}^{\infty} d(x\cdot P)e^{i\omega x\cdot P}f_2(x\cdot P)x\cdot P. \tag{3}$$

$f_2(x\cdot P) \equiv f_2(0, x\cdot P)$ and the integral is on the surface of the light cone. Inverting the Fourier transform and using the evenness of $F_2(\omega)$, Jackiw, Van Royen, and West[8] obtained the equation

$$f_2(x\cdot P) = \frac{1}{4\pi^2}\int_0^1 d\omega\, \frac{\sin(\omega x\cdot P)}{\omega x\cdot P}F_2(\omega). \tag{4}$$

Equation (3) gives $F_2(\omega)$ as a one-dimensional integral over all possible interaction times $x_0 \equiv \tau$ (or ranges $|\vec{x}| \equiv R$) in the laboratory frame.[4] The function $f_2(x\cdot P)$ which can be related to the data by Eq. (4) is the function of interest in describing quantitatively the relative importance of the different laboratory distances of the photon's interaction. We can show the following general properties of $f_2(x\cdot P)$ if $F_2(\omega)$ is smooth enough:

$$f_2(0) = (4\pi^2)^{-1}\int_0^1 d\omega\, F_2(\omega), \tag{5}$$

$$[d/d(x\cdot P)]f_2(x\cdot P)|_{x\cdot P=0} = 0, \tag{6}$$

and $df_2(x\cdot P)/d(x\cdot P) < 0$ in the region $0 < x\cdot P < \alpha \simeq 4$ (where α is the first positive solution of the equation $\sin\alpha/\alpha - \cos\alpha = 0$).[10] Moreover, if $F_2(0) \neq 0$, we have

$$\lim_{x\cdot P\to\infty} f_2(x\cdot P) = \frac{1}{8\pi}\frac{F_2(0)}{x\cdot P}. \tag{7}$$

However, if $F_2(\omega)$ goes to zero as ω^n when $\omega\to 0$, and if n is the order of the highest derivative of $F_2(\omega)$ that exists in $0\leq\omega\leq 1$, then $f_2(x\cdot P)$ falls off as $(x\cdot P)^{-(n+1)}$ when $[n]+1\leq n$ and at least as fast as $(x\cdot P)^{-(n+2)}$ when $[n]+1>n$ as $x\cdot P\to\infty$. The precise form of $f_2(x\cdot P)$ has to be obtained by substituting the data (Fig. 1) in Eq. (4).

In determining the form of $f_2(x\cdot P)$, one is faced with the experimentalists' ignorance of the precise behavior of $F_2(\omega)$ as $\omega\to 0$. As shown in Eq. (7), the behavior of $F_2(\omega)$ at $\omega = 0$ governs the nature of $f_2(x\cdot P)$ as $x\cdot P\to\infty$. Now there are

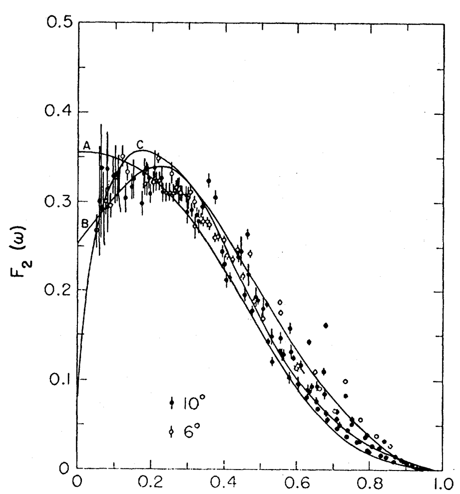

FIG. 1. Fits of the scale function $F_2(\omega)$.

three different possibilities of interest in the behavior of $F_2(\omega)$ as $\omega \to 0$: (A)[11] $F_2(\omega)$ may stay constant at about 0.35 as $\omega \to 0$; (B)[12] $F_2(\omega)$ may go through a maximum and decrease slightly as $\omega \to 0$; and (C)[13] $F_2(\omega)$ may go through a maximum and fall rather rapidly to a very small value or to zero as $\omega \to 0$. We use three different algebraic expressions for $F_2(\omega)$ which adequately fit the present data but correspond, respectively, to the three possibilities mentioned above:

$$F_2(\omega) = (0.07)\{(8.4\omega^2+2)^2+1\}e^{-8.4\omega^2}, \tag{A}$$

$$F_2(\omega) = \frac{1-\omega}{12\omega+4(1-5\omega)(1-\omega)}, \tag{B}$$

$$F_2(\omega) = (0.084)[(1-\omega)^2 - 19.8\omega(\ln\omega + 1-\omega)]. \tag{C}$$

These fits have been drawn in Fig. 1 in comparison with the data and labeled with the letters A, B, and C, respectively. In Fig. 2(a) we have plotted $4\pi^2 f_2(x\cdot P)$ against $x\cdot P$ following Eq. (4) for the three fits. This figure focuses on the short-distance interaction of the virtual photon in the laboratory frame. Figure 2(b) is the same graph as 2(a) on a semilogarithmic plot and shows more clearly the long tail. $f_2(x\cdot P)$ is insensitive to the type of fit.

We can make the following comments on the nature of the curves shown in Figs. 2(a) and 2(b):

(1) The function $f_2(x\cdot P)$ has a long tail and is $\gtrsim 1\,\%$ of its peak value even at $x\cdot P = 100$ or a laboratory distance of interaction $R = 20$ F. For $F_2(0) \neq 0$, this tail is given by $f_2(x\cdot P) \sim (8\pi)^{-1}F_2(0)/x\cdot P$ [vide Eq. (7)] and is presumably diffractive in origin. It corresponds to the data for relatively small $|q^2|$ ($\geqslant 0.5$ GeV^2) and very high ν and is exactly the long-range interaction visualized by Ioffe.[2]

(2) $f_2(x\cdot P)$ falls to half its maximum value and goes through a point of inflection around $x\cdot P \simeq 6$ or in the region of $R \approx 1.3$ F. [This is also the region where the function $x\cdot Pf_2(x\cdot P)$, which is proportional to the Fourier transform of $F_2(\omega)/\omega$, turns out to have a maximum.] The nature of $f_2(x\cdot P)$ for $x\cdot P$ below this value originates from relatively short-range interactions.

(3) Apart from the long tail, the shape of the range function f_2 is reminiscent of the Fermi-like charge distribution in nuclei—perhaps somewhat distorted by the relativistic motion of the constituents. This suggests that for distances $\lesssim 1.3$ F the function f_2 measures the density of hadronic matter inside the proton at rest. The curve has an intercept on the ordinate [$4\pi^2 f_2(0) = \int_0^1 d\omega\, F_2(\omega)$] that is equal to the mean squared

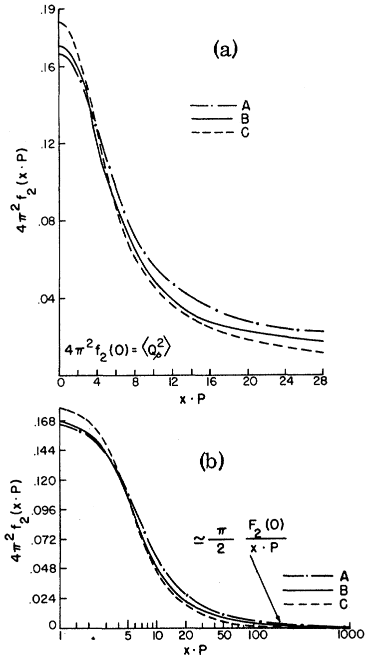

FIG. 2. Range function on (a) linear plot, (b) semilogarithmic plot.

charge of the constituent of the proton.[14]

(4) The variable R refers (after averaging over the angles around the direction of the photon's momentum) to the distance between the point of absorption and the point of emission of the virtual photon in forward virtual Compton scattering in the laboratory frame. Using time symmetry, we can say that on the average the distance between the point of absorption of the photon and the center of the proton target in inelastic ep scattering is going to be $\frac{1}{2}R$. Hence in the lab frame the characteristic distance that separates the regime of long-range interactions [mediated by the hadronic vacuum fluctuations of the photon and illustrated in Figs. 3(a) and 3(c)] from that of short-range interactions [locally between the photon and the constituents of the proton and illustrated in Figs. 3(b) and 3(d)] is $\simeq 1.3$ F in asymptotic forward virtual Compton scattering, and $\simeq 0.65$ F in deep inelastic ep scattering.

(5) We cannot relate this characteristic distance quantitatively to the charge radius of the proton[15] since we are dealing here with a highly relativistic inelastic process. However, in view of our remark (4), we qualitatively expect the two things to be of the same order of magnitude.

We feel that our considerations are of significance on several counts. First of all, it is physically satisfying that the scale function $F_2(\omega)$ in

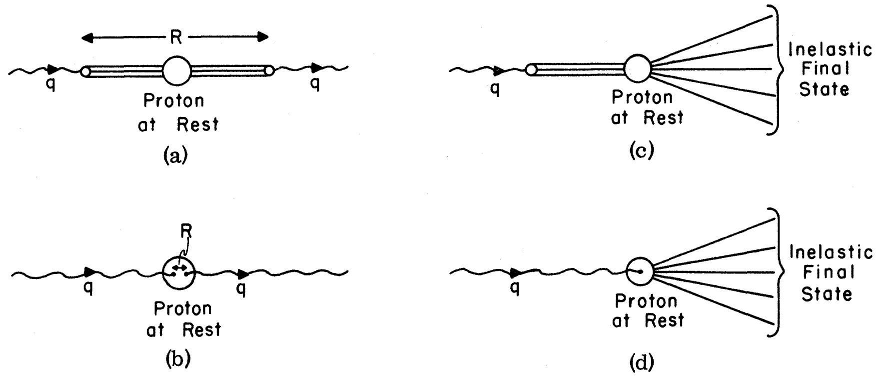

FIG. 3. Virtual photon-proton scattering in the laboratory frame: (a) long-range forward Compton scattering, (b) short-range forward Compton scattering, (c) long-range inelastic *ep* scattering, and (d) short-range inelastic *ep* scattering.

inelastic *ep* scattering can be interpreted as a one-dimensional integral on the surface of the light cone, the variable of integration being the laboratory range of the virtual photon. Secondly, in the laboratory frame the relative importance of both a short-range interaction and a long-range interaction is now quantitatively understood and the shape of the range function is explicitly obtained as a Fermi-like distribution with a long tail. Finally, it is interesting but perhaps not too surprising that the characteristic distance separating these two regions is found to be comparable with the charge radius of the proton.

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[1]V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Yad. Fiz. 2, 768 (1965) [Sov. J. Nucl. Phys. 2, 549 (1966)].

[2]B. L. Ioffe, Phys. Lett. 30B, 123 (1969). See also J. D. Bjorken, in *Eighth Annual Eastern Theoretical Physics Conference, October 10-11, 1969,* edited by F. Rohrlich (Syracuse Univ., Syracuse, N. Y., 1969), p. 225; K. Gottfried, Cornell Univ. Report No. CLNS-87 (unpublished).

[3]Unless otherwise stated, our notations for the inelastic electron scattering parameters are the same as those of J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).

[4]The frame dependence of this statement has to be emphasized. As $\nu \to \infty$, the surface of the light cone is the only region in configuration space that contributes to the Fourier transforms defining the structure functions. Hence in this limit the range of interaction R equals the time of interaction τ. In the infinite-momentum electron-proton center-of-mass frame, for the scale region, τ is much less than the lifetime of virtual constituents of the proton and R is much less than the distances traversed by those states. Thus there is not necessarily any contradiction between the presence of a long laboratory range of the photon and a relatively short-range interaction in the infinite-momentum center-of-mass frame as envisaged in the parton model (*vide* Bjorken and Paschos, Ref. 3).

[5]See R. E. Taylor, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969,* edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 251, and references therein. A more up-to-date report is given by R. E. Taylor, Stanford Linear Accelerator Center Report No. SLAC-PUB-740 (to be published).

[6]M. Keppel-Jones, thesis, Cornell Univ., 1970 (unpublished). Writing the most general form of the commutator in terms of the four-vectors x and P, Keppel-Jones has demonstrated this result explicitly. Note that the required presence of derivatives of δ functions in the commutator does not necessarily imply the presence of such derivatives in $c_{1,2}(x^2, x\cdot P)$.

[7]A refinement of the Ioffe argument is possible given the results of R. Jackiw, R. Van Royen, and G. B. West, Massachusetts Institute of Technology Report No. CTP-118 (to be published). This is because one can write

$$\lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} \nu\omega_2(q^2,\nu) \propto \frac{q^2}{\nu}\int d(x\cdot P)x\cdot P \times \exp\left(-\frac{iq^2x\cdot P}{2\nu M_p}\right) f_2(0, x\cdot P).$$

[8]Jackiw, Van Royen, and West, Ref. 7.

[9]H. Leutwyler and J. Stern, CERN Report No. Th-1138 (to be published); R. Brandt, Phys. Rev. Lett. 23, 1260 (1969), and to be published; D. G. Boulware and L. S. Brown, as cited by Jackiw, Van Royen, and

West, Ref. 7.

[10]We can also show that $[d/d(x\cdot P)]f_2(x\cdot P)<0$ for $x\cdot P>0$ if $F_2(\omega)$ is a monotonically decreasing function.

[11]H. T. Nieh, unpublished.

[12]G. B. West, Phys. Rev. Lett. 24, 1206 (1970).

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RULES FOR CONSTRUCTING DUAL AMPLITUDES*

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The projected propagator of the dual-resonance model is presented. It is used to give an operator proof of duality and to construct various tree and loop operators.

An attractive attitude towards the n-point functions of the dual-resonance model is that they provide the Born terms of a theory of hadrons. Much has already been done to implement this idea. Multiparticle tree graphs have been factorized,[1] thereby yielding the level structure of the model as well as operator expressions for vertices and propagators. All one-loop diagrams have been constructed[2] and renormalized[3,4] and multiloop diagrams have been classified[5] in terms of four primitive loop operators. Despite these achievements, the construction of a complete theory has been impeded by the technical difficulties associated with so-called spurious states.[1,6] Their contributions must be eliminated from the basic operators before a completely dual theory can be formulated. In this paper new forms for the projection operator and the projected propagator are presented. These, together with a vertex operator previously obtained,[7] constitute the complete set of operators required to construct arbitrary dual-resonance diagrams. The projected propagator is used to present an operator proof of duality, to construct multiparticle tree diagrams for arbitrary external states, and to construct simple and useful expressions for the primitive loop operators of the model. These are all the ingredients required to construct any multiloop diagram.[8] Most of the calculational details and some of the basic formulas will be left for a later publication.[9]

The states of the dual-resonance model can be described by vectors in the Hilbert space generated by four-vector creation operators $a_\mu^{\dagger(n)}$, $n=1,2,\cdots$. Some of these states are spurious in that they do not couple to any number of the original on-shell external scalar particles. (These scalar particles are described by the ground state of the Hilbert space.) The spurious states are generated by the operator[6]

$$A^\dagger(-p)=L_0(p)-L_+(p) \tag{1}$$

acting on an arbitrary state, where

$$L_0=R-\tfrac{1}{2}p^2=\sum_{n=1}^{\infty} n a^{\dagger(n)}\cdot a^{(n)}-\tfrac{1}{2}p^2$$

and

$$L_+(p)=L_-^{\dagger}(-p)=p\cdot a^{\dagger(1)}+\sum_{n=1}^{\infty}[n(n+1)]^{1/2}a^{\dagger(n+1)}\cdot a^{(n)}.$$

This is due to the fact that $A(p)$ annihilates any vector in the Hilbert space that describes a tree

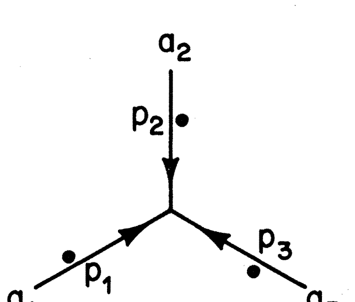

FIG. 1. The symmetric vertex describing the coupling of three arbitrary states.

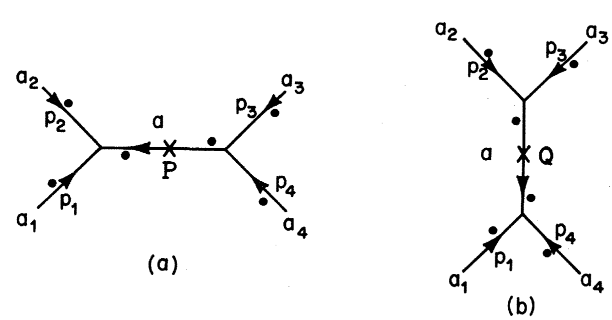

FIG. 2. The four-point operator in two dual configurations.