*gies, Liverpool, England, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), Abstract No. 165 and p. 7. At these conferences, we reported on a preliminary run of this experiment done at lower muon energies. Subsequent analysis indicated a background in the lowest energy bin of about 10% to 20% of the Bethe-Heitler rate, and negligible background in the higher energy bins. This background also varied strongly with peak bremsstrahlung energy. The size of the phi signal then observed agrees quite well with our present result for $C_\varphi$, indicating that the background, while troublesome, did not effect our sensitivity to such a narrow resonance. It should be noted that the branching ratio quoted at those conferences *assumed* the correctness of the production cross sections as quoted in S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968*, edited by J. Prentki and J. Steinberger (CERN) Scientific Information Service, Geneva, Switzerland, 1968), p. 64; G. McClellan *et al.*, Cornell University Laboratory of Nuclear Studies Reports No. CLNS-69 and No. CLNS-70, September 1969 (unpublished), presented by A. Silverman in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 83.

[4]This form is the result of an empirical fit by us to preliminary data on the angular distribution of phi mesons photoproduced from carbon. These data come from an unpublished experiment done by N. Mistry and others at the Cornell synchrotron. The form of the coherent term is consistent with well established results of coherent $\rho$ photoproduction. The incoherent term makes only a small contribution to our results when larger $t$ events are excluded. We thank Dr. Mistry for the use of these data.

[5]P. J. Biggs *et al.*, Phys. Rev. Lett. 24, 1197 (1970).

[6]G. McClellan *et al.*, Phys. Rev. Lett. 22, 377 (1969).

[7]J. E. Augustin *et al.*, Phys. Rev. Lett. 20, 126 (1968).

[8]We have assumed the validity of quantum electrodynamics (QED) for this mass range. This is justified by the results of Ref. 2.

[9]J. Perez-y-Jorba, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 213.

[10]Ting, Ref. 3.

[11]McClellan *et al.*, Ref. 3.

[12]R. Talman and N. Mistry, private communication.

[13]We have spent some time in searching for normalization errors in our experiment. In particular, three independent calculations of our expected $\varphi$ rate using Eq. (1) agreed. We point out that the excellent agreement with the absolute Bethe-Heitler rates indicated in Table I is itself a check on all our normalization factors.

[14]It should be noted that the preliminary measurement of the $\varphi$ photoproduction cross section from hydrogen in Ref. 11 is higher by about a factor of 2 than the value reported by a group at Stanford Linear Accelerator Center [R. Anderson *et al.*, Phys. Rev. D 1, 27 (1970)].

[15]U. Becker *et al.*, Phys. Rev. Lett. 21, 1504 (1968). This gives $B_{\varphi ee} = (3.1 \pm 0.9) \times 10^{-4}$. The large quoted uncertainty in this result makes it compatible both with our results and those of Ref. 7, 10, and 11.

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# DECAY OF SOME HIGH-MASS BOSONS TO $\pi^+\pi^0$ †

S. L. Kramer, H. R. Barton, Jr., L. J. Gutay, S. Lichtman, D. H. Miller, and J. H. Scharenguivel
Department of Physics, Purdue University, Lafayette, Indiana 47907

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We present evidence for three boson structures with masses 1.65, 1.97, and 2.16 GeV/$c^2$ and widths less than 80 MeV/$c^2$ from a study of the reaction $\pi^+p \to p\pi^+\pi^0$ at 13.1 GeV/$c$. These structures are shown to have $I^G = 1^+$, therefore excluding even $J$ assignments. This result favors the interpretation of the 1.97 state as a member of the $A_2$ daughter trajectory.

In this Letter we present evidence for the production of three narrow-width charged boson structures with $I^G = 1^+$ in the reaction

$$\pi^+p \to p\pi^+\pi^0 \qquad (1)$$

at 13.1 GeV/$c$. These states occur in the mass regions of the $R_1$ ($g$), $S$, and $T$ seen with the CERN missing mass spectrometer (CMMS),[1] although significant differences from the $S$ and $T$ are noted below. Henceforth, they will be denoted in this Letter by $R_1$ ($g$), $S'$, and $T'$.

The data presented were obtained from an 8.0-event/$\mu$b exposure of 13.1-GeV/$c$ $\pi^+$ mesons incident in the 82-in. hydrogen bubble chamber, using the rf separated beam from the Stanford linear accelerator. The film was scanned twice for interactions with two charged final-state particles and no associated strange-particle decays. The events found were measured on scanning and measuring projectors and conventional film-plane measuring machines[2] and processed using the TVGP-SQUAW system. Every event included in this analysis has an identifiable proton track which was checked by physicists tobe consistent with ionization estimates from momentum.[3] Elastic events were removed[4] from this sample with a residual contamination estimated to be less than 2%. Reaction (1) was selected by demanding (a) that the neutral missing mass be within the experimental resolution of the $\pi^0$ mass, and (b) that the one-constraint $\pi^0$ hypothesis fit with a confidence level of 3% or better.[5]

The effective-mass spectrum for the $\pi^+\pi^0$ combination of Reaction (1) is presented in Fig. 1(a). Three significant structures are observed having masses above 1.4 GeV/$c^2$ and widths less than 80 MeV/$c^2$. The only other significant structures observed in Reaction (1) are $\rho^+$ (Fig. 1) and $\Delta(1238)^{++}$ production (not shown) which account for 13.6±3% and 13.0±2.8%, respectively, of Reaction (1). The $R_1$ ($g$), $S'$, and $T'$ structures are not associated with $\Delta(1238)^{++}$ production and are produced quite peripherally as indicated in Fig. 1(b), where the open histogram has a $\Delta^{++}$ cut defined by $1.12 \leq M(p\pi^+) \leq 1.32$ GeV/$c^2$ and the shaded histogram has a $t' = |t - t_{\rm min}| \leq 0.3$ (GeV/$c$)$^2$ cut in addition to the $\Delta^{++}$ exclusion. Since the $R_1$ ($g$), $S'$, and $T'$ signals are not associated with any other particular narrow region of the $p\pi^+$ or $p\pi^0$ mass distribution, it is concluded that these structures are not reflections of other resonant combinations.

The possible effects of systematic errors of the beam parameters[6] on these boson structures has been examined. No significant variations in these structures was observed either as a function of vertex position or time of exposure. In addition all of the states shown in Fig. 1(a) have also been observed in the unfitted $\pi^+\pi^0$ spectrum[7] (not shown), with no significant differences between fitted and unfitted values of mass, width, or fraction of the sample.

A fit to the $\pi^+\pi^0$ mass spectrum was attempted using a smooth background plus four Breit-Wigner shapes. The results of our best fit ($\chi^2 = 100.5$ for 75 degrees of freedom) is represented by the solid curve in Fig. 1(a) with the fitted parameters summarized in Table I. The quantity $\Gamma_{\rm obs}$ is the fitted width of the Breit-Wigner shape, and $\Gamma_{\rm phys}$ is the physical width obtained by unfolding the experimental resolution[8] of the $\pi^+\pi^0$ effective-mass combination from the fitted width. The width observed for the $S'$ state is in agreement with the $\pi^+\pi^0$ mass resolution and is therefore presented as an upper limit at a 90% confidence level. The $R_1$ ($g$) and $T'$ are observed to have

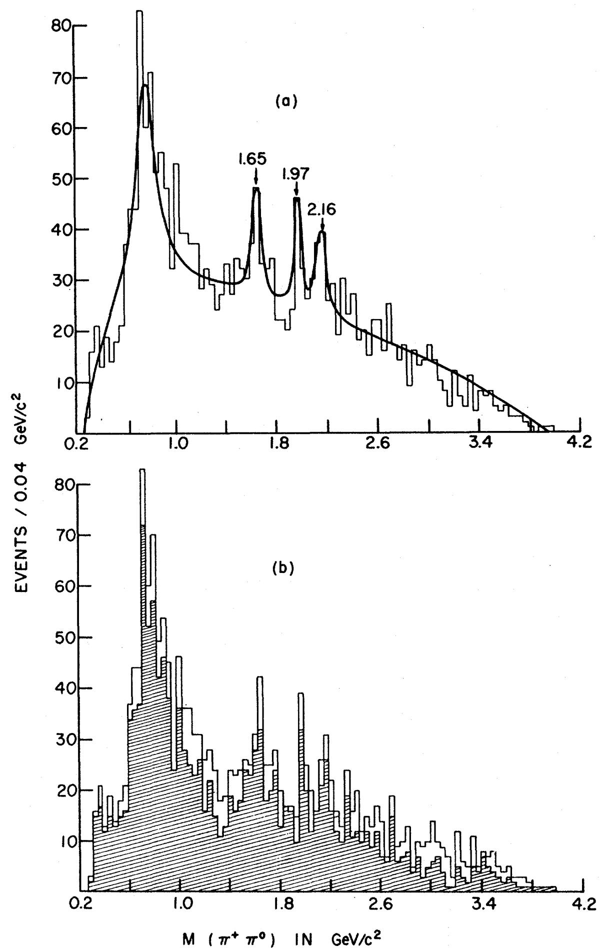


FIG. 1. (a) The $\pi^+\pi^0$ effective-mass spectrum for the total sample of 2165 events. The solid curve represents the fit to this mass spectrum described in the text. (b) The $\pi^+\pi^0$ effective-mass spectrum for events which exclude $\Delta(1238)^{++}$ [$1.12 \leq M(p\pi^+) \leq 1.32$]. The shaded events have $t' = |t - t_{\rm min}| \leq 0.3$, where $t$ is the four-momentum transfer to the proton.

widths ($\Gamma_{\rm phys}$) differing from zero by $1\sigma$ and $3\sigma$, respectively. The cross sections presented in Table I have been corrected for losses due to scanning bias and to the $\pi^0$ selection criteria.

In order to determine the isospin for these states, a preliminary study of the reaction

$$\pi^+p \to \pi^+\pi^+n \tag{2}$$

has been made. Assuming that the $\pi^+\pi^0$ states have $I=2$, $I_z=+1$, then by charge independence one would expect to find a ratio of 4:1 between the $\pi^+\pi^+$ and $\pi^+\pi^0$ signals for these states. The following upper limits (at a 90% confidence level) can be placed on the above ratio:

$$\frac{\pi^+p \to nX^{++} \to n\pi^+\pi^+}{\pi^+p \to pX^+ \to p\pi^+\pi^0} \leq (1.55,\ 0.34,\ 0.49)$$

Table I. The values of the fitted parameters of the $\pi^+\pi^0$ effective-mass spectrum as described in the text. $\Gamma_{obs}$ is the fitted width and $\Gamma_{phys}$ is the physical width obtained by unfolding the resolution from $\Gamma_{obs}$. All quoted errors are statistical.

| State observed | Mass (GeV/$c^2$) | $\Gamma_{obs}$ (GeV/$c^2$) | $\Gamma_{phys}$ (GeV/$c^2$) | Cross section (μb) |
|---|---|---|---|---|
| $\rho^+$ | 0.771 ± 0.025 | 0.118 ± 0.030 | 0.145 ± 0.038 | 54.0 ± 10.3 |
| $R_1{}^+$ | 1.652 ± 0.015 | 0.061 ± 0.020 | 0.040 ± 0.032 | 8.7 ± 2.3 |
| $S'^+$ | 1.975 ± 0.012 | 0.045 ± 0.020 | ≤ 0.052[a] | 6.5 ± 1.8 |
| $T'^+$ | 2.157 ± 0.010 | 0.078 ± 0.018 | 0.068 ± 0.022 | 10.5 ± 3.2 |

[a]At 90% confidence limit.

for the $R_1$ ($g$), $S'$, and $T'$, respectively. It is therefore concluded that the data favor $I=1$ for the $R_1$ ($g$), $S'$, and $T'$ by at least seven standard deviations.

We have attempted to determine the $J^P$ for these states using the method of moments to calculate the Legendre coefficients as a function of $\pi^+\pi^0$ effective mass. Due to the small signals and a high angular-momentum background, a statistically significant determination was not possible. However, we are able to place upper limits on the angular momentum of $J \leq 3$, 4, and 5 (at a 70% confidence level) for the $R_1$ ($g$), $S'$, and $T'$, respectively. Since these states have been shown to have $I^G = 1^+$, even $J$ assignments are excluded. Therefore, we consider only $J^P = 1^-$, $3^-$ for the $R_1$ ($g$) and $S'$, and $J^P = 1^-$, $3^-$, and $5^-$ for the $T'$. In Fig. 2 we present the differential cross sections and the cosine of the $\pi\pi$ scattering angle[9] ($\cos\theta$) distributions for the $R_1$ ($g$), $S'$, and $T'$. The Treiman-Yang angular distributions (not shown) are consistent with isotropy. Although the uncut angular distributions in Figs. 2(d)-2(f) appear to be symmetric, no signal in the $R_1$ ($g$), $S'$, or $T'$ regions is lost when $\Delta(1238)^{++}$ events are excluded. The shaded histograms in Figs. 2(d)-2(f) show the results of this rejection together with a selection on peripherally produced events ($t' = |t - t_{min}| \leq 0.3$ (GeV/$c^2$). The $R_1$ ($g$) remains symmetric, but the $S'$ and $T'$ regions become sharply peaked in the forward direction.

The differential cross sections for the $R_1$ ($g$), $S'$, and $T'$ regions [Figs. 2(a)-2(c)] have been fitted with an exponential function $f(t') = Be^{-At'}$. The values found for the parameter $A$ were 6.2

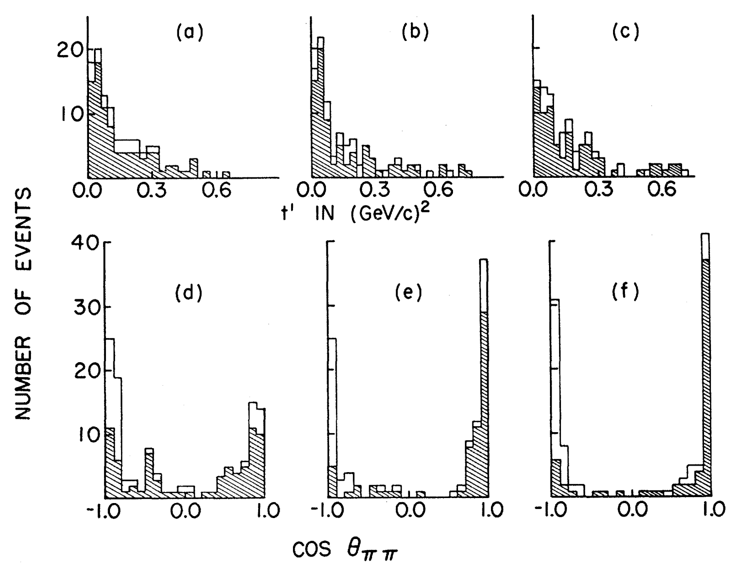


FIG. 2. (a)-(c) Histograms of $t' = |t - t_{min}|$ for events in the $R_1$ [$1.592 \leq M(\pi^+\pi^0) \leq 1.712$ GeV/$c^2$], $S'$ [$1.912 \leq M(\pi^+\pi^0) \leq 2.032$ GeV/$c^2$], and $T'$ [$2.072 \leq M(\pi^+\pi^0) \leq 2.192$ GeV/$c^2$] regions. The shaded events have $\Delta(1238)^{++}$ excluded. (d)-(f) Histograms of the cosine of the $\pi\pi$ scattering angle for the $R_1$, $S'$, and $T'$ regions. The shaded histograms have $\Delta(1238)^{++}$ events excluded together with a $t' \leq 0.3$ (GeV/$c)^2$ cut applied.

$\pm 0.6$, $8.1 \pm 1.4$, and $5.8 \pm 0.7$ $(\mathrm{GeV}/c)^{-2}$ for the $R_1$ $(g)$, $S'$, and $T'$, respectively. The differential cross section for the $S'$ is somewhat steeper than for the $R_1$, $T'$, or the background $[A = 6.1 \pm 0.7\ (\mathrm{GeV}/c)^{-2}]$, suggesting that an exchange mechanism different from that in the $R_1$ and $T'$ regions may be needed to account for the $S'$ state.

The $\pi\pi$ decay mode of the $R_1$ $(g)$ has been well established.[10-14] The mass observed here agrees well with the world average for the $R_1$.[14] Although somewhat narrower than the world average, our width is in agreement with that seen with the CMMS.[1] The most precise $J^P$ determinations[11,12] indicate that $J^P = 3^-$ is favored. This is consistent with the present data and supports the continued association of the $R_1$ $(g)$ with the $J^P = 3^-$ member of the $\rho$ trajectory.[15]

The $S$ also has been seen to have a $\pi\pi$ decay mode[13,16] although of considerably larger width. Our determination of $I^G = 1^+$ for the $S'$ excludes even $J$ assignments, therefore contradicting the current trend[1,15] to associate the $S$ with the $J^P = 4^+$ member of the $A_2$ trajectory. The upper limit on $J$ for the $S'$ found in this experiment favors the association of the $S$ with a daughter of the $A_2$ trajectory.[15] Several other more speculative trajectories are presented by Roos,[15] some of which (IV and first daughter of II in Ref. 15) agree more closely with our mass of the $S'$. However, the possibility of more than one $S$ state is not excluded, especially since a preliminary study of our ratio of one-charged to three-charged particle decay modes[17] (1c/3c) is in considerable disagreement with that observed with the CMMS[1] for the $S$.

The observation of the $T'$ in an $I^G = 1^+$ $\pi\pi$ decay mode, together with the upper limit of $J^P \leqslant 5^-$, is consistent with the association of the $T'$ with the $J^P = 5^-$ member of the $\rho$ trajectory. The observation of an $I^G = 1^-$ state by Kalbfleisch, Strand, and Vanderburg,[18] together with a similar disagreement of our 1c/3c ratio compared with that observed with the CMMS,[1] indicates the possibility of additional states in the $T$ region.

The $U(2375)$ was observed to have a large one-charged-particle decay mode with the CMMS.[1] This result, plus the observation of an $I^G = 1^+$ state at this mass in $\bar{p}p$ experiments,[14,15] suggests that a significant $\pi^+\pi^0$ signal might be expected if $J^P = (\mathrm{odd})^-$. The failure to see any structure in the $U$ region in this experiment is consistent with the even-$J$ prediction from the $A_2$ trajectory.[15]

In conclusion, three narrow-width, $I^G = 1^+$ boson structures have been observed in the $R_1$, $S$, and $T$ mass regions in the $\pi^+\pi^0$ final state, therefore excluding the possibility of $J^P = (\mathrm{even})^+$. This result is consistent with the current interpretation of the $R_1$ and $T$ as Regge recurrences of the $\rho$ meson, but contradicts the association of the $S'$ as a member of the $A_2$ trajectory. The possibility of multiple structures in these mass regions has not been excluded but is in fact suggested by comparison with other experiments.

We wish to thank Dr. R. B. Willmann, Dr. J. A. Gaidos, Dr. J. W. Lamsa, and C. R. Ezell for the opportunity to examine their sample of four-pronged events. We are indebted to the personnel at the Stanford linear accelerator, and especially to Dr. J. J. Murray, for their assistance and cooperation. The diligent efforts of the Purdue measuring staff, particularly Mrs. R. Dauby, are gratefully acknowledged.

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[2]Approximately one tenth of our data was measured on film-plane measuring machines. A study of systematic differences between the two measuring devices has been made. They were found not to affect the results presented in this Letter.

[3]Proton tracks were found to be identifiable for a proton momentum of less than 1.4 GeV/$c$ for nondipping tracks. This places an implied momentum-transfer cut of $\Delta^2 \leqslant 1.45$ $(\mathrm{GeV}/c)^2$ on all data presented in this Letter.

[4]An event was considered elastic if a successful four-constraint fit $[\chi^2(4\mathrm{C}) \leqslant 60]$ was achieved. Approximately 14% of the elastic events failed to attain a successful four-constraint fit because of either measurement problems or small-angle scatters of the outgoing tracks. Such events were identified if they lay within the experimental resolution for at least two characteristics of two-body scattering [e.g., coplanarity, $M(\pi^+\mathrm{MM}^0)^2 = m_{\pi^+}{}^2$, $M(\pi^+p)$ = total center-of-mass energy, etc.]. The remaining elastic contamination is estimated to be less than 2% of Reaction (1) and negligible in all other inelastic channels.

[5]The largest source of contamination to Reaction (1) is due to the $p\pi^+\pi^0\pi^0$ final state. The magnitude and effect of this contamination was examined by fitting $\pi^+p \to p\pi^+\pi^+\pi^-$ events to Reaction (1). This study indicates that the $\pi^0\pi^0$ contamination in Reaction (1) is less than 10%, in agreement with a study of the number of $\gamma$ rays observed to convert in the bubble chamber. The structures observed in Reaction (1) are not associated with this contamination. Contamination due to the final state $pK^+K^0$ plus missing neutrals was studied using events with a visible $K^0$ decay. This contamination is estimated to be less than 1% of Re-

action (1) with no significant boson structures. Another possible source of contamination, which is usually disregarded as small, is that due to the reaction $\pi^+ p \to p\pi^+\gamma$. We are unable to quantitatively estimate this possible contamination, but can find no basis for associating it with the structures observed in Reaction (1).

[6]The momentum bite of the beam is $\Delta p/p = \pm 2.5\%$. However, the dispersion characteristics of the beam provide a momentum resolution of 0.25% full width at half-maximum (FWHM). For additional information on the Stanford Linear Accelerator Center beam refer to S. Flatté, Lawrence Radiation Laboratory Internal Memo No. 664 (unpublished).

[7]The difference between the fitted and unfitted $\pi^+\pi^0$ effective mass was found to have a mean value consistent with zero and a width of less than 15 MeV/$c^2$ FWHM.

[8]The experimental resolution of the $\pi^+\pi^0$ effective mass is primarily determined by the measurement accuracy of the proton track. The error calculation has been checked by comparing it with the observed width of the $\pi^+$ mass in the elastic sample. The resolution was found to vary from approximately 110 MeV/$c^2$ FWHM in the $\rho^+$ region to 40 MeV/$c^2$ FWHM in the $T'$ region. This was found to be relatively insensitive to the uncertainty in the beam momentum. Even so large an error in beam momentum as 1% (see Ref. 6) would result in only an 8-MeV/$c^2$ mass error in the $S'$-$T'$ region.

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[15]M. Roos, Lett. Nuovo Cimento 3, 257 (1970).

[16]C. Caso, paper No. 325, presented to the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968 (unpublished).

[17]Using the 1c/3c ratio for the $S$ and $T$ observed with the CMMS, it is estimated that from the $\pi^+\pi^0$ decay mode alone one would expect a three-charged-particle decay mode of about 583 and 1200 events for the $S'$ and $T'$, respectively. Although the four-prong data are still being analyzed, no signals of this magnitude are apparent.

[18]G. Kalbfleisch, R. Strand, and V. Vanderburg, Phys. Lett. 29B (1969).

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# UNIVERSALITY OF DEEP INELASTIC LEPTON-HADRON SCATTERING*

G. Mack
Center for Theoretical Studies, University of Miami, Coral Gables, Florida 33124
(Received 1 June 1970)

In Wilson's theory of scale invariance at short distances, the Callan-Gross integral for electroproduction comes out to depend on the target hadron only through a factor of target mass. Experimental tests of this result, or of related results by Gatto and collaborators, will have a bearing on the theoretical question of anomalous dimensions of length.

Recently, Wilson published a study of non-Lagrangean models of current algebra.[1] He proposed a set of hypotheses which include (i) the existence of operator product expansions, (ii) scale invariance at short distances, and (iii) an enumeration of the fields of low dimension of length, $-l \leq 4$. Based on these hypotheses, Cicciarello, Gatto, Sartori, and Tonin[2] have derived electroproduction and neutrinoproduction sum rules. We wish to point out that their results can be generalized to arbitrary hadronic targets including nuclei. Their results [Eqs. (11)-(17) of Ref. 2] remain true as they stand; $c_1$ and $c_2$ are universal constants, and the only target dependence comes through the factors of target mass $M$. This amounts to strong universality properties. In particular, the Callan-Gross (CG) integral for electroproduction[3] is simply proportional to the target mass. A test of this prediction will be possible as soon as data on electron-deuteron scattering become available.

To derive these universality properties, let