cause the soft-phonon-mode frequencies behave in a similar way.

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$g_{9/2}$-$d_{5/2}$ INTERACTIONS IN Nb^{96} AND Nb^{92} †

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The reaction $Zr^{96}(He^3,t)Nb^{96}$ was studied and six states belonging to the $(\pi g_{9/2})(\nu d_{5/2})^{-1}$ configuration were identified. The energies and spin assignments agree well with those derived by use of the Pandya transformation from the known states of Nb^{92} belonging to the $(\pi g_{9/2})(\nu d_{5/2})$ configuration.

The low-lying states of ${}^{96}_{41}Nb_{55}$, previously unreported, should be well represented by the $(\pi g_{9/2})(\nu d_{5/2})^{-1}$ configuration, since both the f-p proton shell ($Z=40$) and the $d_{5/2}$ neutron subshell ($N=56$) are reasonably complete in Zr^{96}. The spectrum of low-lying states in Nb^{96} would therefore yield a set of $g_{9/2}$-$d_{5/2}$ matrix elements. The comparison of these with the corresponding $g_{9/2}$-$d_{5/2}$ matrix elements from Nb^{92} affords a rare opportunity for the study of the importance of changing configuration admixtures.

We have chosen to study the low-lying states in Nb^{96} by the charge-exchange (He^3, t) reaction on Zr^{96}. The (He^3, t) reaction has recently been studied for nuclei in the $1f_{7/2}$ shell,[1-4] and the $A \approx 90$ region.[5,6] This reaction selects proton-particle, neutron-hole excitations of the target ground state. The reaction $Zr^{90}(He^3, t)Nb^{90}$, for instance, strongly excites nine low-lying states that were identified[5] with the nine $T=4$ states expected from the $(\pi g_{9/2})(\nu g_{9/2})^{-1}$ configuration. Thus the low-lying Nb^{96} states that should be strongly excited in the reaction $Zr^{96}(He^3, t)Nb^{96}$ are those of the $(\pi g_{9/2})(\nu d_{5/2})^{-1}$ configuration. Admixtures of configurations involving $\pi(g_{9/2})^2$ and $\nu(d_{5/2})^{-2}(s_{1/2})^2$ in the ground state of Zr^{96} could give rise to weak excitations of other states below 1.5 MeV excitation in Nb^{96}. Configurations involving $(\nu g_{9/2})^{-1}$ or $(\pi d_{5/2})$ components should occur at excitation energies of 3 MeV or more because of the gap in single-particle states at $N, Z=50$.

We have studied the reaction $Zr^{96}(He^3, t)Nb^{96}$ with the 21-MeV He^3 beam from the Argonne tandem Van de Graaff accelerator. The target was a rolled foil of Zr metal enriched[7] to 85% in Zr^{96}. The thickness of the foil was 220 $\mu g/cm^2$, representing a target thickness of ~30 keV. Tritons were detected in the Argonne split-pole spectrograph by means of photographic emulsions which were later scanned in a computer-controlled automatic plate scanner.[8] A typical spectrum for the excitation region below 0.9 MeV is shown in Fig. 1. The peaks are not completely resolved, but a least-squares fitting program has enabled us to extract meaningful information on the energies and yields of individual peaks.

The angular distributions for the six most prominent states below 2 MeV are displayed in Fig. 2. It is clear that the six strongly excited states are reasonable candidates for those expected in the $\pi g_{9/2}(\nu d_{5/2})^{-1}$ multiplet. The angular distributions are remarkably similar for two pairs of states, and different for the remaining two states.

In our attempt to ascertain the spin values for these states, we have been guided by empirical rules noted in previous (He^3, t) reaction studies: (1) The shapes of the angular distributions for even-parity states of odd J are similar to those for states with the next larger even J; and (2) in

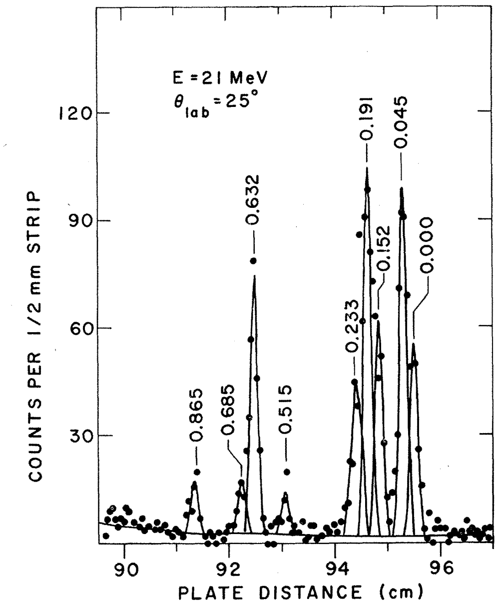

FIG. 1. Spectrum of tritons from the reaction $Zr^{96}(He^3, t)Nb^{96}$. The lines represent fits to the data by a least-squares analysis program. The resolution is limited by target thickness.

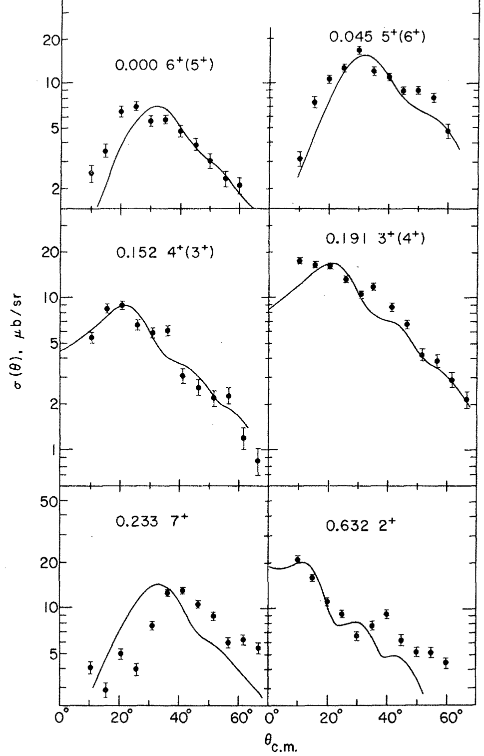

FIG. 2. Angular distributions of tritons from the reactions $Zr^{96}(He^3, t)Nb^{96}$ at 21 MeV. Absolute cross sections are accurate to ~20 %, relative cross sections to 5 %, unless otherwise indicated. The distorted-wave Born approximation calculations are the pure l transitions and apply only to even-J states; they are shown with the odd-J data only for purposes of comparison.

the reactions $Ca^{48}(He^3, t)Sc^{48}$ and $Zr^{90}(He^3, t)Nb^{20}$,[3,5] the odd-J states showed higher yields at the peaks of their angular distributions than did those for reactions leading to states with the next higher even J. Microscopic calculations using the distorted-wave Born approximation (DWBA) have been reasonably successful in fitting the shapes of the experimental angular distributions, although the odd-J final states could be fitted only when a tensor term was included in the interaction potential.[9]

DWBA calculations were performed by use of the program DWUCK,[10] assuming optical-potential parameters that fit the elastic He^3 scattering on Zr^{90}. No tensor interaction was used and only the even-J transitions were calculated. The comparison in Fig. 2 clearly identifies the 632-keV state as the best candidate for the 2^+ member of the multiplet and the 233-keV state for the 7^+ member. The shapes of the angular distributions and their relative intensities imply that the ground state and 45-keV state have $J^\pi = 6^+$ and 5^+, respectively, and the 152- and 191-keV states have $J^\pi = 4^+$ and 3^+. The 191-keV state also shows a tendency for forward peaking, which we interpret as evidence for a possible $l = 2$ component supporting a 3^+ assignment. Thus, we are able to identify all six members of the multiplet; but since these assignments depend on empirical rules they must, of course, be re-

Table I. Energies of $g_{9/2}d_{5/2}$ multiplets.

	Excitation energies (keV)		
J^π	Nb^{92}, obs.	Pred.[a]	Obs.[b]
2^+	135	755	632
3^+	286	186	191
4^+	479	156	152
5^+	357	36	45
6^+	500	0	0
7^+	0	219	233
$(2J+1)$-weighted centroid	412 ± 20[c]	(−412)	-337 ± 20[d]

[a] Predicted from Nb^{92} by the Pandya transformation.
[b] Relative energies, accurate to ±5 keV.
[c] With respect to two-particle energy from the ground-state energies of Nb^{91} and Zr^{91} (Ref. 17).
[d] With respect to the particle-hole energy predicted from the ground-state energies of Zr^{95} and Nb^{97}.

Table II. Violation of the Pandya transformation for various multiplets.

Nuclei	Multiplet	Rms deviation between spectra (keV)	Difference between $(2J+1)$-weighted centroids[a] (keV)
Cl^{38}-K^{40} [b]	$d_{3/2}f_{7/2}$	30	60
Sc^{42}-Sc^{48} [b]	$f_{7/2}f_{7/2}$	230	540
Nb^{92}-Nb^{96} [c]	$d_{5/2}g_{9/2}$	40	75

[a]With respect to single-particle states in adjacent nuclei (Ref. 17).
[b]Data summarized in Ref. 17 are used.
[c]Present data from Table I.

ported as tentative. Recent results from the reaction $Mo^{98}(d, \alpha)Nb^{96}$ are entirely consistent with these multiplet and spin assignments.[11]

The low-lying states of Nb^{92} have been studied in a number of experiments.[12-15] Six states were seen and identified with the $(\pi g_{9/2})(\nu d_{5/2})$ configuration. This spectrum can be transformed into the $(\pi g_{9/2})(\nu d_{5/2})^{-1}$ particle-hole matrix elements by way of a simple shell-model calculation, namely the Pandya[16] transformation. This transformation should work even if the proton wave function is not well described by a closed shell coupled to a $g_{9/2}$ proton, provided that the admixtures remain the same in Nb^{97} as in Nb^{91}. The predicted and observed excitation energies and centroid shifts[17] are listed in Table I. The agreement between the two spectra is remarkably good. It should also be noted that most of the deviation originates in the ~120-keV discrepancy in 2^+ energies. Since the 2^+ state in Nb^{96} is by far the highest in excitation energy, it is most likely to be admixed with 2^+ states of other configurations. Omitting the 2^+ state reduces the rms deviation between the relative excitation spectra from 40 to 7 keV.

The applicability of the Pandya transformation is open to some question in view of the fact that the wave function of the proton core has been found[14] to change between Zr^{90} and Zr^{96}, i.e.,

$$\psi_\pi(Zr^{90}) = (0.6)^{1/2}(p_{1/2})_0^{\,2} + (0.4)^{1/2}(g_{9/2})_0^{\,2}$$

and

$$\psi_\pi(Zr^{96}) = (0.9)^{1/2}(p_{1/2})_0^{\,2} + (0.1)^{1/2}(g_{9/2})_0^{\,2}.$$

The transformation can be applied to data in only two other pairs of nuclei. Each of these pairs includes one multiplet based on the rather questionable closed shells at Ca^{40}. Table II lists both the relative rms deviation and the centroid shift for each pair. The deviation for the Sc^{42}-Sc^{48} pair is an order of magnitude larger than for the Cl^{38}-K^{40} and the Nb^{92}-Nb^{96} pairs. The large violation in the Pandya transformation for the Sc^{42}-Sc^{48} case was analyzed by West and Koltun[18] and attributed to three-body effects. However, it is not at all clear from their analysis why the violation should be so much smaller in the other two pairs. It would be especially interesting to understand the circumstances under which an apparent two-body spectrum reflects the effective two-body matrix elements, and to what degree the validity of the Pandya transformation is relevant to this.

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[17]We calculate the centroid shift (the monopole term) in the manner of M. Moinester, J. P. Schiffer, and W. P. Alford [Phys. Rev. 179, 984 (1969)]. The spin-weighted centroid energy is computed with respect to the combined $1g_{9/2}$ single-proton energy in Nb^{91} and the $2d_{5/2}$ single-neutron energy in Zr^{91}. This should be equal to magnitude and opposite in sign to the centroid shift in the Nb^{96} multiplet relative to the energies of the Zr^{95} and Zb^{97} ground states.

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EXCITATION ENERGY DEPENDENCE OF SHELL EFFECTS ON NUCLEAR LEVEL DENSITIES AND FISSION FRAGMENT ANISOTROPIES

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It is shown that the nuclear shell effects on the level density disappear even at medium excitation energies of the order of 40 MeV. Fission-fragment anisotropy data for actinide nuclei having double-humped fission barriers are shown to contain evidence for this effect. The implication of this effect on the production of superheavy nuclei is pointed out.

The inclusion of a shell correction to the liquid-drop model (LDM) deformation energy of a nucleus, obtained from the deviation of the distribution of single-particle states in the nucleus from a uniform distribution, has led to the prediction of a pronounced double-humped fission barrier[1] for nuclei in the actinide region. Experimental support[2] for these concepts has recently come from several features of low-energy fission, such as the discovery of a large number of spontaneously fissioning isomers, sub-barrier resonance groups in slow-neutron–induced fission, and systematics of near-threshold fragment anisotropies. However, an important question arises as to how the ground-state nuclear shell corrections influence the observables such as the fragment anisotropies and fission excitation functions in the case of a "hot" nucleus, having an excitation energy much above the fission threshold. It is known[3-5] that the fission probability and the fragment angular distributions depend on the properties of the transition-state nucleus, which, by definition, corresponds to the deformation where the nuclear entropy S is minimum. If one uses the Fermi gas expression $S=2(aE_x)^{1/2}$, the point of minimum entropy becomes identical with the point of minimum excitation energy E_x and the transition state corresponds to the nuclear shape at the top of the fission barrier as assumed in all previous work. This expression, however, which is valid only for a system having a uniform spacing of single-particle levels, should be suitably modified to include the nuclear shell effects, now known to be present even at large deformation, and the transition state should be redetermined from considerations of nuclear entropy. In other words, the interpretation of fission data in the statistical region is closely related to the question as to how the shell effects on nuclear entropy (or level density) vary with excitation energy. It is shown in this note that the shell effects on nuclear entropy disappear even at medium excitation energies of the order of 40 MeV, and as an evidence for this the fragment anisotropy data in medium-energy fission are shown to be consistent with the above conclusion.

A brief description of the present calculations of nuclear entropy S is given below. For a system of noninteracting Fermions, containing N particles with total energy E, the following relations hold:

$$N=\sum n_k \tag{1}$$

$$E=\sum n_k \epsilon_k \tag{2}$$

where ϵ_k is the energy of the kth single-particle state. The Fermi-Dirac distribution function n_k and the entropy S are given by

$$n_k=\frac{1}{1+\exp[(\epsilon_k-\mu)/T)]} \tag{3}$$