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REMOTE FEEDBACK STABILIZATION OF THE ION-SOUND INSTABILITY BY A MODULATED SOURCE AT THE ELECTRON-CYCLOTRON RESONANCE FREQUENCY

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Feedback stabilization results are presented for an ion-sound instability. Sensing of the instability was achieved by a photodiode outside the plasma, and the suppressor element was an electron-cyclotron resonance structure remote from the plasma. Radiation from this structure heats the electrons locally and feedback is achieved by amplitude modulation of the electron-cyclotron resonance at the instability frequency. A theory is developed which predicts the variation of the instability amplitude and frequency in terms of the change in electron temperature ΔT, fractional modulation f, and phase change in the loop, φ. Comparison between theory and experiment shows good agreement.

Recently, there have been a number of papers which have shown feedback stabilization of various plasma instabilities.[1,2] Most of the previous work on feedback stabilization of various plasma instabilities has been performed with feedback suppressor elements in contact with the plasma. However, if feedback control of instabilities is to be a feasible method in any thermonuclear plasma, remote sensor and suppressor elements are essential. Lately, experiments have been performed,[3] in which a remote modulated microwave source, irradiating the plasma at the upper hybrid frequency, was used as the feedback element. Here, experiments are reported in which an $m=0$ ion-sound instability has been suppressed by using a remote source of energy at the electron-cyclotron resonance (ecr) frequency, modulated at the instability frequency. The effect of the energy at ecr is to cause a local increase in the temperature ΔT which in this case is small compared with the steady-state electron temperature T_e.

The stability of the plasma has been considered using the "two-fluid" model, in which the axial magnetic field B_0 is taken in the z direction, and only spatial variations of the form $e^{ik_z z}$ are considered, where k_z is the axial wave number of the instability. The density n is considered of the form $n=n_0+n_1$, where n_0 is the zero-order density and n_1, the perturbed value, and V_1 and v_1 are taken as the potential and ion-velocity perturbations, respectively. If the main effect of the ecr is taken to be local heating of the electrons, the z component of the electron equation of motion reduces in the low-frequency approximation to the form

$$eV_1=T_e(n_1/n_0)+f\Delta T[n_1(\tau)/n_0], \qquad (1)$$

where f is the fractional modulation produced by the feedback signal and $n_1(\tau)$ represents the density perturbation delayed by a time τ. Here, $\omega_0\tau=\varphi$ the phase delay.

The ion equation of motion is

$$\frac{d\vec{v}_i}{dt}=-\frac{e}{M_i}\nabla V_1-\vec{v}_i\nu+\frac{e}{M_i}\vec{v}_i\times\vec{B}_0, \qquad (2)$$

where ν is the ion-neutral collision time, and the equation of continuity is

$$\partial n/\partial t+\nabla\cdot(n\vec{v}_1)=S_i(n_1). \qquad (3)$$

S_i is a source term due to ionization etc., caused by large-amplitude oscillations present in the plasma. This source term is taken to be of the form

$$S_i = \alpha n_1 - \gamma n_1^2 - \beta n_1^3, \qquad (4)$$

where $\beta n_i^2 \ll \gamma n_i \ll \alpha \ll \omega_0 = k_z C_s$, and $C_s = (T_e/M_i)^{1/2}$ is the ion-sound velocity. After eliminating v_i between (2) and (3) and substituting for S_i from (4), in the approximation that ν is small, the equation for n_1 reduces to

$$\frac{d^2 n_1}{dt^2}\frac{dn_1}{dt}[\alpha - 2\gamma n_1 - 3\beta n_1^2] + \omega_0^2 n_1 + \omega_0^2 f\left(\frac{\Delta T}{T_e}\right) n_1(\tau) = 0 \qquad (5)$$

which is the equation of Van der Pol[4] including a feedback term. A solution of the form $n_1 = a(t) \times \cos\omega t$ may be tried in a similar manner to that of Keen.[5] This results in the following conditions:

$$a_0^2 - a^2 = -f(\Delta T/T_e)(\omega_0^2/\alpha\omega)a_0^2 \sin\varphi \qquad (6)$$

and

$$\omega^2 = \omega_0^2[1 + f(\Delta T/T_e)]\cos\varphi, \qquad (7)$$

where a_0 and a are the instability amplitudes, and ω_0 and ω the frequencies, with and without feedback, respectively. Equation (6) shows that as the fractional modulation of the ecr frequency or the change in temperature ΔT is increased, the amplitude a will increase or decrease according to the sign of $\sin\varphi$. Optimum suppression is achieved for

$$\sin\varphi = -1 \text{ [i.e., } \varphi = -90° \text{ or } +270°]. \qquad (8)$$

Then,

$$f(\Delta T/T_e) = \alpha/\omega_0. \qquad (9)$$

The apparatus was similar to that employed previously.[6] The plasma was the positive column of a neon arc discharge and had a peak density $\sim 3\times 10^{11}$ cm^{-3} and an electron temperature $T_e = 5.4$ eV, and was contained in a magnetic field of ~180 G. A remote photodiode detector was employed to sense the instability, which had a $m = 0$ azimuthal mode number and a frequency ~7.5 kHz independent of magnetic field. It was identified as an ion-sound instability. Power at the ecr frequency was applied to the plasma from a resonant structure around the glass tube,[7] from a uhf oscillator and power amplifier. The temperature change ΔT produced was linearly proportional to the power input to the cavity in the range used in these experiments. A signal proportional to the density perturbations was fed from the photodiode, via a phase shifter and a low-frequency amplifier, to modulate the uhf oscillator at the ecr frequency. The effect of the feedback was observed by displying the output from a separate probe on a spectrum analyzer.

With the ecr power on, the phase angle φ in the feedback loop was varied until minimum instability amplitude was achieved. Then the fractional modulation f was set ~1.0 and the ecr power level was varied until suppression just occurred. At this power level and $f = 0$, the change in electron temperature was $\Delta T = 0.6 \pm 0.1$ eV, as measured on a double probe and by the change in floating potential technique.[8] At this phase angle and ecr power level, the amplitude a was measured as a function of the fractional modulation, f. Figure 1 shows $(a/a_0)^2$ plotted versus f, and it is seen that a good linear relationship is obtained, as predicted by Eq. (6). In Fig. 1, photograph (a) shows the instability when $f = 0$, and photograph (b) shows the effect when it is suppressed with $f = 1.0$.

At this same phase angle, and with the fractional modulation maintained at 1.0, the instability

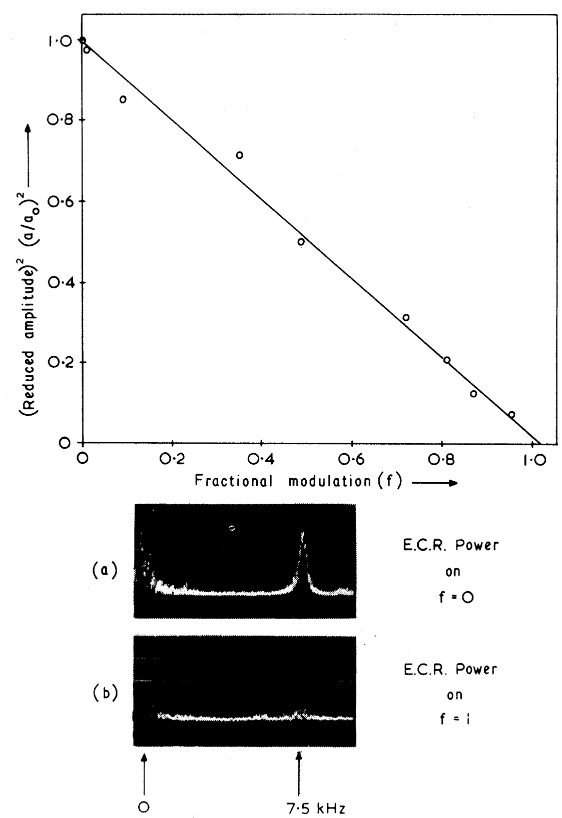

FIG. 1. The square of the reduced amplitude $(a/a_0)^2$ versus the fractional amplitude modulation f. Photograph shows the spectrum analysis of the instability for (a) $f = 0$, and (b) $f = 1.0$.

amplitude a was measured as a function of the input power at the ecr frequency ($\propto \Delta T$). This is shown plotted in Fig. 2 as $(a/a_0)^2$ versus ecr power ($\propto \Delta T$), and shows a linear relationship as predicted by Eq. (6). Figure 3 shows the square of the reduced amplitude $(a/a_0)^2$ plotted against phase angle φ in the feedback loop, and it is seen that suppression occurs at a phase change of $-105° \pm 20°$ or $255° \pm 20°$ as compared with the predicted value [Eq. (8)] of $-90°$ or $270°$. The full line in Fig. 3 is the theoretical predicted variation of Eq. (6) shifted so that its minimum is at $-105°$ instead of $-90°$. Here, again, good agreement is obtained.

From Eq. (6) it is possible to obtain a value for α, the linear growth rate. At the position for optimum suppression ($a = 0$) and $\alpha/\omega_0 = f(\Delta T/T_0) = 0.11 \pm 0.02$. This value was checked by directly measuring the growth and decay rate of the instability amplitude. This was effected by suppressing the instability by the method of "asynchronous quenching."[8] By gating the asynchronous frequency, the instability was turned on and off at periodic intervals. Analysis of the resulting signal gave $\alpha/\omega_0 = 0.12 \pm 0.03$, in good agreement with the former value. A further check was afforded by the use of Eq. (7). If the frequency shift $\Delta\omega = \omega - \omega_0$ is measured as a function of phase angle, the maximum frequency shift $\Delta\omega_m$ is given by $2\Delta\omega_m/\omega_0 = f(\Delta T/T_e) = \alpha/\omega_0$. The measured maximum change in frequency was $\Delta\omega_m = 0.3 \pm 0.1$ kHz and thus the value $\alpha/\omega_0 = 0.08 \pm 0.03$ resulted, which is not in such good agreement.

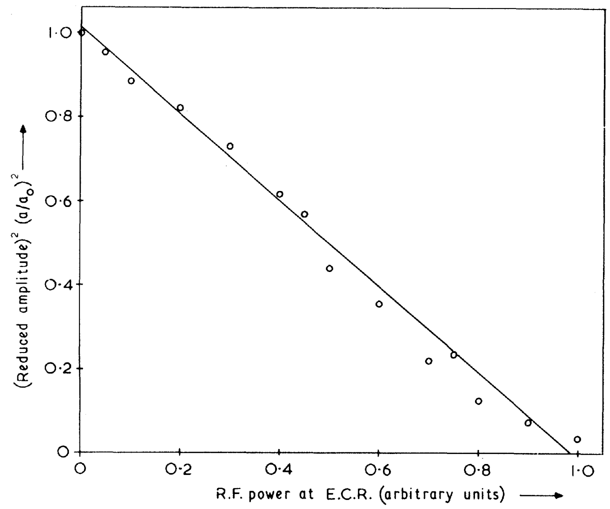

FIG. 2. The square of the reduced amplitude $(a/a_0)^2$ plotted versus the input power at the ecr frequency ($\propto \Delta T$).

Summarizing, it is seen that theoretical relationships can be obtained which predict the change in instability amplitude a and frequency shift $\Delta\omega$ in terms of the change in electron temperature ΔT, its fractional modulation f, and phase change in the loop, φ. The resulting measurements show the predicted variations and a consistent value for the growth rate α is obtained, within experimental error.

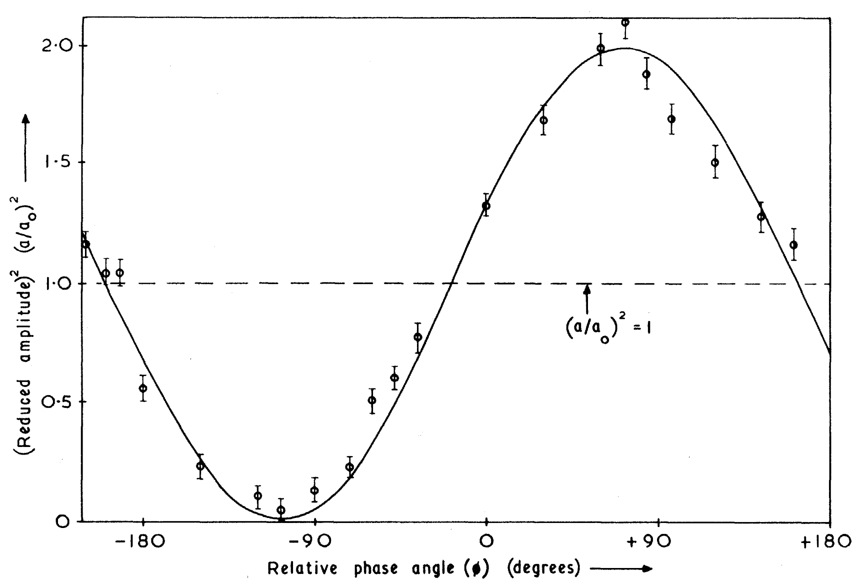

FIG. 3. The square of the reduced amplitude $(a/a_0)^2$ plotted versus phase angle φ, in the feedback loop.

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POSSIBILITY THAT ANOMALOUS PLASMA DIFFUSION DEPENDS ON GEOMETRIC FACTOR*

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Turbulent diffusion coefficients for an assumed spectrum are derived using fluid equations, a crucial assumption being that the ion motion parallel to the magnetic field is hindered by the collision of ions with neutrals. The result shows that the turbulent diffusion coefficients depend on geometric factors, as well as on the level of fluctuations. This dependence on geometric factors may explain the experimentally observed difference in confinement times in different toroidal plasma confinement devices.

In recent years a number of experiments have been carried out to establish the relationship between low-frequency fluctuations in plasmas and the anomalous plasma loss across a magnetic field. It is fair to say that the evidence is almost conclusive that fluctuations do cause the anomalous loss in some plasmas. In other plasmas, however, the observed fluctuations appear to cause an insignificant amount of plasma loss. One of the most striking examples of the latter case is a Tokamak plasma.[1] From observations such as those made in stellarators[2] and Zeta,[3] Tokamak plasmas are also expected to have fluctuations in potential and density. In fact, recent measurements reported by Mirnov at the Dubna conference on toroidal plasmas show that magnetic field fluctuations exist in the T-3 Tokamak. In view of the similarity of his measurement and that made in the C stellarator by Young,[4] density and potential fluctuations are also to be expected. Theoretically, a number of low-frequency instabilities are possible in Tokamaks.

Yet the plasma confinement times in Tokamak T-3 are approximately 100 times the Bohm confinement time τ_B. Here,

$$\tau_B = [B(\text{kG})/T_e(\text{eV})][r_0(\text{cm})/5]^2 \text{ msec},$$

where r_0 is the plasma radius. This Bohm time has been shown by Hinnov and Bishop[2] to correspond approximately to the confinement time observed in the C stellarator.

It is natural to consider the possibility that the level of fluctuation is similar in the stellarator and the Tokamak but the transport coefficient (calculated from the fluctuation level) is smaller in the Tokamak. In the following, a simplified consideration for deriving the transport coefficient is presented; a more detailed description will be found elsewhere.[5]

The method previously used to derive an anomalous electron diffusion coefficient will be used, including the motion of the ions.[6] In order to allow for the motion of ions across the magnetic field, collisions between ions and neutrals are included. In Tokamak plasmas there may not be enough neutrals; however, ion motion across the magnetic field could be caused by effects such as finite Larmor radius or ion viscosity. The end result does not depend on the ion-neutral collision frequency if ambipolar diffusion is assumed.

Initially we assume that the entire space is filled by a plasma having the density

$$n(\vec{r}) = n_0 + \sum\nolimits_{\vec{k}}' n_{\vec{k}} \exp i(\vec{k}\cdot\vec{r}) = n_0 + \tilde{n}, \qquad (1)$$

where the summation $\sum'$ means the sum of all the Fourier components of n except $\vec{k}=0$. This density perturbation, $\tilde{n}$, is assumed to be statistically homogeneous. A uniform magnetic field B is imposed in the z direction. Electrons are assumed to be in thermal equilibrium. The pattern of the density perturbation is assumed to be time independent. This assumption of time independence can be justified approximately if the

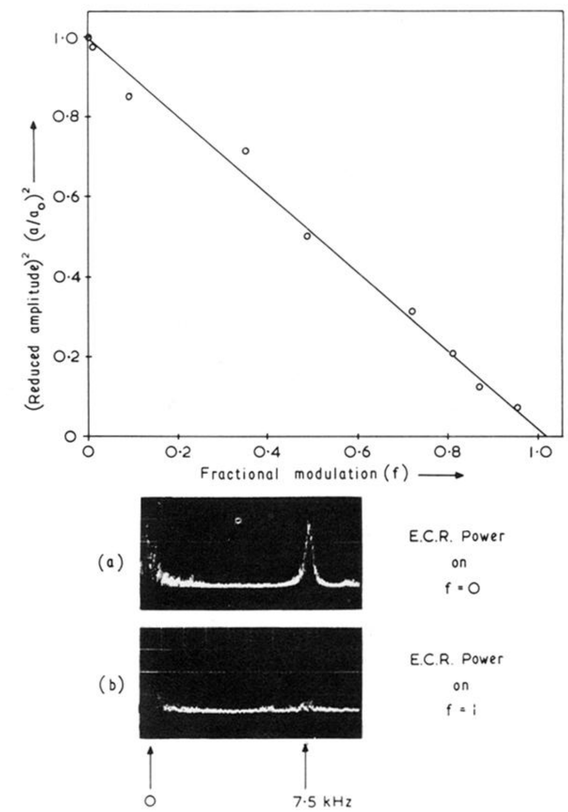

FIG. 1. The square of the reduced amplitude $(a/a_0)^2$ versus the fractional amplitude modulation f. Photograph shows the spectrum analysis of the instability for (a) $f=0$, and (b) $f=1.0$.