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ION-WAVE INSTABILITIES AND ANOMALOUS RESISTIVITY*

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A theory of ion-wave instabilities is presented which takes into account the self-consistent inhomogeneities generated by currents flowing along magnetic field lines. It is found that for $T_e \sim T_i$, the minimum unstable current is reduced from the electron to approximately the ion thermal speed. The result is attributed to the drift-wave instability caused by the inhomogeneities produced by the driving current.

For many years it has been known that, for $T_e \sim T_i$, the minimum current for ion-wave instabilities (or the so-called two-stream current instabilities) predicted by the existing theories[1] is too high in comparison with experimental results.[2] In this Letter, we present a theory which takes into account the self-consistent inhomogeneities generated by currents flowing along the magnetic lines of force. In our model, the inhomogeneities are produced by the driving current alone. The plasma is uniform when the driving current vanishes. It should be noted that our model is different from Kadomtsev's[1] model in that the inhomogeneities in his model are independent of the driving current. As has been noted by Bernstein et al.,[1] it is inconsistent to assume currents along the magnetic lines of force and, at the same time, a uniform plasma. The importance of the self-consistent inhomogeneities in a current-carrying plasma is clearly demonstrated by our results. For $T_e \sim T_i$, our theory predicts a minimum unstable current of the order of the ion thermal speed rather than the electron thermal speed predicted by the existing theories.[1] Much of the stellarator data, particularly current work on anomalous resistivity, shows good agreement with our results.[3] It may be noted that the stability criterion predicted by our theory can also be interpreted as the stability criterion for a low-β sheet pinch.

In this communication, we can only outline the general approach and present the main results of our work. Full details will be given in a separate paper. Consider a low-β (β = kinetic pressure/magnetic pressure) collisionless plasma, consisting of electrons and protons, in a constant external magnetic field ($B_0\hat{z}$) with current along the field lines. We consider a one-dimensional model in which all quantities may only depend on the x coordinate. Under three reasonable boundary conditions, it can be shown by extending Harris' solution[4] that for a low-β plasma the most probable self-consistent equilibrium distribution in the presence of a uniform external magnetic field is given by

$$f_{0j} \approx \frac{N_0}{\pi^{3/2} v_{0j}{}^3 \cosh^2(x_0/L)} \left[1 - \frac{2\tanh(x_0/L)}{L}\left(x + \frac{v_y}{\Omega_j} - x_0\right)\right] \exp\{-v_{0j}{}^{-2}[(v_\parallel - V_j)^2 + v^2]\}, \tag{1}$$

where $L = [k(T_e+T_i)/(2\pi N_0 e^2)]^{1/2} c/V$ characterizes the width of the current layer, $V = |V_i - V_e|$ is the uniform relative streaming speed, c the speed of light, $v_{0j} = (2kT_j/m_j)^{1/2}$ the thermal speed, $\Omega_j = q_j B_0/(m_j c)$ the Larmor frequency, and T_j, m_j, and q_j the temperature, the mass, and the charge of jth species, respectively. N_0 is the maximum number density. The boundary conditions are these: (i) The external current is of finite extent in one dimension (x) and is unidirectional throughout the plasma; (ii) the average relative velocity between the two species along the field lines (V) is position independent; (iii) the source of the external uniform magnetic field ($B_0\hat{z}$) does not vary with time.

In accordance with the local approximation,[5] x_0 is chosen to be the location of the maximum density gradient at $x=x_0=0.66L$. The equilibrium electric field is zero in the frame where $V_i/T_i=-V_e/T_e$. In view of the low-β assumption, the effect of the shear field $B_y=B_{y0}\tanh(x/L)$ and the field of the drift current can be safely neglected. In deriving the dispersion relation for low-frequency ($\omega \ll \Omega_i$) perturbations about the equilibrium state described by Eq. (1), we make the electrostatic approximation and the local approximation for a weakly inhomogeneous medium.[5] The resulting dispersion relation is

$$D=k^2/k_{Di}^2+(1+1/\tau)+\exp(-\lambda_i)I_0(\lambda_i)[\alpha_i+(v_{di}/v_{0i})\tan\theta]z(\alpha_i)+(1/\tau)[\alpha_e-(v_{de}/v_{0e})\tan\theta]z(\alpha_e), \qquad (2)$$

where $z(\alpha)=i\pi^{1/2}\exp(-\alpha^2)-2\alpha Y(\alpha)$ is the plasma dispersion function,[6] θ is the angle between the wave vector $\vec{k}$ and the external magnetic field, $\tau=T_e/T_i$ is the temperature ratio, $v_{dj}=L^{-1}bv_{0j}^2/|\Omega_j|$ is the magnitude of the gradient drift, $b=\tanh(0.66)$, $\lambda_j=\frac{1}{2}\sin^2\theta(\omega/\Omega_j)^2(\omega/kv_{0i})^{-2}$, $\alpha_j=(\omega/kv_{0j})/\cos\theta-V_j/v_{0j}$, $k_{Di}^{-1}=v_{0i}\omega_{pi}^{-1}/\sqrt{2}$ is the Debye length for the ions, and I_0 is the modified Bessel function. The dispersion relation, Eq. (2), has the same general form as the dispersion relation for the drift waves,[5] but our expression has the unique feature that the gradient drifts ($v_{dj}=L^{-1}bv_{0j}^2/|\Omega_j|$) appearing in our dispersion relation depend on the driving current via L.

To analyze the stability of the system, we apply the Nyquist criterion to Eq. (2). The Nyquist diagram is formed on the complex D plane with ω/kv_{0i} as the running parameter. We find that for a given τ ($=T_e/T_i$), there is a minimum unstable current $(V/v_{0e})_m$ for every θ. The overall minimum unstable current of the system, denoted by $(V/v_{0e})_{mm}$, is therefore given by the minimum of $(V/v_{0e})_m$ for θ in the range $0 \le \theta < \pi/2$, i.e.,

$$(V/v_{0e})_{mm}=\min_{0\le\theta\le\pi/2}(V/v_{0e})_m.$$

The angle at which $(V/v_{0e})_{mm}$ is reached is denoted by θ_m. We plot $(V/v_{0e})_{mm}$ vs τ in Fig. 1 with θ_m indicated on the curve (solid line). We have also plotted in Fig. 1 the curve of $(V/v_{0e})_m$ for $\theta=0$ (dashed line). The curve for $\theta=0$ agrees with the result of Fried and Gould[1] as it should.

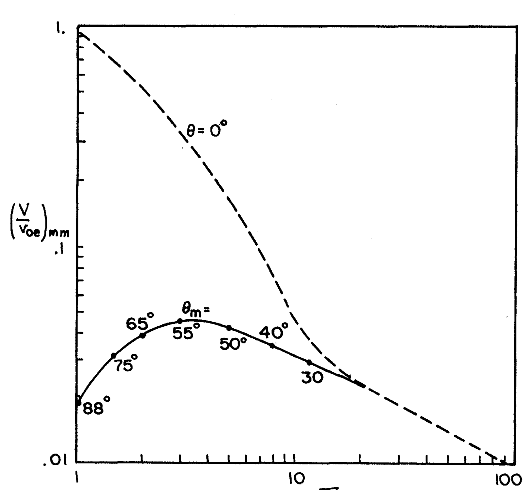

FIG. 1. Critical streaming velocity for ion-wave instabilities versus the electron-ion temperature ratio.

In conclusion, our theory predicts that for $T_e \sim T_i$, the minimum unstable current is reduced from the electron to approximately the ion thermal speed. The result is attributed to the drift-wave instability caused by the self-consistent inhomogeneities produced by the driving current.

Our results show that there are two roles played by the external currents in causing instabilities. The first is that the external currents directly destabilize waves propagating along the lines of force through resonant interaction; the second role is that the currents can indirectly destabilize waves propagating at an angle to the lines of force by producing a diamagnetic current which will interact resonantly with these waves. The physical picture of the two distinct roles played by the current in our theory is always correct as long as the current is of finite extent. The importance of the second role in current instabilities is clearly demonstrated by our results. The existing theories[1] take into account the first, but neglect the second role of the current. Our theory should serve to correct this misconception.

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REMOTE FEEDBACK STABILIZATION OF THE ION-SOUND INSTABILITY BY A MODULATED SOURCE AT THE ELECTRON-CYCLOTRON RESONANCE FREQUENCY

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Feedback stabilization results are presented for an ion-sound instability. Sensing of the instability was achieved by a photodiode outside the plasma, and the suppressor element was an electron-cyclotron resonance structure remote from the plasma. Radiation from this structure heats the electrons locally and feedback is achieved by amplitude modulation of the electron-cyclotron resonance at the instability frequency. A theory is developed which predicts the variation of the instability amplitude and frequency in terms of the change in electron temperature ΔT, fractional modulation f, and phase change in the loop, φ. Comparison between theory and experiment shows good agreement.

Recently, there have been a number of papers which have shown feedback stabilization of various plasma instabilities.[1,2] Most of the previous work on feedback stabilization of various plasma instabilities has been performed with feedback suppressor elements in contact with the plasma. However, if feedback control of instabilities is to be a feasible method in any thermonuclear plasma, remote sensor and suppressor elements are essential. Lately, experiments have been performed,[3] in which a remote modulated microwave source, irradiating the plasma at the upper hybrid frequency, was used as the feedback element. Here, experiments are reported in which an $m=0$ ion-sound instability has been suppressed by using a remote source of energy at the electron-cyclotron resonance (ecr) frequency, modulated at the instability frequency. The effect of the energy at ecr is to cause a local increase in the temperature ΔT which in this case is small compared with the steady-state electron temperature T_e.

The stability of the plasma has been considered using the "two-fluid" model, in which the axial magnetic field B_0 is taken in the z direction, and only spatial variations of the form $e^{ik_z z}$ are considered, where k_z is the axial wave number of the instability. The density n is considered of the form $n=n_0+n_1$, where n_0 is the zero-order density and n_1, the perturbed value, and V_1 and v_1 are taken as the potential and ion-velocity perturbations, respectively. If the main effect of the ecr is taken to be local heating of the electrons, the z component of the electron equation of motion reduces in the low-frequency approximation to the form

$$eV_1=T_e(n_1/n_0)+f\Delta T[n_1(\tau)/n_0], \tag{1}$$

where f is the fractional modulation produced by the feedback signal and $n_1(\tau)$ represents the density perturbation delayed by a time τ. Here, $\omega_0\tau=\varphi$ the phase delay.

The ion equation of motion is

$$\frac{d\vec{v}_i}{dt}=-\frac{e}{M_i}\nabla V_1-\vec{v}_i\nu+\frac{e}{M_i}\vec{v}_i\times\vec{B}_0, \tag{2}$$

where ν is the ion-neutral collision time, and the equation of continuity is

$$\partial n/\partial t+\nabla\cdot(n\vec{v}_1)=S_i(n_1). \tag{3}$$

S_i is a source term due to ionization etc., caused by large-amplitude oscillations present in the plasma. This source term is taken to be of the