the naive values $\frac{1}{3}$ and $\frac{2}{3}$ due to the residual interaction between droplets (excluded-volume effects, short-range part of $E^{(1,2)}$).

[9]Numerical computation of the integral in Eq. (10) shows that $\ln C_K^{\rm sing}$ begins to deviate from the straight line $-\alpha' \ln t$ at $t \approx 10 t_0$ and is nearly independent of $\ln t$ for $t \lesssim 10^{-1} t_0$. The author is indebted to Dr. H. Horner for help in these calculations.

PROTON-PROTON SCATTERING AT 9.690, 9.918, AND 13.600 MeV†

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(Received 27 April 1970)

Adjustments to proton-proton elastic scattering data at 9.690 and 9.918 MeV have apparently resolved a discrepancy between the data and current energy-dependent phase-shift analyses. New data at 13.600 MeV are also presented and are compatible with the phenomenological analysis. These results indicate that the energy-dependent fitting of $p+p$ data in the region of 10 MeV is now satisfactory.

A strong discrepancy in the phenomenological analysis of proton-proton scattering near 10 MeV has been pointed out by Holdeman, Signell, and Sher[1] (HSS). They indicate that the consensus of experimental information[2-4] on scattering cross sections near 10 MeV results in a $^1S_0$ phase that is markedly below reasonable phenomenological predictions,[1,5] that in order to fit the data, serious readjustment of fits to a number of well-accepted data at other energies would be necessary (see Figs. 2 and 3 of Ref. 1).

To help resolve this discrepancy we undertook two courses of action. First, since the $^1S_0$ phase is strongly affected by the absolute values of the data, we planned a thorough examination and recalibration of all experimental parameters that affect the absolute normalization in our previous measurements[2] at 9.690 and 9.918 MeV. We also restudied the assumptions and approximations made in the reduction of the data, especially at small angles. Secondly, we measured an accurate angular distribution at a nearby energy (13.6 MeV) to help tie down the absolute value and search for possible energy-dependent systematic errors.

With one exception, the recalibrations and remeasurements produced no significant results outside expected errors. We did find a gross systematic error in the device used to measure the width of the slits in the detector; and the value of the G factor and the cross sections are directly affected. The correction increases the absolute values about 2%; the final corrected values are given in Table I. As can be seen in Fig. 3 of Ref. 1, this correction brings our absolute values close to the predicted values of the multienergy analysis of Sher, Signell, and Heller.[6] The error of the absolute scale is slightly smaller because of an improved method of slit measurement. The relative relation of the values and the relative errors have not changed.

The experimental method used for the 13.600-MeV data given in Table II is the same as presented in Ref. 2 except that the geometry-factor accuracy has been improved to ±0.20%.

Table I. Differential cross sections for $p+p$ elastic scattering.

9.918 MeV

$\theta_{\rm lab}$ deg.	$\sigma(\theta)_{\rm lab}$ mb/sr	$\theta_{\rm c.m.}$ deg.	$\sigma(\theta)_{\rm c.m.}$ mb/sr	Relative Error %	Absolute Error %
10.00	296.22	20.05	74.83	0.80	0.90
12.50	212.58	25.06	54.18	0.41	0.56
15.00	194.83	30.08	50.20	0.39	0.54
17.50	191.72	35.09	50.04	0.37	0.52
20.00	189.03	40.10	50.09	0.37	0.52
25.00	185.92	50.12	51.11	0.41	0.55
30.00	180.31	60.13	51.91	0.34	0.50
35.00	172.44	70.14	52.53	0.34	0.50
40.00	161.56	80.15	52.69	0.34	0.50
45.00	148.37	90.15	52.46	0.36	0.52
50.00	135.58	100.15	52.78	0.36	0.51

9.690 MeV

$\theta_{\rm lab}$ deg.	$\sigma(\theta)_{\rm lab}$ mb/sr	$\theta_{\rm c.m.}$ deg.	$\sigma(\theta)_{\rm c.m.}$ mb/sr	Relative Error %	Absolute Error %
13.00	215.29	26.06	54.98	0.41	0.55
15.00	201.79	30.07	51.99	0.39	0.53
20.00	196.07	40.09	51.96	0.36	0.52
25.00	192.56	50.11	52.94	0.40	0.54
30.00	186.90	60.13	53.82	0.41	0.55

Table II. Differential cross sections for $p+p$ elastic scattering.

13.600 MeV					
θ_{lab} deg.	$\sigma(\theta)_{lab}$ mb/sr	$\theta_{c.m.}$ deg.	$\sigma(\theta)_{c.m.}$ mb/sr	Relative Error %	Absolute Error %
10.00	180.44	20.07	45.50	0.94	1.00
12.50	141.88	25.09	36.09	0.57	0.66
15.00	137.18	30.10	35.28	0.51	0.61
17.50	135.37	35.12	35.28	0.58	0.67
20.00	136.02	40.13	35.99	0.31	0.45
25.00	134.00	50.16	36.79	0.35	0.48
30.00	130.12	60.18	37.43	0.51	0.61
35.00	123.15	70.19	37.49	0.35	0.48
40.00	115.29	80.20	37.58	0.36	0.48
50.00	96.24	100.20	37.48	0.38	0.50
55.00	85.15	110.19	37.20	0.38	0.51

To gain a rough idea of the import of the data in Tables I and II, we have compared the data with predictions based on the phase-shift analysis of MAW-X (Ref. 5). For the 9.918-MeV data we obtain a χ squared of 14 for 11 degrees of freedom. For this comparison, the normalization of the absolute scale of our experimental data was taken as a separate degree of freedom, calculated from the data such that the sum of the differences between the data points and the predictions was zero. This χ-squared value is very acceptable. In particular, the normalization comes out to be -0.42% which is compatible with the absolute scale error for the 9.918-MeV data which was $\pm 0.38\%$. Before the correction, the normalization was on the order of 2%, or more than 5 standard deviations out. Since the $^1S_0$ phase depends strongly on the absolute normalization of the data, these results indicate that the previous discrepancy with the $^1S_0$ predictions will no longer exist. In the above analysis, the 10° (lab) datum was discarded (as was also done by HSS) because the χ squared was 21 for this point alone.

Similar results are obtained for the 13.6-MeV data. A χ-squared value of about 17 resulted for 12 degrees of freedom. Included this time is the 10° (lab) datum which also has a high value (χ squared of 3.1 for that point alone) but not so far out that it could be arbitrarily discarded. The normalization for the 13.6-MeV case is $+0.38\%$, comparable with the absolute scale standard deviation of $\pm 0.33\%$.

P. Signell and J. Holdeman have analyzed[7] the 9.918-MeV data of Table I [omitting again the 10° (lab) datum] and computed a value for $^1\delta_0^E$ of 55.23 ± 0.13 degrees in good agreement with their prediction of Ref. 1. The value of $^3\Delta_c^E$ changes little as was expected: -0.033 ± 0.030 degrees. They calculate a χ squared per point of 0.8 comparing our 9.918-MeV data with their prediction.

Our overall conclusion is that our $p+p$ scattering data in the region of 10 MeV now agree with the detailed analysis of HSS and disagree with the Berkeley data in both shape and absolute value. We anticipate that a more detailed analysis will show that the present work resolves the discrepancies in the phenomenological description of proton-proton scattering in the 10-MeV region.

We wish to thank Peter Signell and John C. Hopkins for their comments and the staff of the accelerators for their continued help.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

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