without resorting to a many-body model. One such model we considered in detail was the electrostricted cluster model[9] which would produce a higher-than-ambient density of helium atoms in the neighborhood of the positron. In addition such a model would have the attractive feature of predicting an increase in decay rate with decreasing temperature and an increase in the stability of the cluster with increasing gas density. It is understood that for the onset of cluster formation the positron must be initially localized since otherwise its velocity, even at thermal energies, would be too great for the helium atoms to collect about it. We can show, using a Born-Oppenheimer approximation, that it is also necessary for the positron to be localized by some mechanism, other than the potential well presented by the cluster, after the cluster has been formed in order to arrive at a self-consistent solution to the problem.[10]

We thus find that in order to make any quantitative use of the cluster model we would have to postulate either attachment of the positron to a single helium atom or a sudden reduction of the mobility of the positron which could result from fully taking into account the quantum mechanical aspects of the positron in a system of many atoms. Such an effect has been predicted for electrons in a random system of hard-core scatterers in a recent paper by Neustadter and Coopersmith.[11] Although the positron–helium-atom interaction cannot be represented by hard-core repulsion, two main features of the Neustadter-Coopersmith calculation (the inclusion of multiple scattering to all orders and making use of the randomness of the spatial distribution of the scatterers) would be retained if a similar calculation for positrons in gaseous helium were made.

In conclusion, the observed critical behavior of the annihilation spectra of positrons appears to require the consideration of many-body effects for an understanding of the phenomena reported in the paper.

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[1] A combination source holder and moderator with cylindrical geometry instead of the conventional foil sandwich for the Na^{22} positron source made measurements with such a small sample cylinder feasible. At a density of 0.016 g/cm^3, for example, 25% of the positrons annihilated in the gas – a factor of 2 to 4 times the fraction of positrons stopped in the gas when a foil sandwich is used.

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DISCONTINUITIES OF IONIC MOBILITIES IN SIMPLE LIQUIDS

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This Letter reports the results of ionic-mobility measurements in the four simple liquids He I, N_2, Ar, and CCl_4. Periodic steps in the mobility, like those found earlier in liquid helium II, were found also in these four liquids. We found also a simple relation connecting the critical velocities to the temperatures of the investigated liquids.

Periodic steps in ionic mobilities have been reported to exist in liquid helium II[1-5] and in liquid argon and nitrogen.[6] The models proposed for the superfluid helium[7,8,3] have not been confirmed by the experiments performed thereafter,[4,5,9] while no models have been proposed for argon and nitrogen.

Because this effect is at present completely unexplained, we decided to investigate several liquids by a technique which enables us to measure the mobility in a wide range of drift velocities, with an accuracy of ±1 %. The details of the method, together with a discussion of the errors can be found in a recently published paper.[10]

We report in this Letter the results of the measurements, performed with the technique described in Ref. 10, in four simple liquids: helium at 4.2°K, nitrogen at 77°K, argon at 86°K, and carbon tetrachloride at 293°K. The current density of the injected charges, produced by a tritium 2-Ci β source, was between 5×10^{-13} and 5×10^{-12} A/cm^2. When the current density exceeded this upper limit, as in CCl_4, the source was screened by a bored metal foil. The gaps between the source, grid, and collector electrodes have been varied from 0.2 to 0.7 cm.

Helium. – The helium gas was condensed in the cell, submerged in cryogenic liquid helium at 4.2°K, through a cooled charcoal trap. We found a positive-ion mobility of 3.5×10^{-2} cm^2/V sec, which was constant for drift velocities up to a critical velocity $v_c = 180$ cm/sec. At this drift velocity the mobility falls to a lower level, and a second fall was found at a drift velocity $\langle v\rangle = 2v_c$. In our previous measurements[4] in liquid helium I at 2.29°K we found a constant mobility between 20 and 100 cm/sec, the minimum and maximum drift velocities we were able to measure with that method and with the rather low mobility involved. The present result is in agreement with our expectation[4] that the steps above the λ point should be eventually found at drift velocities greater than 100 cm/sec.

Nitrogen. – We condensed the gas[11] into the cell, after pumping for many hours. We worked at a fixed temperature of 77°K and various pressures from 1 to 2 atm, for a total of 20 runs. We observed steps at multiple drift velocities, but the critical velocity was not reproducible from run to run, ranging between 0.8 and 1.3 cm/sec. We did not use particular purification of the gas, in that our aim was mainly that of confirming Henson's results.[6] On the other hand the result obtained in He I, where the gas was purified, suggests that the impurities should not play a determining role, apart from the value of the mobility. The results obtained with positive ions at $P = 1.8$ atm, and with negative ions at $P = 1.3$ atm, are shown in Fig. 1.

Argon. – The measurements in liquid argon[12] were made at saturation vapor pressure, at eight different temperatures from 88 to 94°K. We found steps, in agreement with Henson's results,[6] with a critical velocity which was reproducible within ±5% from run to run, around a mean value of 0.68 cm/sec.

CCl_4. – We used a stainless-steel demountable cell, with Teflon spacers and gaskets. The liquid we used was a commercial CCl_4 for electronic uses.[13] After many fillings and washings and electrolytic purification[14] we reached a typical background current of $\sim10^{-12}$ A under an electric field of 1.5 kV/cm. The background current is therefore of the same order of magnitude as the injected current. This was not a big trouble, however, since the recorded wave form is due to the injected current, the background current being a constant signal. We found that the decay wave form for positive ions was the sum of two straight lines. Therefore, two species of positive ions were present in our samples, with different mobility. The measurements reported here concern the faster species, the time of flight of which was taken as the intercept of the two straight lines. When one measures very low drift velocities, care must be taken in order to

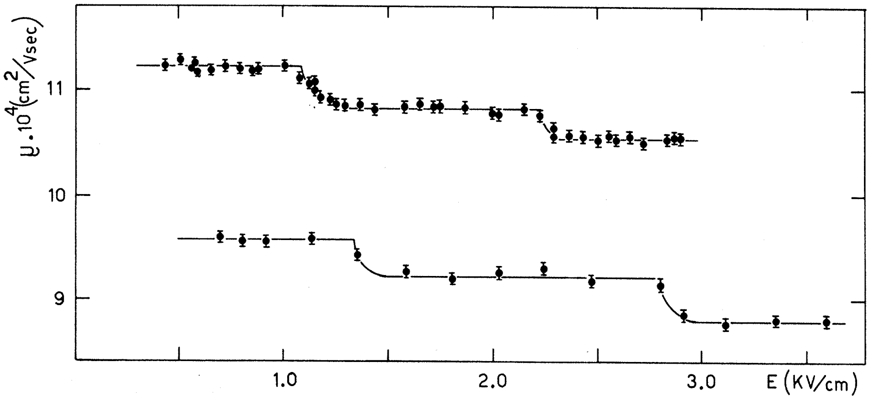

FIG. 1. Two examples of results obtained in liquid nitrogen at 77°K, where the first and the second steps are evident. Upper curve; negative ions, $P = 1.3$ atm. Lower curve; positive ions, $P = 1.8$ atm.

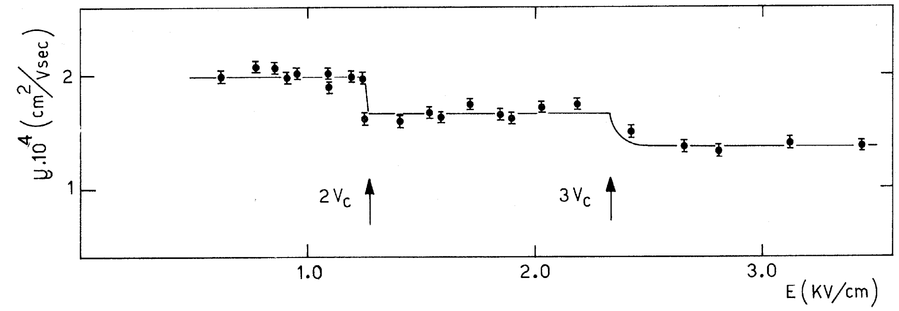

FIG. 2. Positive ionic mobility in liquid carbon tetrachloride at saturation vapor pressure $T = 293°K$. The displayed steps are the second and the third. Every point is the mean of five or more measurements.

minimize the liquid drag effect.[15] The ions lose momentum against the liquid which is forced to move forward in the direction of the mean ion motion. The liquid in turn drags the charges and the measured mobility turns out to be greater than the true one. The presence of this effect can be monitored as a dependence of the mobility on the current density, and as a continuous decrease of the mobility with electric field. To avoid it, one must work at low current density. The results of 10 runs with positive ions gave steps which occur at drift velocities that are multiples of 1.2×10^{-1} cm/sec. An example of our results is shown in Fig. 2.

The results of our measurements are in agreement with those performed in liquid helium II, which gave steps in three different laboratories[1,3,4] and with two different methods.[16,2] They are also in agreement with Henson's results,[6] which showed steps in liquid nitrogen and argon, with a method similar to the present one, but with a field emission source.

We have no explanation of the step effect. At present the simplest way we found to interrelate the four critical velocities is to plot them as a function of temperature, as shown in Fig. 3. This plot cannot be the most significant one, but it is striking that the description is so simple. The dependence of v_c on the various physical properties of the liquids is in some way summarized by a simple temperature dependence. We think, however, that experiments in many other liquids are needed in order to connect v_c to the physical properties of the liquid state.

We want to remark here that the critical velocities in N_2, Ar, and CCl_4 are very small compared with the mean thermal velocities. Therefore, if the steps were due to some kind of inelastic ion collision (as proposed, for example, in the Cope and Gribbon model) one must see a continuous decrease of the mobility as a function of the drift velocity. On the other hand we found steps, and therefore the explanation must be found in a physical effect which involves times much greater than the relaxation times.

This point suggests that the steps should be a hydrodynamical effect, in agreement with our conclusion about our results in liquid helium II,[4] where we found that $\langle v_c \rangle$ is strongly dependent on the normal-superfluid hydrodynamics.

The hydrodynamical nature could probably explain the instability of the effect observed in liquid helium.[1] We found instability in the normal

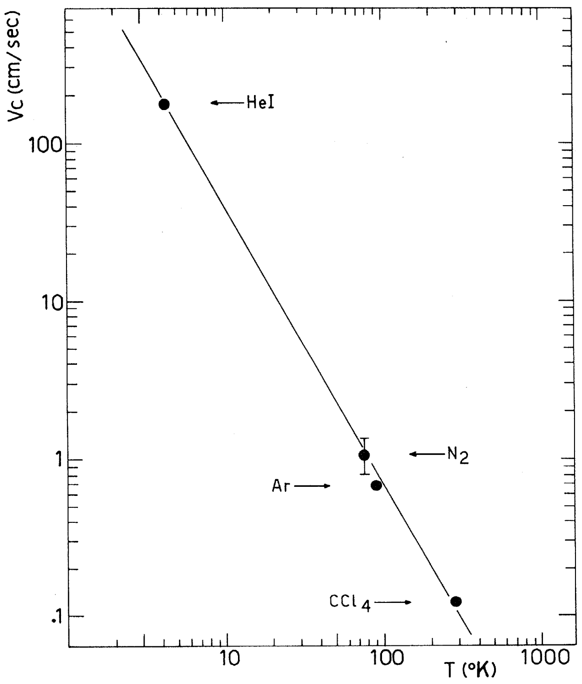

FIG. 3. Critical velocity v_c for the first discontinuity of positive-ion mobility versus temperature. The four experimental points correspond, respectively, to He I, N_2, Ar, and CCl_4.

liquid also, in the sense that sometimes the first step only was found, or the first and the third but not the second.

Very recently, measurements of mobility in liquid helium at 1.14°K have been reported by Schwarz,[17] without evidence of steps. He measured with a method which is practically the same as the present one, and ascribes the steps to spurious effects due to the method used previously.[1,3,4] We think that it seems very unlikely that an effect of the method could give results simply related to the normal superfluid densities[4] and radically different below and above the λ point.[18] Moreover, the present results and those reported by Henson[6] rule out any explanation of the disagreement in terms of trival spurious effects due to the method of measurement.

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THEORY OF STRUCTURE IN THE SUPERFLUID HELIUM SPECTRUM CONSIDERING ROTON-ROTON RESONANCES*

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Interactions between excitations in superfluid helium are investigated by Green's function techniques. In our model, a sharp peak appears in the two-excitation spectrum corresponding to a resonance of two rotons. Interaction of the resonance with the single-particle spectrum results in a hybridization and splitting of the one-particle spectrum into two distinct branches. These theoretical results are in agreement with recent neutron-scattering and Raman-scattering experiments.

The excitation spectrum of superfluid He^4 has attracted considerable interest in the last thirty years following the pioneering work of Landau.[1] Recent developments in experimental techniques have made possible the observation of anomalous structure in the energy spectrum with reasonable accuracy. The aim of the present work is to explain some of these anomalies as a result of the roton-roton interaction.

Recent neutron-scattering experiments[2] have exhibited two particularly strange features which are shown in Fig. 1(a) by the solid lines: (a) the peculiar flatness of the spectrum for large momenta ($K < 2k_0$) at an energy approximately equal to twice the single roton energy, and (b) the existence of another branch of the spectrum (for $K < 2k_0$) which is continuously connected to the free helium spectrum at very large momenta ($K > 2k_0$).[2]

The possibility of observing two rotons by neutron scattering was first suggested by Pitaevskii.[3] Later Anderson proposed that the flat part of the spectrum near 2Δ may be associated with the