dependent solution of the Livermore paper VII. The convergence test function suggested by Cutkosky in Ref. 8 was found to work well. We also found indications that a generalization to the n-p case will be helpful in determining the phase-shifts in the isospin-0 channel.

I am indebted to Professor R. E. Cutkosky for suggesting this problem and for guidance throughout. I also thank Professor B. B. Deo and Professor L. Wolfenstein for very useful suggestions and enlightening discussions.

---

*Work supported in part by the U. S. Atomic Energy Commission.

†Address after August 1970: Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark.

[1]R. E. Cutkosky and B. B. Deo, Phys. Rev. Lett. 20, 1272 (1968), and Phys. Rev. 174, 1854 (1968).

[2]R. E. Cutkosky and B. B. Deo, Phys. Rev. D 1, 2547 (1970).

[3]M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 182, 1714 (1969). References to their earlier work are given there.

[4]M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

[5]H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. 105, 302 (1956).

[6]The same was done in Ref. 2.

[7]We are indebted to Professor M. H. MacGregor for sending us the matrices in the form of punched cards.

[8]R. E. Cutkosky, Ann. Phys. (New York) 54, 350 (1969). The convergence test function involves ratios of successive coefficients in a Tschebyscheff polynomial expansion, the minimization of which constrains the convergence of higher terms in the expansion to be more like what, theoretically, is expected.

[9]Since we used the Livermore second-derivative matrices, all χ^2 values given in this paper are only differences from their $\chi_{\min}{}^2$. Thus, with ten phase shifts (ten terms in optimized polynomials, correspondingly), the theoretically best values of χ^2 and Φ would be 4 and 5, respectively.

MODEL FOR SCATTERING AT ALL ANGLES*

Shu-Yuan Chu and Archibald W. Hendry
Physics Department, Indiana University, Bloomington, Indiana 47401

(Received 29 April 1970)

We present a simple absorptive optical model for all scattering angles. For simplicity, we discuss the case of pion-nucleon charge-exchange scattering and show how the model contains in a unified way the dip structures in both forward and backward directions. For comparison with experiment, we present a fit to elastic π^+p scattering at 3 GeV/c where data are available at all angles.

A large number of models have been devised to explain the many structures observed in scattering cross sections.[1] However, this is always done in a piecemeal fashion. For example, high-energy forward scattering is usually approximated by t-channel Regge poles plus cuts or absorption, while backward scattering is described separately in terms of u-channel exchanges. In the present paper, we present a model for all scattering angles; in particular, we show that it successfully correlates observed structures in the forward and backward directions. The model is based on the familiar optical and absorptive models.[2]

We shall first of all illustrate the model by discussing the uncluttered case of pion-nucleon charge-exchange scattering. Since no all-angle data are available for this reaction except at low energies, we then use the model, with the appropriate modifications for elastic scattering, to obtain a fit to elastic π^+p scattering at 3 GeV/c where there are data at all angles.

The well-known features in the differential cross section for pion-nucleon charge-exchange (CEX) scattering are[3] (1) a dip at $t=0$, (2) a valley near the forward direction at $t\approx-0.6$ (GeV/c)^2, and (3) a valley near the backward direction at $u\approx-0.2$ (GeV/c)^2, accompanied by a sharp rise on approaching 180°. These features are more or less energy independent. We shall now show how they arise in a direct-channel picture.

From Jacob and Wick,[4] the s-channel helicity amplitudes may be expressed in the form

$$f_{++}(E,\theta)=k^{-1}\sum_j(j+\tfrac{1}{2})T_{++}{}^j(E)d_{\frac{1}{2}\frac{1}{2}}{}^j(\theta),$$
$$f_{+-}(E,\theta)=k^{-1}\sum_j(j+\tfrac{1}{2})T_{+-}{}^j(E)d_{-\frac{1}{2}\frac{1}{2}}{}^j(\theta),$$

where k is the center-of-mass momentum. The experimental feature (1) above indicates that the helicity-flip amplitude of $f_{+-}(\theta)$ is the dominant amplitude for this process, at least near the forward direction. In fact, we shall show that features (2) and (3) also come about naturally from $f_{+-}(\theta)$; so, for the time being, let us concentrate on this amplitude.

At this stage, we invoke the ideas of the optical and absorptive models. This means crudely that, in the summation over j, there is little or no contribution from the low values of j since they are absorbed, while above a certain value of j there is likewise no contribution since these waves are unscattered. Hence, for any given energy, there is a particular band of j values in the above summation which is particularly important and gives almost all of the scattering.

Suppose, therefore, for the sake of discussion, that at any given energy only one value of j is important. One can then see directly how features (2) and (3) come about by considering the Bessel-function approximation for the d function,

$$d_{\lambda\mu}{}^{j}(\theta) \sim J_{\mu-\lambda}((2j+1)\sin\tfrac{1}{2}\theta)$$

for large j and small θ, and using the relations[4]

$$d_{\lambda\mu}{}^{j}(\theta) = (-1)^{j+\lambda} d_{\lambda,-\mu}{}^{j}(\pi-\theta) = d_{-\mu,-\lambda}{}^{j}(\theta).$$

Forward direction. – Here

$$f_{+-}(\theta) \propto J_1((2j+1)\sin\tfrac{1}{2}\theta) \approx J_1(b(-t)^{1/2}),$$

where b is the impact parameter and t the momentum transfer. This form clearly embodies the forward-scattering features (1) and (2), these being associated with the first two zeros of $J_1(x)$ which occur at $x=0$ and $x=3.83$, respectively. The second zero coincides with $t \approx -0.6$ $(\mathrm{GeV}/c)^2$ provided that the impact parameter $b \approx 5$ $(\mathrm{GeV}/c)^{-1} \equiv 1$ F, an eminently believable number.

Backward direction. – Here

$$f_{+-}(\theta) \propto J_0((2j+1)\cos\tfrac{1}{2}\theta) \approx J_0(b(-u)^{1/2}),$$

where we have taken $(M^2-\mu^2)^2/s$ to be negligible. This form is consistent with the backward-scattering features (3); the sharp falloff from the 180° direction, followed by the pronounced valley, resembles a $J_0(x)$ behavior. Using the impact parameter $b \approx 1$ F from the forward direction, we can now calculate where the dip position in the backward direction should be. Since the first zero of $J_0(x)$ occurs at $x=2.41$, an impact parameter $b \approx 1$ F gives the dip position at $u \approx -0.2$ $(\mathrm{GeV}/c)^2$! This agrees quantitatively with the experimental value. Thus the same d function gives both the forward and the backward dips.

It is very encouraging that this model is able to correlate the forward and backward structures of πN CEX scattering. It has been remarked before, in particular by Ross and co-workers at Michigan[5] (as well as by Harari[6]), that these features may be related to the oscillations of the J_0, J_1 Bessel functions. They have furthermore pointed out the absorptive optical model as the probable physical picture. Here, however, we have illustrated how these structures arise naturally from a general expansion over d functions. Both forward and backward dip positions are consistent with scattering from an absorbing disk of radius about 1 F, the main contribution coming from the edge of the disk. The direct use of the d functions, moreover, has the important advantage that it provides a continuous extrapolation between forward and backward directions, which is not possible if we consider the Bessel-function approximations. The experimental dip structures are to be associated with zeros of d functions.

So far, the discussion has been in terms of only one j value. One would certainly expect several j values to be important, these values for any given energy being centered around $j+\frac{1}{2} \approx kb$, with $b \approx 1$ F. In fact it is necessary to use several adjacent values of j in order to get the correct ratio of forward-backward magnitudes. In the forward direction for the above example, all the $d_{\frac{1}{2}\frac{1}{2}}{}^{j}$ functions are positive, since

$$d_{-\frac{1}{2}\frac{1}{2}}{}^{j}(\theta) = (j+\tfrac{1}{2})^{-1}\sin\tfrac{1}{2}\theta(P_{j+\frac{1}{2}}{}' + P_{j-\frac{1}{2}}{}'),$$

while in the backward direction they alternate in sign with successive j. Thus using several adjacent j values causes a strong enhancement in the forward direction, but brings about a cancelation in the backward direction. Also, since the zeros of adjacent $d_{-\frac{1}{2}\frac{1}{2}}{}^{j}$ functions near the forward and backward directions are approximately at the same places, the dip positions in these regions will remain approximately the same [$t=-0.6$, $u=-0.2$ $(\mathrm{GeV}/c)^2$ for CEX] even when several adjacent j values are used. The situation at intermediate angles is more complicated, the resultant shape depending more sensitively on the proportions of the various j's.

As regards the energy dependence of the dip structure, with b taken constant (the corresponding dominant j values at each energy coming from $j+\frac{1}{2} \approx kb$) or a slowly varying function of the energy such as $\ln s$, the positions of the dips near the forward and backward directions [where the Bessel functions $J_n(b(-t)^{1/2})$, $J_n(b(-u)^{1/2})$, $n=0$, 1 for πN scattering, are good approximations for the d functions] will be more or less constant in t and u, respectively. Dips at intermediate angles are expected to move slowly with energy.

Calculations to obtain fits to the experimental data for pion-nucleon scattering are in progress. The case of πN CEX is much simpler to discuss due to the fact that f_{+-} dominates over f_{++} (which behaves as J_0, J_1 in the forward and backward directions, respectively), the latter amplitude being primarily responsible for filling in the valleys of f_{+-}. But there are no data for this reaction at intermediate angles except at low energies. We therefore apply our model to elastic π^+p scattering, where there are preliminary all-angle data[7,8] in the 3- to 5-GeV/c energy range. The theoretical situation for elastic scattering is slightly more complicated. Both f_{++} and f_{+-} are now important. Moreover, the low partial waves which are absorbed out of nonelastic reactions give an appreciable contribution (through unitarity) to elastic scattering, particularly in the forward diffraction peak.

We present, in Fig. 1, a fit to the π^+p data[7,8] at 3 GeV/c where the data for all angles are good. Seven partial waves, $l = 0, 1, \cdots, 6$, have been included, though some are more important than others. For the sake of clarity as to how the model accounts for the experimental features, which vary in magnitude over four decades, we have drawn the separate contributions from the s-channel helicity-nonflip and helicity-flip amplitudes. As is anticipated from our discussion above in terms of the Bessel-function approximations, the forward region is dominated by f_{++} and the backward region by f_{+-}, the pronounced structures in the differential cross section arising from the oscillations of these amplitudes. The valley at $t \approx -2.6$ (GeV/c)^2 is so deep because this position is close to dips in both helicity amplitudes. This striking fit[9] for all angles from forward to backward provides substantial justification for the simple picture described above.

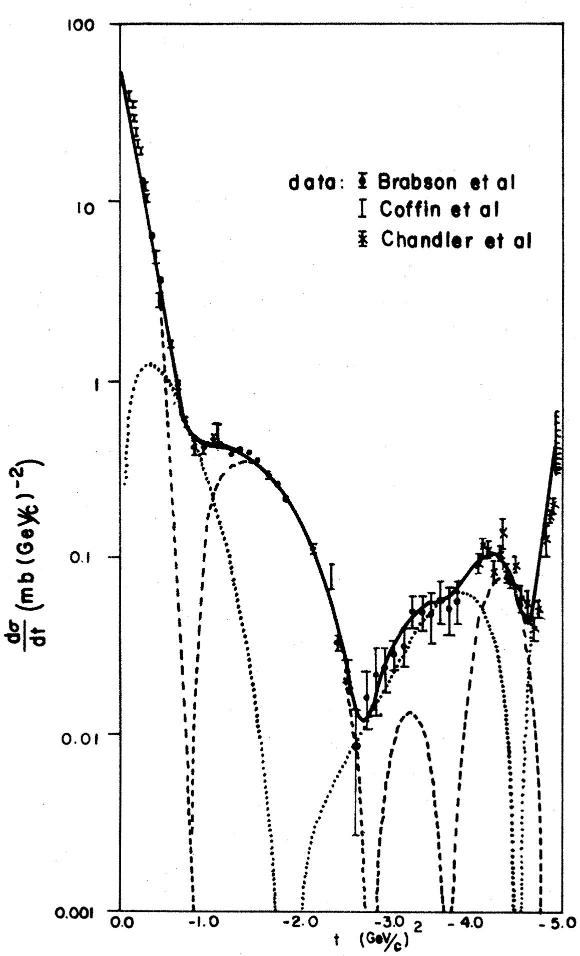

FIG. 1. Fit to the π^+p differential cross section at 3 GeV/c for all angles. Data from Refs. 7 and 8. The dashed and dotted curves are the individual contributions from the s-channel helicity-nonflip and helicity-flip amplitudes, respectively.

This model can obviously be applied to other two-body reactions between particles of any spin, the only difference being that there may be more helicity amplitudes with different d functions. But the structure in t or u should come from zeros of d functions, just as in the πN case.

The model is considerably different from the usual Regge-pole model and the Veneziano model, where the dips come about through nonsense, wrong-signature factors.[1] Whether the two models are complementary or mutually exclusive remains to be investigated. Certainly in the model above, energy dependence has not yet been incorporated, though this is at present being studied. Nevertheless, a model which, as it stands, has embodied in it the correct angular dependence will clearly have a wide range of applications in high-energy phenomenology.

We would like to thank our colleagues at Indiana University for many helpful discussions. We especially thank Professor Marc Ross for stimulating conversations and discussions about similar work being done at Michigan.

---

*Work supported in part by the National Science Foundation.

[1]See, for example, J. D. Jackson, in *Proceedings of the Lund International Conference on Elementary Particles*, edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1970).

[2]F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, Phys. Rev. 182, 1579 (1969); R. C. Arnold, Phys. Rev. 153, 1523 (1967); J. N. J. White, Nucl. Phys. B13, 139 (1969); A. Dar and W. Tobocman, Phys. Rev. Lett. 12, 511 (1964); A. Dar, T. L. Watts, and V. F. Weisskopf, Nucl. Phys. B13, 477 (1969); S. C. Frautschi and B. Margolis, Nuovo Cimento 56A, 1155 (1968); C. B. Chiu and J. Finkelstein, Nuovo Cimento 57A,

649 (1968).

[3]A. V. Stirling *et al.*, Phys. Rev. Lett. 14, 763 (1965); P. Sonderegger *et al.*, Phys. Lett. 20, 75 (1966); R. C. Chase *et al.*, Phys. Rev. Lett. 22, 1137 (1969).

[4]M. Jacob and G. C. Wick, Ann. Phys. (New York) 7, 404 (1959).

[5]M. H. Ross, in Proceedings of the Regge Pole Conference, University of California, Irvine, Calif., 1969 (unpublished), and private communication.

[6]H. Harari, in Proceedings of the Regge Pole Conference, University of California, Irvine, Calif., 1969 (unpublished).

[7]B. B. Brabson *et al.*, "Intermediate Angle πp Elastic Scattering from 3 to 5 GeV/c" (to be published).

[8]C. T. Coffin *et al.*, Phys. Rev. 159, 1169 (1967); J. P. Chandler *et al.*, Phys. Rev. Lett. 23, 186 (1969).

[9]Details of this fit, as well as similar fits for the other energies, will be given in a separate article.

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s\to\infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s\to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.

Feynman's parton model[1] for deep-inelastic weak or electromagnetic processes is an expression of the impulse approximation as applied to elementary-particle interactions. In order to apply the impulse approximation we demand the following. We analyze the bound system—be it a nucleon or nucleus—in terms of its constitutents, called "partons." Nucleons are the "partons" of the nucleus and the "partons" of a nucleon itself are still to be deciphered. If we specify the kinematics so that the partons can be treated as instantaneously free during the sudden pulse carrying the large energy transfer from the projectile (or lepton) then we can neglect their binding effects during the interaction and we can treat the kinematics of the collision as between two free particles, the projectile and the parton. Moreover, if we are in a kinematic regime so that energy is approximately conserved along with momentum across the interaction vertex of the parton with the weak or electromagnetic current, the conditions for applying the impulse approximation are satisfied.

The Bjorken limiting region[2] satisfies this condition for the deep inelastic electron scattering from protons as viewed from a certain class of $P\to\infty$ or infinite-momentum frames. The "partons" constituting a proton are strongly bound together as viewed in the rest frame. However, if their bound state can be formed primarily by momentum components that are limited in magnitude below some fixed maximum—i.e., if there exists a finite $k_{\max}$—then as viewed in an infinite-momentum frame these parton states are long-lived by virtue of the characteristic time dilatation. The derivation of this intuitively appealing picture from a canonical quantum field, modified by imposing a maximum constraint on $k_\perp$, has been discussed as well as its applicability to the particular class of amplitudes with "good currents."[3] In particular, the ratio $Q^2/2M\nu$, where $Q^2>0$ is the negative of the square of the invariant momentum transfer and $q\cdot P=M\nu$, measures the fraction $x\equiv Q^2/2M\nu$ of the longitudinal momentum on the parton from which the electron scatters and is a finite fraction $0<x<1$ in the Bjorken limit.

It is easy to show that the ratio x must be finite in order to apply the impulse approximation. Otherwise as x approaches very close to 0 or 1 we will be forced to deal with very slow partons in the $P\to\infty$ system, or, as seen in the rest system of the proton, with the high-momentum extremities of the bound-state structure, and for these the impulse approximation breaks down.

The beauty of the electron scattering is that it allows us to "tune" the mass of the virtual photon line as we choose to probe finite x. However when we return to the world of only real external hadrons, we have no large mass since $Q^2\to M^2$ while $2M\nu\to s$, the total collision energy. In this case x becomes very small,[1] or "wee." Our condition for applying the impulse approximation also fails and the value of the parton con-