1968 (International Atomic Energy Agency, Vienna, 1968), p. 179; O. Nathan, *ibid.*, p. 191, and references to experiments therein.

[4]M. Oothoudt, P. Vedelsby, and N. M. Hintz, Bull. Amer. Phys. Soc. 14, 509 (1969); John Williams Laboratory, University of Minnesota, Annual Report No. C00-1265-83, 1969 (unpublished), p. 71.

[5]J. R. Maxwell, G. M. Reynolds, and N. M. Hintz, Phys. Rev. 151, 1000 (1966); J. H. Bjerregaard, O. Hansen, O. Nathan, and S. Hinds, Nucl. Phys. 86, 145 (1966); W. McLatchie, J. E. Kitching, and W. Darcey, Phys. Lett. 30B, 529 (1969); K. Yagi, Y. Aoki, J. Kawa, and K. Sato, Phys. Lett. 29B, 647 (1969).

[6]G. J. Igo, P. D. Barnes, and E. R. Flynn, Phys. Rev. Lett. 24, 470 (1970).

[7]T. H. Braid, R. R. Chasman, J. R. Erskine, and A. Friedman, Phys. Rev. C 1, 275 (1970).

[8]J. R. Erskine and R. H. Vonderohe, to be published.

[9]We are indebted to Dr. B. Bayman for this computer program.

[10]Bjerregaard, Hansen, Nathan, and Hinds, Ref. 5.

[11]C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (Wiley, New York, 1967), 6th ed.; E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear Properties of the Heavy Elements* (Prentice-Hall, Englewood Cliffs, N. J., 1964); S. Bjornholm, thesis, Institute for Theoretical Physics, University of Copenhagen, 1965 (unpublished).

[12]S. Bjornholm, J. Dubois, and B. Elbek, Nucl. Phys. A118, 241 (1968).

[13]F. E. Durham, D. H. Rester, and C. M. Class, Phys. Rev. Lett. 5, 202 (1960); see also Bjornholm, Ref. 11.

[14]J. O. Rasmussen, Nucl. Phys. 19, 85 (1960).

[15]See, for instance, O. Mikoshiba, R. K. Sheline, T. Udagowa, and S. Yoshida, Nucl. Phys. A101, 202 (1967).

[16]V. G. Soloviev, At. Energy Rev. 3, 117 (1965); R. R. Chasman, Phys. Rev. 138, B326 (1965).

[17]It would be most interesting to study the (t,p) reaction in the actinides. Unfortunately, the one published result on $U^{238}(t,p)U^{240}$ [R. Middleton and H. Marchant, in *Proceedings of the Second International Conference on Nuclidic Masses, Vienna, Austria 1963* (Springer, Vienna, Austria, 1964)], gives angular distributions only for the ground state and first excited state. In the one spectrum shown, the excitation region of interest is obscured by impurities.

SEPARATION OF DIRECT-REACTION AND COMPOUND-NUCLEUS CONTRIBUTIONS TO (p,p') REACTIONS IN Sn ISOTOPES*

B. L. Cohen, G. R. Rao, C. L. Fink, J. C. Van der Weerd, and J. A. Penkrot
University of Pittsburgh, Pittsburgh, Pennsylvania 15213
(Received 30 March 1970)

By comparing energy spectra of protons from (p,p') reactions on various tin isotopes, a clean separation between direct-reaction and compound-nucleus contributions is made and the two processes are studied independently. The compound-nucleus results are in good agreement with predictions from the Gilbert-Cameron level densities. The direct-reaction angular distributions become quite isotropic for large $-Q$; the direct reaction cross section at 17 MeV is about one-third of the total reaction cross section.

Measurements were made of energy distributions of protons emitted at various angles following bombardment of various Sn-isotope targets with 17-MeV protons. Typical results are shown in Fig. 1. The cross lines on the curves there show the maximum proton energy for which subsequent neutron emission is energetically possible; the increased intensity to the left of these cross lines in the spectra from Sn^{112} and Sn^{114} are clearly due to (p,np) reactions, so we will temporarily ignore that part of the spectrum. The peaks in the spectra for $Q \leq -9$ MeV are due to $(p,n\tilde{p})$ reactions from excitation of isobaric analog states in direct (p,n) reactions, and the peaks in the spectra for $Q \gtrsim -6$ MeV are due to excitation of individual collective states by direct reactions, but these are of no concern here. The aspect of Fig. 1 we emphasize here is the difference between the spectra from light and heavy isotopes, and its regular progression with mass number in the region above and a short distance below the (p,np) threshold where there are no peaks or where these peaks can be averaged over.

There are two well-known processes leading to (p,p') reactions in this mass region, direct reaction (DR) and compound nucleus (CN). The DR process is controlled by the relationship between the nuclear structure of the ground and excited states, and this relationship is extremely similar for all even-A Sn isotopes; we therefore expect the DR contribution from these isotopes to be virtually identical. The cross section for the (p,p') CN process, on the other hand, is very sensitive to competition from (p,n), and hence is highly sensitive to the Q value for the latter reaction. In the heaviest isotopes where $Q(p,n)$ is near zero [the value for $Q(p,p')$], neutron

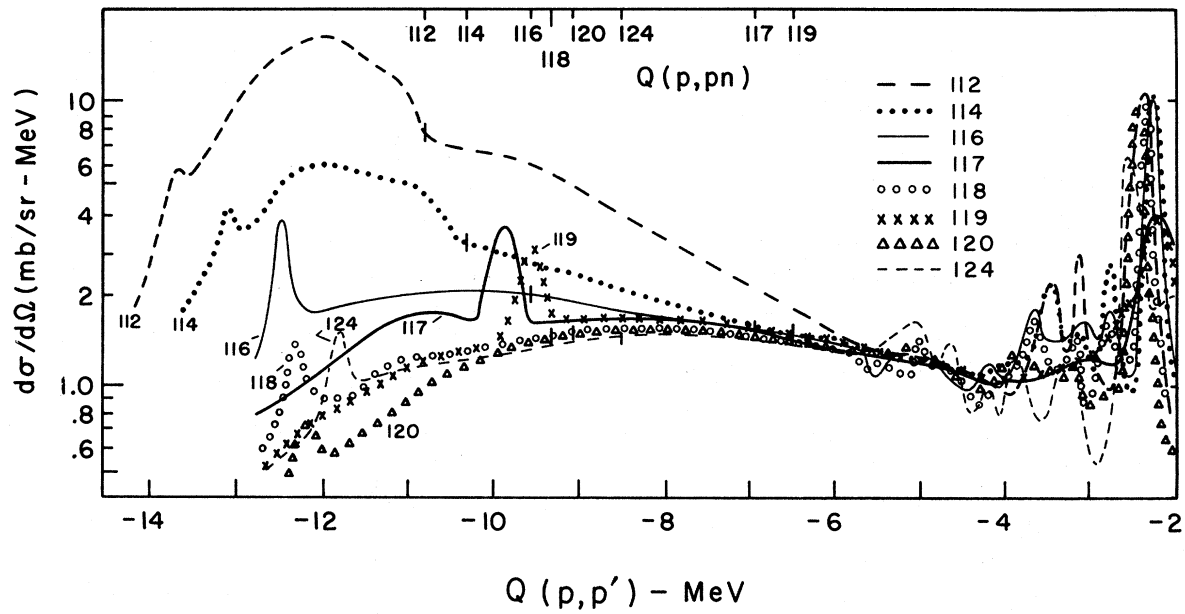

FIG. 1. Energy spectra of protons emitted at 75° following bombardment of various isotopes of Sn with 17-MeV protons. Background effects are not negligible for $Q<-10$ MeV. Energies are expressed as the Q values of (p,p') reactions that would give the observed laboratory energies. Vertical cross lines show the maximum proton energy which allows a neutron to be emitted subsequently; this is $Q(p,pn)$. The treatment given in this paper ignores the peaks at $Q\leq -9$ MeV [due to $(p,n\bar{p})$ reactions], and ignores or averages over the higher-energy peaks.

emission predominates and we expect very little CN contribution to (p,p'), whereas in the light isotopes where $Q(p,n)$ becomes progressively more negative with decreasing A, the CN contribution to (p,p') should increase progressively. Since the spectra for all isotopes with $A>117$ in Fig. 1 are essentially identical in the region of interest described above, we may conclude that the CN contribution is entirely negligible for the heavier isotopes; we therefore take the results for them to be the DR contribution, and assume it to be the same for all isotopes. The DR contribution may then be subtracted from the spectra for the light isotopes to give the CN contribution in them. Thus we are able to make a clean separation between DR and CN contributions and to study each independently. Previously, such a clean separation has only been possible after long and painstaking fluctuation experiments, and the difficulties with that technique have precluded its use to study the two processes in detail.

In the remainder of this Letter, we summarize some of the conclusions we have obtained from these studies.

Compound-nucleus process. – Due to obvious intensity considerations, CN was studied principally in Sn^{112}, and to a lesser extent in Sn^{114}. The energy spectra were analyzed in the usual way by plotting $\ln(N/\sigma_p E)$ vs E, where N is the measured intensity at energy E (in the center-of-mass system) and σ_p is the total reaction cross section for protons at that energy as obtained from optical-model calculations. Values of σ_p were found to be very insensitive to the choice of optical-model parameters from among those reported in the literature. These plots were then compared with plots from statistical theory[1] for various values of the level-density parameter a. The experimental data are in agreement with these for

$$a=(0.16\pm 0.03)A \quad \text{(expt)},$$

where A is the mass number. This is in reasonable agreement with the value given by Gilbert and Cameron,[1]

$$a=0.142A \quad \text{(G-C, theor)}.$$

A much more sensitive test of statistical theory is the prediction of the relative probability for emission of protons and neutrons, f_p/f_n, in CN reactions.[2] In terms of these, the cross sections for (p,p') and (p,n) reactions via CN (including reactions in which other particles are subsequently emitted) are

$$\sigma(p,p')=\sigma_p(\text{CN})f_p/(f_p+f_n),$$
$$\sigma(p,n)=\sigma_p(\text{CN})f_n/(f_p+f_n),$$
$$\sigma(p,p')/\sigma(p,n)=f_p/f_n. \qquad (1)$$

The (p,n) reaction, with the exception of excitation of the isobaric analog state by DR which is easily subtracted off in the analysis here, is known[3] to be almost exclusively CN and to lead predominantly to the emission of low-energy neutrons. In Sn^{112}, where $(p,2n)$ reactions are energetically impossible, all of these reactions become (p,np) and their cross section may be estimated from their contribution to the proton spectrum for Sn^{112} below $Q=-11$ MeV. This was done by smoothly extrapolating the curve for (p,p') with due attention to the shapes of the curves for the other isotopes, and assuming that the remaining protons are from (p,np).[4] While the above extrapolation cannot be very accurate, it does not introduce a large error into the (p,np) cross section since the latter reaction has a much larger cross section (note that the ordinate in Fig. 1 is logarithmic), and it does not cause large uncertainties in the (p,p') cross section since the major contribution does not come from this energy region. Thus we can directly determine the ratio f_p/f_n for Sn^{112}. The result is

$$f_p/f_n \simeq 1.1 \pm 0.3 \quad (Sn^{112}, \text{ expt}).$$

The compound-nucleus cross sections for (p,p') and (p,n) are about 340 and 310 mb, respectively.

The ratio f_p/f_n may be calculated from statistical theory. The expression for f_p is

$$f_p(E_{0p}) = \int_0^{E_{0p}} \sigma_p(E) E \omega(E_{0p}-E) dE,$$

where E_{0p} is the maximum energy available to an emitted proton (aside from a pairing correction), $\sigma_p(E)$ is the cross section for formation of a compound nucleus by a proton of energy E, and $\omega(\epsilon)$ is the density of levels at excitation ϵ in the nucleus left after emission of a proton. The expression for f_n is completely analogous although the values are quite different because σ_p includes a Coulomb-barrier penetrability while σ_n does not. Since $\omega(\epsilon)$ increases exponentially with increasing ϵ, f_n is a very sensitive function of E_{0n} (it approximately triples for each 1-MeV increase in E_{0n}). Since $E_{0n} = 17 \text{ MeV} - Q(p,n)$, whereas E_{0p} is approximately the same for all isotopes, the ratio f_p/f_n is quite sensitive to $Q(p,n)$. For Sn^{112}, $Q(p,n)$ is not accurately known, but making a reasonable estimate $[Q(p,n) = -9.3 \text{ MeV}]$, and using the Gilbert-Cameron level-density prescriptions, we find

$$f_p/f_n \simeq 1.5 \pm 0.5 \quad (Sn^{112}, \text{ theor}).$$

Considering the complexity and sensitivity of the theory, this is a very satisfactory agreement.

For Sn^{114}, $(p,2n)$ reactions are energetically possible so we cannot determine f_p/f_n directly; it can, however, be estimated from the first reaction of (1). We take

$$\sigma_p(\text{CN}) = \sigma_p - \sigma(p,p')_{\text{DR}},$$

where σ_p is obtained from optical-model calculations, and the last term is the cross section for (p,p') via direct reactions, the determination of which will be described below. This gives

$$f_p/f_n \approx 0.19 \pm 0.04 \quad (Sn^{114}, \text{ expt}),$$

which compares reasonably well with the theoretical estimate,

$$f_p/f_n \approx 0.10 \quad (Sn^{114}, \text{ theor}).$$

A much cruder experimental estimate for Sn^{116} gives $f_p/f_n \approx 0.04$ with about a factor of 2 uncertainty. This compares well with the theoretical estimate, $f_p/f_n \approx 0.02$. For Sn^{120} and Sn^{124}, the theoretical predictions are $f_p/f_n \approx 0.002$ and 0.000 25, respectively. If these are valid, the CN contribution to the spectrum in Fig. 1 for these is entirely negligible, as we have assumed. The extremely rapid variation of f_p/f_n between different isotopes indicates how impressive the agreement between theory and experiment really is.

Direct-reaction process.—Data like those in Fig. 1 for Sn^{120} and Sn^{124} may be used directly for studies of the DR process. One interesting question is how the angular distribution of protons from DR (p,p') processes varies with Q value. Some data on this are shown in Fig. 2, where we see that angular distributions become more isotropic with decreasing proton energy. At angles beyond ~75° for $-Q > 9$ MeV, angular distributions are not less isotropic than those for CN reactions. This indicates that angular distributions may not be used to separate CN and DR; this method was somewhat popular at one time.

An interesting question about DR concerns its cross section. This was not carefully studied in these experiments because forward-angle contributions, which are quite significant, could not be measured due to background from reactions of elastically scattered protons in the detector and to difficulties with H, C, and O impurities. If we take the 75° data to represent a true average of all angles, we find $\sigma(\text{DR}) \simeq 280$ mb, of which 200 mb is due to reactions with $Q < -3$ MeV. If the 50° data are taken to represent this average, we find $\sigma(\text{DR}) \approx 500$ mb. It therefore seems most

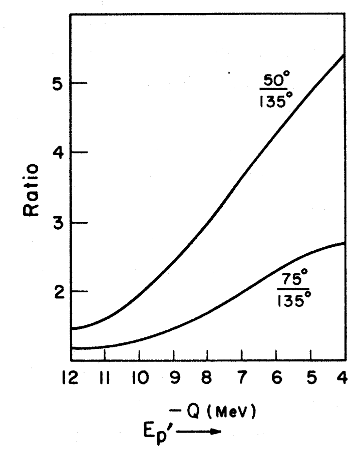

FIG. 2. Angular-distribution data on direct reactions. These curves represent the average intensity ratios between the angles shown for (p,p') reactions on Sn^{118}, Sn^{119}, Sn^{120}, and Sn^{124} versus their Q value. The energies of emitted protons are a few tenths of an MeV less than (17 MeV$-Q$), so the abscissa is essentially a scale of emitted proton energy increasing from 5 to 13 MeV.

likely that σ(DR) is at least 300 mb, and may be much larger. A basically more sound although practically less accurate approach is to subtract the CN contribution for Sn^{112} from the total reaction cross section as calculated from the optical model. The latter is ~1000 mb and the former is 650 mb, which would give $\sigma(\mathrm{DR}) \sim 350 \pm 150$ mb. The uncertainty here represents the sum of the uncertainties in the two numbers which are being subtracted.

By either estimate, it seems probable that DR accounts for at least one-third of the total reaction cross section at 17 MeV. This would seem to be a surprisingly large result for so low a bombarding energy.

---

*Work supported by the National Science Foundation.

[1]A. Gilbert and A. G. W. Cameron, Can. J. Phys. 43, 1446 (1965).

[2]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[3]R. M. Wood, R. R. Borchers, and H. H. Barschall, Nucl. Phys. 71, 529 (1965).

[4]$(p,2p)$ is also energetically favorable in Sn^{112}, but a search for this reaction using coincidence detection of the protons showed its cross section to be less than 10 mb.

APPLICATION OF OPTIMIZED POLYNOMIAL EXPANSIONS TO THE ANALYSIS OF p-p SCATTERING*

Yung-An Chao†
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 27 April 1970)

The optimized polynomial expansions of p-p amplitudes represented by a chosen set of five invariant amplitudes are given and the transformation matrix between them and the conventional partial waves is worked out. Through this matrix, we have re-examined the results of the elastic p-p phase-shift analysis below 400 MeV done by the Livermore group and found very significant improvements.

The method of optimized polynomial expansion[1] was found very helpful by Cutkosky and Deo in analyzing the K^+-p scattering data.[2] Here we apply it to analyze p-p scattering data [indirectly, as explained in Sec. (2) of this Letter] to see how many fewer phase shifts will be needed, compared with the analysis by the Livermore group,[3] and if the π^0-p coupling constant can be determined better in a single-energy analysis. We conformally mapped the cut $x = \cos\theta_{\mathrm{c.m.}}$ planes of analyticity of the real and imaginary parts of the scattering amplitudes into two separate ellipses[1] (because of the different domains of analyticity) and called the new variable, in either case, z. The prescription for obtaining $z = z(x)$ is given in Ref. 1. We then expanded in powers of z the following five invariant amplitudes:

$$A = 2(1+\beta)A' - \beta C - 2\beta x E = \sum_{\text{even } n} a_n z^n, \tag{1a}$$

$$B = 2(F_1 + F_3) = \sum_{\text{even } n} b_n z^n, \tag{1b}$$

$$C = F_1 - 2F_3 - F_5 = \sum_{\text{even } n} c_n z^n, \tag{1c}$$

$$D = F_2 + F_4 = \sum_{\text{odd } n} d_n z^n, \tag{1d}$$

$$E = F_2 = \sum_{\text{odd } n} e_n z^n, \tag{1e}$$

where $A' = F_1 - 3F_3$, $\beta = (p/M)^2$, p = c.m. momentum, M = mass of the proton, and the F's are invariant amplitudes defined by Goldberger, Grisaru, MacDowell, and Wong[4] (which are assumed