might expect the emitted radiation to be polarized so that the polarization vector lies in the plane determined by $\vec{K}$ and $\hat{n}_0$. The polarization must have a nonvanishing component along $\vec{K}$ so one would expect the intensity of radiation back along $-\vec{K}$ always to be zero.

The above classical analysis shows that a sharply defined optical electric field or a sharply defined notch in such a field can produce bunching in an electron beam. This may partly account for the effect Schwarz and Hora observed. Even though the density of electrons in their experiment was only one electron per fifty "bunches" in the primary beam, there is still time coherence between electron beam charge and current density at the screen and the electric field at the interaction region.

However, a quantum mechanical treatment of the problem now being developed seems to indicate that true quantum phenomena may be present. In particular, our wave equations predict that the electron bunching for monochromatic electrons obeys the classical Bessel-function relationship only if $\hbar\omega_0^2 y/(4eV_0 v) \ll \pi/2$. For $\hbar\omega_0^2 y/(4eV_0 v) \gg \pi/2$ and $(4eE_0 v/\hbar\omega_0^2)\sin(\omega_0 d/2v) \ll 1$, the bunching appears to vary sinusoidally with distance and thus not decay to zero as $y\to\infty$. If further study supports this finding or predicts other nonclassical effects, the quantum mechanical analysis will be reported in a future paper.

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## ATTENUATION OF FIRST SOUND NEAR THE LAMBDA TRANSITION OF LIQUID HELIUM

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The attenuation of first sound near the lambda point can be interpreted as being due to two phenomena: (1) a relaxation process described by Pokrovskii and Khalatnikov occurring only below the lambda point, and having a relaxation time of $\xi/C_2$ where $C_2$ is the velocity of second sound and $\xi$ is a coherence length of magnitude $1.36\times10^{-8}\ (T_\lambda - T)^{-2/3}$ cm; (2) a critical attenuation which is nonsingular and symmetrical about $T_\lambda$.

In an earlier investigation Barmatz and Rudnick[1] (BR) in an effort to determine the thermodynamic first-sound velocity near $T_\lambda$ made measurements at the low frequency of 22 kHz. These measurements were sufficiently accurate, and the approach to $T_\lambda$ sufficiently close, that attenuation and dispersion effects were measurable. Because of the low frequency, the results were necessarily inaccurate. The purpose of the present investigation was to use essentially the same apparatus over as wide a frequency range as possible, and we report here the results of the attenuation measurements.

The frequency range is 16.8 kHz to 3.17 MHz in He II, and 600 kHz to 3.17 MHz in He I. With this frequency range, and microdegree temperature resolution, the measurements yield greater detail about the nature of the attenuation than has previously been reported. In particular, at a frequency $\omega$, the maximum attenuation is unambiguously shown to occur at a temperature $T$, below $T_\lambda$, such that $\omega\epsilon^{-1}=$const, where $\epsilon=|T_\lambda - T|$. Recently there has been much theoretical speculation on the nature of first sound at lambda transitions, resulting in a bewildering array of sometimes conflicting results. While there are instances where there is partial agreement between such predictions and our data, we know of none which completely account for the results. In view of this, we make no attempt at a conscientious comparison of the data with existing theory, except as regards the relaxation theory first suggested by Landau and Khalatnikov.[2] Our results, and this situation, point up the need for a unified and complete theory of the attenuation of sound near the lambda point of helium.

The apparatus, except for minor modifications to improve the acoustic response, is described by BR. The acoustic element is a copper cylindrical resonator 2.5 cm long and 2.5 cm in diameter, sealed at both ends by capacitative-type transducers. The active element of the transducer is aluminum-coated Mylar 0.000 63 cm thick. The operating frequency of the transducers is continuously variable up to about 10 MHz. The pulse-echo technique is used to continuously measure the attenuation at frequencies above 600 kHz. Below 300 kHz the attenuation maximum is determined using the resonant method described by BR. The temperature resolution is better than a microdegree. The measurements are made during controlled upward and downward temperature drifts at a rate of less than 0.1 μdeg/sec near $T_\lambda$. Except immediately above the transition, the temperature is measured with a germanium resistor mounted axially inside the resonator. The germanium chip is 0.125 cm by 0.23 cm and is located 0.38 cm above the bottom transducer. $T_\lambda$ is determined by using the heat-transport anomaly described by BR. An added check on $T_\lambda$ is obtained by observing the point at which the minimum in the low-frequency (4 kHz) velocity of first sound occurs. This should occur about 1 μdeg below the value of $T_\lambda$ determined by the transport anomaly and it does. There are limitations in our certainty about the assignment of $T_\lambda$. One arises from the fact that due to the effect of gravity the transition temperature at the bottom of the resonator is 3.3 μdeg less than that at the top.[3] Another is due to the poor thermal conductivity of the He I which leads to spatial temperature variations of several microdegrees. In this region immediately above $T_\lambda$ we are currently using the strong temperature dependence of the first-sound velocity to measure the average temperature in the resonator. This velocity thermometer has been used systematically for the 600 kHz, 1 MHz, and 1.75 MHz measurements. The consistency of these results encourages us to regard the acoustic thermometer as potentially the most accurate device for microdegree measurements near $T_\lambda$. It has the virtue of measuring the average temperature in the resonator and of having the property that the sensitivity increases as $T$ approaches $T_\lambda$. All things considered, we believe our $T_\lambda$ assignment to be accurate to somewhat better than 2 μdeg at the position of the germanium chip.

Figure 1 shows the attenuation, $\alpha$, as a function of temperature for four frequencies both above and below the transition. A temperature-independent background (due to extraneous sources of attenuation) which is usually less than 30% of the peak attenuation is subtracted out in order to study only the attenuation associated with the lambda transition. The background attenuation is equal on both sides of the transition, within uncertainties given in Fig. 1, and is taken to be that which occurs at $\epsilon = 15\times10^{-3}\,°\mathrm{K}$.

He I.—A common theoretical prediction is that as $T$ approaches $T_\lambda$ the attenuation is proportional to $\epsilon^{-n}$, and a distinguishing feature of these theoretical results is the value of $n$ predicted. First of all it is clear from Fig. 1 that the singular nature implied by the power law is not present when $\epsilon$ is small. Excluding small values of $\epsilon$, we find that the results can be fitted with values of $n$ ranging from $\frac{1}{2}$ to 1 depending on the minimum value of $\epsilon$ included in the fitting procedure. With the proviso that deviations for large $\epsilon$ ($\cong 10^{-2}\,°\mathrm{K}$) never exceed the estimated uncertainty (see caption Fig. 1) it is found that very small variations of the assumed background attenuation (within this same uncertainty) are also crucial in determining $n$.[4] One expects that the attenuation should scale as $\omega^2$ (a feature of ordinary fluid hydrodynamics) if the neighborhood of $T_\lambda$ is excluded. We believe it is significant that under conditions where the best fit is obtained with $n = 1$, the resultant expression for the attenuation scales as $\omega^2$. The excluded neighborhood of $T_\lambda$ is greatest for $n = 1$ with significant departures (few percent) from an $\omega^2\epsilon^{-1} = \mathrm{const}$ dependence occurring when $\omega\epsilon^{-1}$ exceeds approximately $4\times10^{10}$ $(°\mathrm{K\ sec})^{-1}$. This is about one third of the value of $\omega\epsilon^{-1}$ at which $\omega\tau = 1$ for the relaxation process to be described in He II. Therefore it is reasonable to conclude that if a relaxation time characterizes sound attenuation in He I it has a value comparable (if not equal) to that existing in He II. Since the total attenuations in He I and He II are not very different, a similar statement can be made about the relaxation strengths. We can add that a $\omega^2$ dependence is not found when other values of $n$ are used.

There is also interest in the frequency dependence of the attenuation exactly at the transition. We find that the attenuation scales as $\omega^{1.3}$ from 16 kHz to 1 GHz. If the high-frequency data[5,6] are not used, then we obtain $\omega^{1.15}$ from 16 kHz to 3.17 MHz. It is noteworthy that Kawasaki[7] predicts $\omega\epsilon^0$ in approximate agreement with our results. This is also the limiting behavior of Ahlers' trial function No. 2.[8] We have had limited success in fitting the attenuation at all frequencies and temperatures by this trail function with the relaxation time and strength for He II

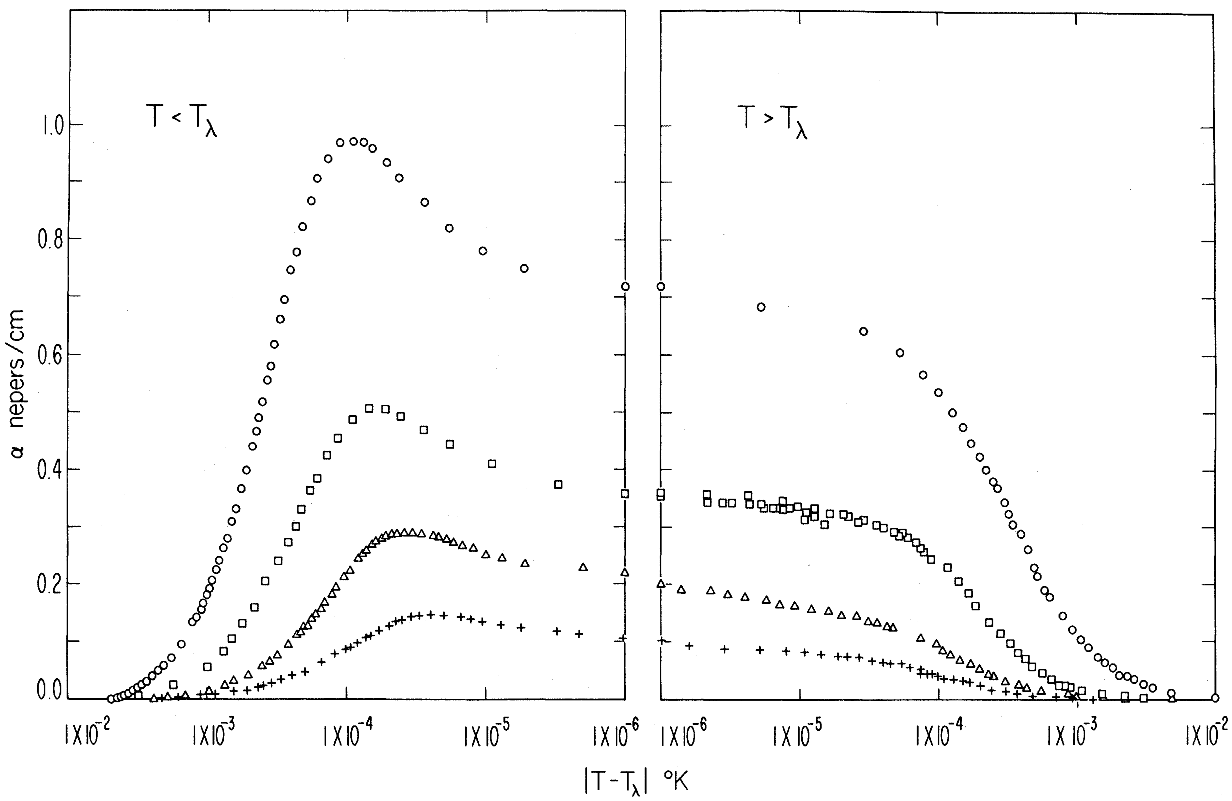


FIG. 1. Attenuation of first sound, as determined from pulse-echo measurements, near the lambda transition of liquid helium. Beginning with the lowest curve the frequencies are 600 kHz, 1 MHz, 1.75 MHz, 3.17 MHz. A temperature-independent background attenuation has been subtracted out in order to study only the attenuation associated with the transition. Note that the attenuation remains finite, is asymmetric about the transition, and exhibits a maximum on the low-temperature side. Based on the scatter of data on different runs, which include upward and downward drifts of temperature, we estimate that the error in assignment of the background attenuation (in Np/cm) to be less than $0.6\times10^{-2}$, $1\times10^{-2}$, $2\times10^{-2}$, $2.5\times10^{-2}$ for the four frequencies, in order of ascending frequency. The uncertainties in $\alpha$ increase near $T_\lambda$ and are estimated to be 5% or less in He II, and may be as much as 15% in He I, near $\epsilon=10^{-6}$ °K.

to be described in the next section.

He II. — The presence of a peak in attenuation was first predicted by Landau and Khalatnikov[2] and was associated with an order-parameter relaxation process. They concluded that the relaxation time $\tau$ should be proportional to $\epsilon^{-1}$, and while this has been observed, their prediction was based on an incorrect temperature dependence for $\rho_s$, and when their result includes the correct temperature dependence an unacceptable value for $\tau$ is obtained. This has been emphasized by Pokrovskii and Khalatnikov[9] who ascribe the relaxation process to a coupling of first and second sound and find

$$\tau=\xi/C_2, \tag{1}$$

$$\alpha=\frac{U_\infty-U_0}{U_\infty U_0}\frac{\omega^2\tau}{1+\omega^2\tau^2}, \tag{2}$$

where $U_0$ and $U_\infty$ are, respectively, the sound velocities in the low- and high-frequency limits, $C_2$ is the velocity of second sound, and $\xi$ is a coherence length.[10] The coupling leads to an irreversible exchange of energy between the two modes. Far from $T_\lambda$, in the region $\omega\tau\ll1$ the energy exchange between first and second sound occurs in almost reversible processes, and the loss in first-sound wave energy is small. Very near $T_\lambda$ in the region $\omega\tau\gg1$ second sound cannot be generated since its wavelength is less than the coherence length, and so the energy loss from the first-sound wave is again small. The peak in attenuation occurs when $\omega\tau=1$. Now $\xi\sim\rho_s^{-1}$ and $C_2\sim\rho_s^{1/2}$. When $T\to T_\lambda$ one expects, since $\rho_s\sim\epsilon^{2/3}$, that $\tau\sim\epsilon^{-1}$. Thus a maximum attenuation occurs at $\omega\epsilon^{-1}=\text{const}$. The temperature dependence of the specific-heat term in $C_2$ is neglected in this analysis since its effect is within experimental error.

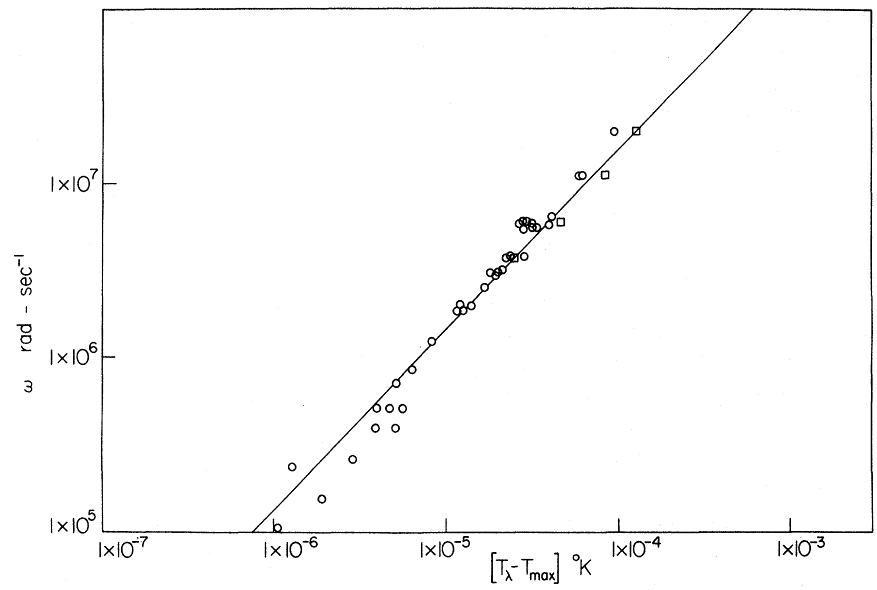


FIG. 2. Position of attenuation maximum as a function of frequency. Circles are determined from data of the type shown in Fig. 1. Squares are determined from the best fit relaxation curves of Fig. 3. The line is $\omega\tau = 1$ where the relaxation time is $\tau = \xi/C_2$, $C_2$ is the velocity of second sound, and $\xi = 1.36\times10^{-8}\epsilon^{-2/3}$ cm is one half of the healing length given in Refs. 11 and 12.

Figure 2 presents the results of a systematic determination of the temperature at which the peak occurs as a function of frequency from 16.8 kHz to 3.17 MHz. The temperature dependence of the data is very close to that expected. The principal departures occur for values of $\epsilon$ less than $10^{-5}$ °K where errors of one microdegree would be very important and cannot be ruled out. The line drawn in Fig. 2 is $\omega = \tau^{-1} = C_2/\xi$ and corresponds to a value of $\xi = 1.36\times10^{-8}\epsilon^{-2/3}$ cm which is half the value of the healing length proposed by Mamaladze[11] and is also half that found to be consistent with the depletion in $\rho_s/\rho$ measured in third-sound experiments.[12] Comparable values of $\xi$ have been quoted in other connections.[13,14] We would emphasize that while it is not unreasonable to expect that $\xi$ in Eq. (1) and the healing length have the same temperature dependence, no exact correspondence between $\xi$ and the healing length is established in the theory. If it were, or is in the future, these experimental results yield an independent determination of the healing length of superfluid helium, and in any case establish the magnitude of the coherence length appearing in Eq. (1).

With the temperature dependence given in Eq. (1) the attenuation of Eq. (2) should be very nearly logarithmically symmetric about the maximum and have the shape shown by the curves of Fig. 3.

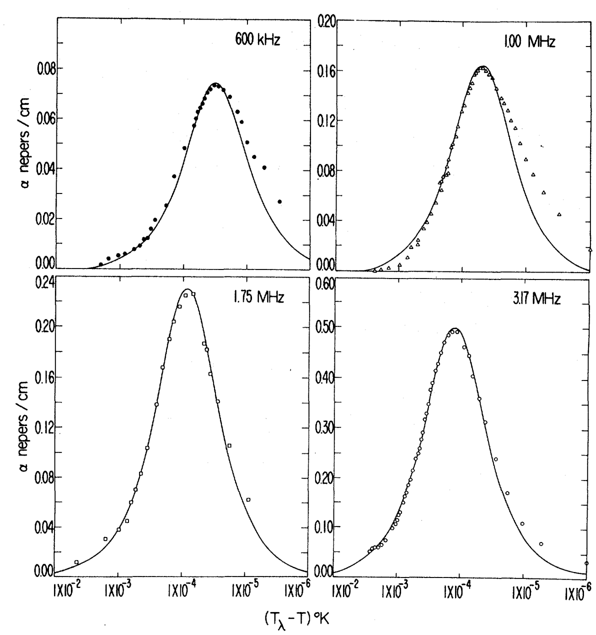


FIG. 3. The points were obtained by subtracting the attenuation in He I from that in He II at the same value of $\epsilon = |T_\lambda - T|$ using the data of Fig. 1. The curves are plots of Eq. (2) with $\xi = 1.36\times10^{-8}(T_\lambda - T)^{-2/3}$ cm and values of $U_\infty - U_0$ given in the text. Our conclusion based on the reasonable agreement between the data and the curves is contained in the last paragraph of the text.

This relaxation attenuation is predicted to approach zero as $T \to T_\lambda$. This clearly does not occur in Fig. 1 and our results indicate that there must be additional attenuation which grows as $T_\lambda$ is approached. Arguments can be advanced for expecting the attenuation associated with critical points to have the same singular dependence on $\epsilon$ on both sides of the transition. With this in mind we determine the values of $\alpha_{\mathrm{He\,II}}(\epsilon) - \alpha_{\mathrm{He\,I}}(\epsilon)$ and these are plotted in Fig. 3 for the four frequencies 600 kHz, 1MHz, 1.75 MHz, and 3.17 MHz. The interesting result is that Eq. (2) fits the points with the relaxation time previously given, and $U_\infty - U_0$ (in cm/sec) given by 18.5, 24.6, 20.6, and 23.2, respectively, for the above frequencies. For comparison the BR best-estimate value, which is less reliable, is 28 cm/sec.

Ferrell et al.,[15] Halperin and Hohenberg,[16] Swift and Kadanoff,[17] and Stauffer and Wong[18] predict that for frequencies less than $C_2\xi^{-1}$ the attenuation should vary as $\epsilon^{-1}$ below $T_\lambda$. Moreover, the same temperature dependence is expected above $T_\lambda$. We find in fact that excluding the region $\omega\epsilon^{-1} \gtrsim 4\times10^{10}$ our data can be used to verify this prediction in both He I and He II. In the belief that a preferable way of understanding the phenomena is one which includes the discussed relaxation process in an explicit way, we conclude that our results can be used to support a thesis that the attenuation is due to a combination of that occurring in He II, attributed to the relaxation mechanism resulting in Eqs. (1) and (2), and one which is symmetrical about $T_\lambda$. We were tempted but have resisted claiming that such a thesis must be correct.

We acknowledge the value of informative conversations with Pierre Hohenberg and attach particular weight to his emphasis on the importance of determining the hydrodynamic regime in He I.

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