HYPOTHESIS FOR THE SCHWARZ-HORA EFFECT

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This note shows that the effect observed in the Schwarz-Hora experiment, in which an electron beam passing through a crystal irradiated with laser light proceeds to a nonfluorescent screen and radiates light the same color as the laser, is probably caused by electron bunching. The crystal provides a sharply defined region of electric field amplitude which makes bunching possible.

Schwarz and Hora[1] have reported that 50-kV electrons diffracted through a thin (≃1000 Å) crystal produce luminous spots on a nonfluorescent screen when the crystal is illuminated by a laser beam traveling at right angles to the incident electron beam and polarized so that its E vector is parallel to the electron motion. The spots occupy very nearly the same positions on the screen as the normal Laue spots (observed with a fluorescent screen) and have the same color as the laser beam. The authors suggested that a quantum mechanical effect is responsible; but others[2] have questioned this hypothesis, suggesting instead that the electric field of the light produces a sinusoidal velocity modulation of the electron stream, causing bunching of the electrons at the screen, and, consequently, a periodic excitation of the electrons in the screen—in effect, an optical klystron. Schwarz and Hora themselves had also mentioned the possibility of velocity modulation. The effect is not observed when no crystal is present.

One purpose of this note is to suggest that the function of the crystal is merely to provide a short region with sharply defined edges within which the electric field is reduced by the dielectric constant. Since the electric field is normal to the surface of the thin crystal, the electric displacement is continuous across the surface and the field inside will be $1/\epsilon_r$ times as strong as the field outside.

Consider a laser beam polarized in the y direction and having an intensity profile $P(y)$. The electric field amplitude is then

$$E(y) = [2Z_0 P(y)]^{1/2}, \tag{1}$$

where Z_0 is the impedance of free space. An electron crossing this beam at position x with velocity v, as shown in Fig. 1(a), will experience an instantaneous field

$$e(t) \approx E(vt)\cos[\omega_0(t+t_0)-k_0 x], \tag{2}$$

where ω_0 is the optical frequency, $k_0 = \omega_0/c$, and t_0 is the time at which the electron is at $y=0$. The velocity increment produced by this field is $-(e/m)\int e(t)dt$ and may be written as

$$\Delta v = (e/m)[F_e(\omega_0)\cos(\omega_0 t - k_0 x) - F_o(\omega_0)\sin(\omega_0 t_0 - k_0 x)], \tag{3}$$

where e is the magnitude of the electronic charge and $F_e(\omega)$ and $F_o(\omega)$ are the Fourier transforms of the even and odd parts of $E(vt)$, respectively. Relativistic effects have been neglected. Suppose the light beam has uniform intensity for $|y| < d/2$ and is zero for $|y| \geq d/2$. Then $E(vt)$ has the value E_0 for $|t| < d/2v$ and is zero outside this interval and

$$\Delta v = E_0 \frac{-2e}{m\omega_0} \sin\frac{\omega_0 d}{2v} \cos(\omega_0 t_0 - k_0 x). \tag{4}$$

The amplitude of the velocity increment has a maximum,

$$\Delta v_{\max} = 2eE_0/m\omega_0, \tag{5}$$

whenever

$$d = \pi(v/\omega_0)n \quad (n \text{ an odd integer}). \tag{6}$$

Now suppose the electric field amplitude is

$$e(vt) = E_0 \exp(-v^2t^2/2\sigma^2). \tag{7}$$

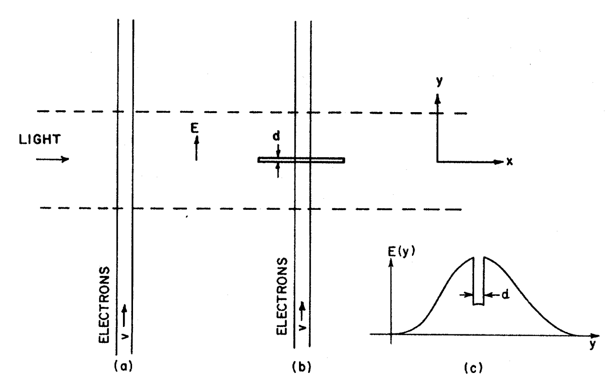

FIG. 1. (a), (b) The experimental arrangements and coordinate system without and with the crystal, respectively. (c) The notch in the electric field amplitude caused by the dielectric constant of the crystal.

Then

$$\Delta v = E_0 \frac{-e}{m}\frac{\sigma}{v}(2\pi)^{1/2}\exp\left(-\frac{\sigma^2\omega_0^2}{2v^2}\right) \times\cos(\omega_0 t_0 - k_0 x). \quad (8)$$

The ratio of the amplitude of (8) to (5) is $(\pi)^{1/2}ue^{-u^2}$ where $u=\sigma\omega_0/\sqrt{2}v$. In Schwarz and Hora's experiment, $\omega_0 = 3.86\times10^{15}$/sec and $v = 1.237\times10^8$ m/sec. If we assume that their beam had a Gaussian distribution with $\sigma = 10^{-5}$ m, then we find $u \simeq 220$. Clearly no observable interaction should be expected in this case, since the $\Delta v_{\max}$ given by (5) is approximately the largest obtainable with a given E_0.

On the other hand, with the crystal in place, as in Fig. 1(b), the electric field profile has a sharp-edged notch in the center where the field is reduced by the factor $1/\epsilon_r$ as in Fig. 1(c). For this case, since $d \ll \sigma$, we can neglect the exact field variation with y in the crystal and write:

$$E(vt) \approx E_0 \exp(-v^2t^2/2\sigma^2) - \begin{cases} (1-1/\epsilon_r)E_0, & |vt| < \frac{1}{2}d, \\ 0, & |vt| \geq \frac{1}{2}d. \end{cases} \quad (9)$$

The output is then simply (8) minus $(1-1/\epsilon_r)$ times (4). Since (8) is negligible, the expected modulation with the crystal in place is

$$\Delta v = -\left(1-\frac{1}{\epsilon_r}\right)E_0\frac{-2e}{m\omega_0}\sin\frac{\omega_0 d}{2v} \times\cos(\omega_0 t_0 - k_0 x). \quad (10)$$

We note that in this and previous expressions

$$\omega_0 d/2v = \pi(c/v)(d/\lambda). \quad (11)$$

In Schwarz and Hora's experiment, $v/c \simeq 0.41$, and they report using crystals for which d/λ ranged from 0.123 to 0.41. Thus, for their experiments, $0.87 \leq \omega_0 d/2v \leq 2.9$. Some of their better results were obtained with 1000 Å thick crystals. For these, $\omega_0 d/2v = 1.56$, which is close to the optimum value of $\pi/2$.

Assume for the moment that the electron beam is much smaller in diameter than the optical wavelength but still large compared with the electron wavelength. The electrons proceed toward the screen and become bunched because of the sinusoidal time dependence of the velocity. The harmonic content of the current at the screen is determined by the bunching factor[3]

$$B = \frac{\Delta v_{\text{peak}}\omega_0 L}{v^2} = \frac{(1-\epsilon_r^{-1})E_0 L}{V_0}\sin\frac{\omega_0 d}{2v}, \quad (12)$$

where L is the distance from the crystal to the screen and eV_0 is the beam energy. In Schwarz and Hora's experiment, $V_0 = 5\times10^4$ V, $L = 0.25$ m, $\epsilon_r \simeq 1.8$, and $E_0 \simeq 2\times10^6$ V/m, giving $B \simeq 4.4 \times\sin(\omega_0 d/2v)$. The ratio of the amplitude of the mth-harmonic current to the dc current is $I_m/I_{\text{dc}} = 2J_m(mB)$, where $J_m(u)$ is the mth-order Bessel function of the first kind. Thus the fundamental component has its first maximum for $B = 1.84$. Since the bunching factor is proportional to L, classical theory predicts that the fundamental current and presumably the emitted light should have the maxima and zeros associated with the Bessel function as L (or V_0) is changed.

The thermal spread of the electron energy leads to debunching. The spreading of the bunch is $\Delta y \simeq \delta v/v$, where $\delta v/v = (1/\sqrt{2})(kT/eV_0)$ is the fractional thermal velocity spread in a beam, k is Boltzman's constant, and T is the absolute temperature of the electron emitter. If we assume that $kT = 0.1$ eV, Schwarz and Hora's experiment gives $\Delta y \simeq 0.35\times10^{-6}$ m. The space between bunches is $\lambda_0 v/c = 0.201\times10^{-6}$ m for the 0.488×10^{-6}-m laser wavelength used. Thus, the fundamental component should be small compared with its maximum value and should fall off very rapidly with decreasing V_0 (or increasing L). The dependence upon V_0 agrees qualitatively with that reported in the experiment.

In the foregoing analysis, we have assumed the velocity of light in the crystal to be c and have neglected bending of the E lines at the crystal surface. With crystals only $\lambda/4$ in thickness, the propagation is probably in the form of a guided wave having a velocity very nearly that of free space; but there may well be a significant phase change with y at a given instant of time. Thus, the above results for the case of the crystal in place should be considered only as approximations. The H component of the light has negligible effect.

Let us now consider the effect of optical phase shift across the profile of the electron beam. First, we assume a parallel beam of electrons for which the beam diameter is much greater than the electron wavelength. If the material defining the sharp edges needed for producing the velocity modulation is an amorphous dielectric, there will be no Laue diffraction and the electron beam will proceed to the screen with essentially no diffraction or change in diameter. The relative optical phase picked up by each part of the beam in the x direction will be preserved in the beam as it goes toward the screen. This is depicted in Fig. 2(a) in which the dashed lines

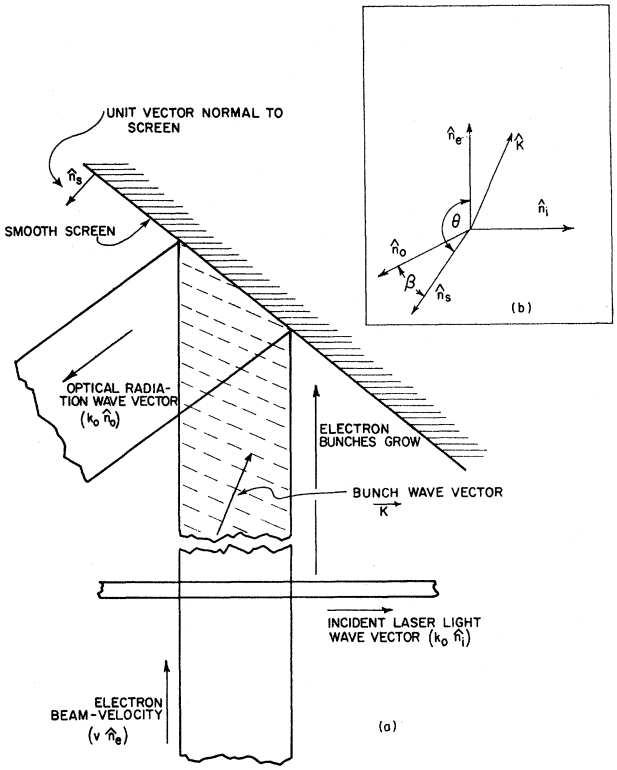

FIG. 2. (a) A bunched beam of parallel electrons interacting with a smooth screen to produce radiation and the unit vectors used in the description. (b) Definition of the angles β and θ for the special case where $\hat{n}_i$, $\hat{n}_e$, and $\hat{n}_s$ all lie in a plane.

represent surfaces of constant interaction phase or bunch phase. If the screen is optically smooth and if we assume that the radiation from each point is coherent with the electron bunches striking there and that it occurs very close to the surface, there should be a coherent beam of radiation produced and directed as shown in the figure with

$$\hat{n}_0 = \left\{\left(\left[\frac{c}{n}(\hat{n}_s\cdot\hat{n}_e) + (\hat{n}_s\cdot\hat{n}_i)\right]^2 - \frac{c^2}{v^2}\right)^{1/2} - \frac{c}{v}(\hat{n}_s\cdot\hat{n}_e) - \hat{n}_s\cdot\hat{n}_i\right\}\hat{n}_s + \frac{c}{v}\hat{n}_e + \hat{n}_i, \qquad (13)$$

where $\hat{n}_0$, $\hat{n}_s$, $\hat{n}_e$, and $\hat{n}_i$ are unit vectors in the direction of the output optical beam, the normal to the surface of the screen, the electron beam, and the incident laser light, respectively, and c and v are the velocities of light and the electrons, respectively. In deriving (13) we assumed that $\hat{n}_e$ and $\hat{n}_i$ were perpendicular so that $\hat{n}_e\cdot\hat{n}_i = 0$. The growing bunches of electrons propagate as a wave. The wave vector $\vec{K}$ of this bunch wave lies in the same plane as $\hat{n}_s$ and $\hat{n}_0$. Also $\vec{K}$, $\hat{n}_e$, and $\hat{n}_i$ lie in a plane. Consider the special case in which $\hat{n}_e$, $\hat{n}_i$, and $\hat{n}_s$ all lie in one plane. Then $\vec{K}$ and $\hat{n}_0$ are also in the same plane and the results become somewhat simpler. The angle β between $\hat{n}_0$ and $\hat{n}_s$ is given by

$$\sin\beta = \cos\theta + (c/v)\sin\theta, \qquad (14)$$

where θ is the angle between $\hat{n}_e$ and $\hat{n}_s$, and $\pi/2 < 2\tan^{-1}(c/v) < \theta < \pi$. Figure 2(b) shows these relations. When $\sin\beta$ is negative, $\hat{n}_e$ and $\hat{n}_0$ are on opposite sides of $\hat{n}_s$. Note that for normal incidence of the electron beam on the screen, $\theta = \pi$ and $\beta = -\pi/2$, so that the radiated light should come off in the plane of the screen in the same direction as the incident laser light. If the surface of the screen is rough, the radiation from each part of the screen should have random phase and there should be radiation in all directions rather than in a beam.

Electron beams that are not parallel should behave in a different fashion. A beam brought to a fine focus at the interaction point with the incident laser light should have essentially no phase shift across the beam profile as the beam diverges and proceeds towards the screen. It is likely that the velocity wavefronts will be spherical and, consequently, the radiation from a smooth screen should also have spherical wavefronts and form a diverging beam. The relationship between the angles is much simpler here since the direction of the incident laser light does not enter. We have $(\sin\beta)/c = (\sin\theta)/v$, where β and θ have the same meanings as before [see Fig. 2(b)]. This is very reminiscent of Snell's law. For $v \simeq c$, the angles are equal just as if the electron beam, radiated beam, and screen behaved as incident light beam, reflected light beam, and mirror, respectively.

A converging beam brought to a fine focus at the screen after passing through the interaction region with the laser light should not have bunches because the phase of the electric field goes through many cycles across the large interaction region.

The effect of using a crystalline material in place of an amorphous dielectric is to produce the Laue spots. Each of the Laue spots should have the same behavior as that indicated in the previous paragraphs for the amorphous material and the light from one spot should be coherent with that from another if the radiation is produced very close to the surface of the screen.

From the symmetry of the system and the probable mechanisms for producing the radiation, one might expect the emitted radiation to be polarized so that the polarization vector lies in the plane determined by $\vec{K}$ and $\hat{n}_0$. The polarization must have a nonvanishing component along $\vec{K}$ so one would expect the intensity of radiation back along $-\vec{K}$ always to be zero.

The above classical analysis shows that a sharply defined optical electric field or a sharply defined notch in such a field can produce bunching in an electron beam. This may partly account for the effect Schwarz and Hora observed. Even though the density of electrons in their experiment was only one electron per fifty "bunches" in the primary beam, there is still time coherence between electron beam charge and current density at the screen and the electric field at the interaction region.

However, a quantum mechanical treatment of the problem now being developed seems to indicate that true quantum phenomena may be present. In particular, our wave equations predict that the electron bunching for monochromatic electrons obeys the classical Bessel-function relationship only if $\hbar\omega_0^2 y/(4eV_0 v) \ll \pi/2$. For $\hbar\omega_0^2 y/(4eV_0 v) \gg \pi/2$ and $(4eE_0 v/\hbar\omega_0^2)\sin(\omega_0 d/2v) \ll 1$, the bunching appears to vary sinusoidally with distance and thus not decay to zero as $y \to \infty$. If further study supports this finding or predicts other nonclassical effects, the quantum mechanical analysis will be reported in a future paper.

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[3]D. R. Hamilton, in *Klystrons and Microwave Triodes*, MIT Radiation Lab. Series Vol. 7 (Massachusetts Institute of Technology, Cambridge, Mass., 1948), Vol. 7, Chap. 9, p. 201.

ATTENUATION OF FIRST SOUND NEAR THE LAMBDA TRANSITION OF LIQUID HELIUM

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The attenuation of first sound near the lambda point can be interpreted as being due to two phenomena: (1) a relaxation process described by Pokrovskii and Khalatnikov occurring only below the lambda point, and having a relaxation time of ξ/C_2 where C_2 is the velocity of second sound and ξ is a coherence length of magnitude $1.36\times 10^{-8}\ (T_\lambda - T)^{-2/3}$ cm; (2) a critical attenuation which is nonsingular and symmetrical about T_λ.

In an earlier investigation Barmatz and Rudnick[1] (BR) in an effort to determine the thermodynamic first-sound velocity near T_λ made measurements at the low frequency of 22 kHz. These measurements were sufficiently accurate, and the approach to T_λ sufficiently close, that attenuation and dispersion effects were measurable. Because of the low frequency, the results were necessarily inaccurate. The purpose of the present investigation was to use essentially the same apparatus over as wide a frequency range as possible, and we report here the results of the attenuation measurements.

The frequency range is 16.8 kHz to 3.17 MHz in He II, and 600 kHz to 3.17 MHz in He I. With this frequency range, and microdegree temperature resolution, the measurements yield greater detail about the nature of the attenuation than has previously been reported. In particular, at a frequency ω, the maximum attenuation is unambiguously shown to occur at a temperature T, below T_λ, such that $\omega\epsilon^{-1}=\text{const}$, where $\epsilon = |T_\lambda - T|$. Recently there has been much theoretical speculation on the nature of first sound at lambda transitions, resulting in a bewildering array of sometimes conflicting results. While there are instances where there is partial agreement between such predictions and our data, we know of none which completely account for the results. In view of this, we make no attempt at a conscientious comparison of the data with existing theory, except as regards the relaxation theory first suggested by Landau and Khalatnikov.[2] Our results, and this situation, point up the need for a unified and complete theory of the attenuation of sound near the lambda point of helium.

The apparatus, except for minor modifications to improve the acoustic response, is described by BR. The acoustic element is a copper cylindrical resonator 2.5 cm long and 2.5 cm in di-