NEUTRINOS VERSUS MUONS IN W-BOSON PRODUCTION*

R. W. Brown†
Brookhaven National Laboratory, Upton, New York 11973

and

A. K. Mann
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

J. Smith
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 1 June 1970)

Detailed calculations show that the theoretical total cross section for the production of the intermediate vector boson W by incident neutrinos is much larger than that for W production by incident muons in the relevant range of National Accelerator Laboratory energies and corresponding W masses.

With the advent of higher energy beams at the National Accelerator Laboratory, renewed efforts will be made to find the intermediate vector boson of weak interactions. The most favorable reactions involve either incident neutrinos or muons which can dissociate into W's by recoiling against the electromagnetic field of a nucleus,[1] i.e.,

$$\nu_\mu\ (\overline{\nu}_\mu) + Z \to W^\pm + \mu^\mp + Z, \quad (1)$$

$$\mu^\pm + Z \to W^\pm + \overline{\nu}_\mu\ (\nu_\mu) + Z. \quad (2)$$

To date, this neutrino reaction has not been seen at Brookhaven National Laboratory[2] or at CERN.[3] Therefore, together with the proton-nucleus collision studies,[4] the only thing we can say now about the W is that if it exists, its mass M_W is probably greater than 2 or 3 GeV.

It is the purpose of this note to point out that, contrary to general expectations and to what has been assumed in planning for muon experiments,[1] the cross section for (1) is significantly larger than that for (2). An order-of-magnitude discrepancy for energies less than 10 GeV and for $M_W \approx 1$ GeV was previously noticed by Berends and West[5] for protons. Our findings show that, indeed, Reaction (1) is between ten and a thousand times more probable for energies less than 300 GeV and for 3 GeV $\lesssim M_W \lesssim$ 12 GeV. Relative to (1), this casts grave doubts on the usefulness of Reaction (2) for W searches—the fact that the parent pion gives most of its energy to the muon notwithstanding.

We consider only proton targets and assume that the W has no anomalous magnetic moment and no electric quadrupole moment. Inclusion of these moments, and the general case of a nucleus with attendant nucleon-motion and exclusion-principle effects and the problem of incoherent versus coherent production, will be discussed elsewhere.[6] These complications, which have previously been studied by a number of authors[7-12] with respect to the neutrino experiments, do not qualitatively change our conclusion. The deep-inelastic contributions to the muon reaction have been estimated to be of the same order of magnitude as the elastic contribution.[6] This, combined with the recent work of Chen,[13] implies that such contributions will likewise not change our conclusion here.

In Fig. 1, (a) and (b) are the W^+-production Feynman diagrams corresponding to the lowest order matrix element of Reaction (1), while (c) and (d) are those diagrams relevant to (2). It is clear from this figure that the muon propagator has a different momentum dependence in the two cases, so the total cross sections are not expected to be the same. What is surprising, however, is the orders-of-mangitude difference that arises in actual calculation. This is true in spite of the approximate equality in phase-space volume at high energies.

Our procedure was to do, analytically, some of the integrations in the c.m. frame of the final-state lepton-boson pair which left us with a two-dimensional numerical integration. For this last step we used a Gaussian quadrature computer program, making sure that the small-momentum-transfer area was covered with a sufficiently fine mesh. Our results for the total cross sections are given in Table I where we have averaged over the muon spin in Reaction (2). (Here, the ν_μ-p and $\overline{\nu}_\mu$-p cross sections are equal, as are the μ^+-p and μ^--p cross sections.) The matrix

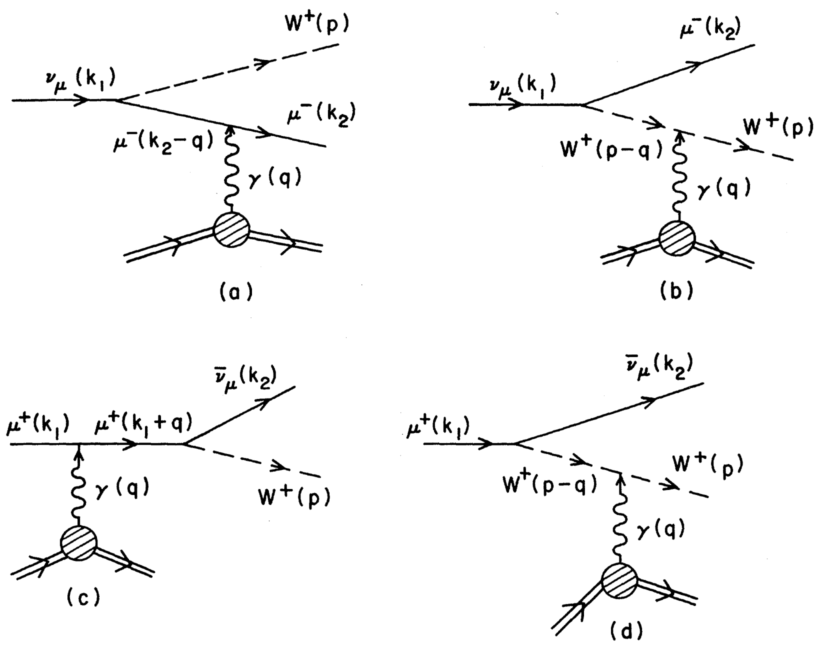

FIG. 1. The lowest order Feynman diagrams for Reactions (1) and (2). The notation $\gamma(q)$ refers to a photon with four-momentum q, etc. The blobs represent the nuclei form factors.

elements and calculational procedures will be discussed in a separate paper[6]; for the proton form factors, we employed the familiar dipole fit.[14]

If we compare the neutrino and muon reactions at the same beam energies, then it is apparent from Table I that neutrino cross sections are two orders of magnitude larger than those for muons in the energy-mass region of interest. Using $M_W = 8$ GeV as an example (motivated mildly by several theoretical model expectations[15]), one finds $\sigma_\nu \sim 10^{-38}$ cm^2 and $\sigma_\mu \sim 5\times10^{-41}$ cm^2 for a 100-GeV projectile. As an important check on our work, the muon calculations of Berends and West[5] and the neutrino calculations of Wu et al.[11] were used. The muon results of Ref. 5 cover a large part of Table I and agree with ours within a few percent. Since the work of Ref. 11 is restricted to the energy region below 10 GeV, we could only check our programs there; however, it was possible to compare a couple of unpublished numbers[16] at higher values of energy and mass. For all of these neutrino results, we obtained agreement at the 1% level—taking into account the slight differences in form factors. Finally, it should be noted that σ_ν and σ_μ become comparable at extremely large energies in accord with the asymptotic formulas of Solov'ev and Tsukerman[8] which imply a ratio of 2 at infinity (merely the spin-average factor difference).

The reason for the large difference between the two cases lies in the fact that the electromagnetic interaction of the final-state muon contributes to σ_ν differently from the way in which the electromagnetic interaction of the incident muon contributes to σ_μ. Looking in the c.m. frame of the muon and W-boson pair, the muon propagator in the former [see Fig. 1(a)] develops a pole at $q^2 = 0$ when the W is parallel to the incident nuetrino and when the muon mass is neglected. This does not happen in the latter [see Fig. 1(c)], however, since we must have $(k_1+q)^2 \geq M_W^2$ there. This distinction is crucial except at extremely large beam energies where the mass difference between the W and the muon becomes less important. (As we move very close to threshold, on the other hand, σ_ν/σ_μ will decrease slighlty since the momentum transfer q^2 is necessarily large then.)

Although the proton form factors limit the reactions to small momentum transfers, which is precisely the region where this propagator effect is important, they do not play a critical role. For example, if we remove them altogether, we find that the σ_ν/σ_μ ratios given in Table I are reduced only by a factor of 2 or 3 on the average. This indicates that the muon-photon interaction should be crucial in the inelastic channels as well.

The principal reason for using muons to pro-

Table I. The theoretical total cross sections σ_ν [Reaction (1)] and σ_μ [Reaction (2)] in units of 10^{-38} cm^2 for free protons. We have also given their ratio at each energy and boson mass. The latter are in GeV.

M_W	E:	10	30	50	70	90	100	150	200	250	300	400	600	1000
	σ_ν	27.9	89.8	132	163	189	200	247	284	314	340	381	445	536
1	σ_μ	1.04	5.48	9.79	13.7	17.3	19.0	26.6	33.0	38.8	44.0	53.1	67.9	89.6
	σ_ν/σ_μ	27	16	13	12	11	11	9.3	8.6	8.1	7.7	7.2	6.6	6.0
	σ_ν	1.49×10^{-2}	9.01	24.2	39.3	53.4	60.1	89.5	114	135	154	186	235	305
3	σ_μ	1.57×10^{-4}	9.05×10^{-2}	.345	.703	1.12	1.35	2.57	3.84	5.09	6.34	8.75	13.2	20.9
	σ_ν/σ_μ	95	100	70	56	48	45	35	30	27	24	21	18	15
	σ_ν		.181	3.00	8.25	14.5	17.8	34.6	50.5	65.3	78.9	103	143	202
5	σ_μ		7.30×10^{-4}	1.53×10^{-2}	5.43×10^{-2}	.117	.156	.415	.749	1.13	1.54	2.42	4.24	7.81
	σ_ν/σ_μ		250	200	150	120	110	83	67	58	51	43	34	26
	σ_ν			1.17×10^{-3}	.169	.948	1.60	6.71	13.5	21.0	28.6	43.6	71.2	116
8	σ_μ			2.86×10^{-6}	3.89×10^{-4}	2.63×10^{-3}	4.85×10^{-3}	2.97×10^{-2}	7.81×10^{-2}	.148	.236	.455	1.01	2.34
	σ_ν/σ_μ			410	430	360	330	230	170	140	120	96	70	50
	σ_ν				8.39×10^{-5}	2.89×10^{-2}	9.82×10^{-2}	1.56	4.71	8.88	13.61	23.8	44.4	81.4
10	σ_μ				1.66×10^{-7}	4.67×10^{-5}	1.69×10^{-4}	3.93×10^{-3}	1.60×10^{-2}	3.78×10^{-2}	6.91×10^{-2}	.158	.417	1.13
	σ_ν/σ_μ				510	620	580	400	290	230	200	150	110	72

duce W's stems from the fact that they generally receive about three-fourths of the pion-beam energy so that in spite of their large electromagnetic background it might be possible to look at heavier W's. But even if we compare, say, a 300-GeV incident muon with a 100-GeV incident neutrino, σ_ν is still an order of magnitude larger than σ_μ for $3 \lesssim M_W \lesssim 8$. Moreover, the higher energy muons come away from the pions with the wrong helicity.[17] The μ^- (μ^+) is right-handed (left-handed) for a pion at rest; the highest laboratory-energy muons preserve this helicity since they are going forward in the rest frame of the π and, as a consequence, cannot initiate the semiweak W production. (For ultrarelativistic muons, the electromagnetic interaction conserves helicity and the $V-A$ $W\mu\nu$ vertex couples only to left-handed μ^-'s and right-handed μ^+'s.)

This last point means that the σ_μ in Table I should really be multiplied by $1+P$ where P is the longitudinal polarization of the muon relative to that needed for Reaction (2) to proceed. For either μ^+ or μ^-, one obtains $P \approx -1$ at the highest energies and $P \approx +1$ at the lowest.

As stated earlier, nuclear effects do not qualitatively change our conclusions. In fact, our studies show that σ_ν/σ_μ is generally larger for coherent scattering than for incoherent.[6] This is due to the more rapid falloff of nuclear form factors; as an example, the cross sections per proton on an iron nucleus assuming a Fermi distribution are ~0.5×10^{-38} cm^2 and ~1×10^{-41} cm^2 for 100-GeV neutrinos and muons, respectively, at $M_W = 5$ GeV.[18] This gives a ratio of ~500 to be compared with ~100 from Table I. Finally, it should be noted that one can already see up to two orders of magnitude difference between Reactions (1) and (2) in the Weizsäcker-Williams coherent calculations of Solov'ev and Tsukerman[8] and of Überall[12] for low incident energies and $M_W \approx 1$ GeV.

We thank Professor T. D. Lee, Professor H. Primakoff, Professor R. L. Schult, and Professor C. N. Yang for essential discussions and encouragement.

*Work supported in part by the U. S. Atomic Energy Commission.

†Address after 1 September 1970: Case Western Reserve University, Cleveland, Ohio 44106.

[1]The advantage of using neutrino beams in W searches was first discussed by B. Pontecorvo and S. Ryndin, in *Proceedings of the Ninth Annual International Conference on High Energy Physics, Kiev, U. S. S. R., 1959* (Academy of Sciences, Moscow, U. S. S. R., 1960). More recently, employment of the muon beam at the National Accelerator Laboratory in these searches has been advocated by L. M. Lederman, National Accelerator Laboratory 1968 Summer Study Report No. B.2-68-74 (unpublished), Vol. 2, p. 55; and by T. Kirk, National Accelerator Laboratory 1969 Summer Study Report No. SS-11 (unpublished), Vol. 4, p. 191.

[2]R. Burns *et al.*, Phys. Rev. Lett. 15, 42 (1965).

[3]G. Bernardini *et al.*, Nuovo Cimento 38, 608 (1965).

[4]R. C. Lamb *et al.*, Phys. Rev. Lett. 15, 800 (1965); R. Burns *et al.*, *ibid.* 15, 830 (1965); P. J. Wanderer, Jr., *et al.*, *ibid.* 23, 729 (1969); G. S. Hicks *et al.*, Bull. Amer. Phys. Soc. 15, 579 (1970).

[5]F. A. Berends and G. B. West, Phys. Rev. D 1, 122 (1970).

[6]R. W. Brown and J. Smith, to be published.

[7]T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Lett. 7, 429 (1961).

[8]V. V. Solov'ev and I. S. Tsukerman, Zh. Eksp. Teor. Fiz. 42, 1252 (1962) [Sov. Phys. JETP 15, 868 (1962)].

[9]J. S. Bell and M. Veltman, Phys. Lett. 5, 94 (1963).

[10]G. von Gehlen, Nuovo Cimento 30, 859 (1963).

[11]A. C. T. Wu, C.-P. Yang, K. Fuchel, and S. Heller, Phys. Rev. Lett. 12, 57 (1964); A. C. T. Wu and C.-P. Yang, Phys. Rev. (to be published).

[12]H. Überall, Phys. Rev. 133, B444 (1964).

[13]According to Chen [H. H. Chen, Phys. Rev. (to be published)], the deep inelastic contributions are of the same order of magnitude as or smaller than the elastic contribution to the neutrino total cross section. Our work (Ref. 6) confirms this result.

[14]D. H. Coward *et al.*, Phys. Rev. Lett. 20, 292 (1968); J. Litt *et al.*, Phys. Lett. 31B, 40 (1970).

[15]See, for instance, S. I. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Lett. 19, 205 (1967); M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969).

[16]A. C. T. Wu, private communication. We thank Professor Wu for providing us with these values. After completing our work, we received a preprint of some higher energy neutrino work performed by Chen [H. H. Chen, Univ. of Calif. at Irvine Report No. UCI-10P19-34 (to be published)]. Although addressed to cosmic-ray analysis, it furnished us with another check on some of the proton σ_ν values in Table I and the agreement was excellent. We thank Dr. Chen for sending these results to us before publication. We believe that the high-energy neutrino compilations of A. V. Berkov, G. G. Bunatyan, E. D. Zhizhin, and Yu. P. Nikitin, Yad. Fiz. 9, 605 (1968) [Sov. J. Nucl. Phys. 9, 348 (1969)], are uniformly *too large* by roughly an order of magnitude.

[17]We thank Professor T. D. Lee for pointing this out to us. This effect is discussed by T. Kirk, F. Pipkin, and J. Sculli, National Accelerator Laboratory 1969 Summer Study Report No. SS-34 (unpublished), Vol. 4, p. 185.

[18]Upon comparison with Table I, we see an important result which will be discussed more extensively in Ref. 6. That is, our computations show that incoherent scattering remains important at higher incident energies for a given M_W than has been assumed in the past.