section as would be predicted by the Glauber formalism when the quadrupole deformation of the deuteron is ignored.

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QUARK-MODEL CLASSIFICATION OF LOW-MASS, $I=\frac{1}{2}$ NUCLEON ISOBARS*

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A comparison is made of recent data on η-meson photoproduction with photoexcitation amplitudes calculated in the nonrelativistic quark model. This leads to definite multiplet assignments for all low-mass $I=\frac{1}{2}$ nucleon isobars. In particular, we find no evidence for a radial sequence beyond $n=2$. The assignment of $P_{11}(1750)$ to an $n=3$ second radially excited $[\underline{56},0^+]$ appears ruled out; instead, we argue that consistency of results from photoproduction and diffraction scattering makes the configuration $[\underline{70},0^+]$ the best choice for this state.

In the framework of the nonrelativistic quark model,[1] with totally symmetric three-quark wave functions for the baryons, we have examined features of the qqq configurations open to experimental test in the photoexcitation of nucleons. Recent data on the reaction

$$\gamma p \to p\eta \tag{1}$$

supply us with information on the $I=\frac{1}{2}$ nucleon isobar channel in the absence of the dominating resonant terms of pion photoproduction.[2]

What are the salient experimental features of η photoproduction?

(1) A large peak immediately above threshold, close to isotropic in angular distribution, is most naturally explained in terms of the isobar $S_{11}(1550)$, with a possible small admixture of the tail of $P_{11}(1460)$, whose peak is below threshold.[3-6]

(2) A dip[7,8] in the mass region around 1650 MeV may be interpreted by means of an S_{11}-P_{11} interference effect.

(3) Substantial polarization values[9] for the recoil proton at 90 deg from threshold to 1700 MeV similarly indicate S_{11}-P_{11} interference.

(4) A second flat peak[8,10] at mass values ~1700-1800 MeV is most naturally attributed[8,11] to $P_{11}(1750)$; a forward dip, reported preliminarily from Daresbury,[12] may hint at the presence of an admixture of $S_{11}(1710)$, or at an effect of the high-energy tail of $S_{11}(1550)$.

The remarkable success of the nonrelativistic quark model in explaining qualitatively, and in most instances quantitatively, all prominent features observed in the photoproduction of pions in the isobar region[13,14] suggests that it should be equally successful in the simpler case of η photoproduction. In particular, the apparent absence of higher-J intermediate states in process (1), in this energy range, opens up the possibility of investigating quark-model assignments for $J=\frac{1}{2}$ states. There are four of appropriate mass, one of which, $S_{11}(1710)$, the model predicts <u>not</u> to be photoexcited off protons.[15] The P_{11} states at 1460 and 1750 MeV have been the object of considerable speculation.[16,17] They share the quantum numbers of the nucleon and might be members of a continuing radial sequence. This would correspond to shell-model states

$P_{11}(940) \rightarrow (1s)^2(1s)$, $N=0$;

$P_{11}(1460) \rightarrow (1s)^2(2s)$, $N=2$;

$P_{11}(1750) \rightarrow (1s)^2(3s)$, $N=4$;

where the high degree of symmetry could be invoked to explain the fact that the $N=2$ and $N=4$ states might be found at such low mass values. (There are N units of excitation, $N=2n+l-2$.) This question is of a twofold interest:

(1) While the far-reaching agreement between the experimentally accessible data and a model as simple as the symmetric-oscillator quark model is certainly puzzling, the analogy has, until now, been probed only with respect to orbital excitations. Does the analogy extend to radial modes as well?

(2) It is well known that orbital quark excitations will lead to states located on the identical trajectory or on a quasidegenerate trajectory of opposite parity. Radial excitations imply the population of daughter trajectories (as suggested by the treatment of the harmonic oscillator or the Coulomb trajectories).

We assume, as usual, that the three quarks are coupled by (spring-like) harmonic forces. By separating out the center-of-mass motion of the system, and introducing relative coordinates $\vec{\xi}$ and $\vec{\eta}$, we treat it as a $\underline{\text{two}}$-oscillator problem with the Hamiltonian

$$H=(2m)^{-1}(\vec{p}_\xi^{\,2}+\vec{p}_\eta^{\,2})+\tfrac{1}{2}m\omega^2(\vec{\xi}^{\,2}+\vec{\eta}^{\,2}), \qquad (2)$$

with m the quark mass. We apply raising operators $\bar{\xi}$ and $\bar{\eta}$ on the qqq ground state $[\underline{56}, 0^+]$. Then the requirement of a totally symmetric wave function is sufficient to make us arrive at the possible multiplets as given in Table I.

It is now easy to calculate the transition matrix elements for the γNN^* vertex, assuming one-quark interactions.[13,18] Assuming the quarks to be Dirac particles with a g factor of $g=1$, the only free parameter is $\alpha^2=m\omega$. We choose α^2 to be 0.14 GeV^2. This value is close to the one found by Copley, Karl, and Obryk[13] from fits to photoproduction data, and can also naively be inferred from isobar mass spacings.[14] The relevant amplitudes are given in Table II. Table II also contains the usual multiplet assignments for all isobars[17]; for the $P_{11}(1750)$ state, it mentions a number of possible representations. We will now check the calculated amplitudes, as given in columns 4 and 5 of Table II, against the experimental features of η photoproduction:

(1) The dominant peak above η threshold is due to the large photoexcitation amplitude of $S_{11}(1550)$. However, the comparison shows that it is too large by a factor of 2. This situation can be remedied by appropriate mixing of this configuration with the 8 ($\frac{3}{2}$) of $[\underline{70}, 1^-]$ normally identified with $S_{11}(1710)$. Since this configuration itself

Table I. Multiplets in symmetric quark model.

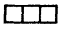
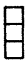
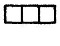
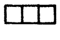
N	Spatial Excitations	Permutation Symmetry	SU(6)⊗O(3) Multiplets
0			$[\underline{56}, 0^+]$
1	$\bar{\eta},\bar{\xi}$		$[\underline{70}, 1^-]$
2	$\bar{\eta}\cdot\bar{\eta}+\bar{\xi}\cdot\bar{\xi}$		$[\underline{56}, 0^+]$
	$\bar{\eta}\cdot\bar{\eta}-\bar{\xi}\cdot\bar{\xi}$, $\bar{\eta}\cdot\bar{\xi}$		$[\underline{70}, 0^+]$
	$\bar{\eta}\bar{\eta}+\bar{\xi}\bar{\xi}$		$[\underline{56}, 2^+]$
	$\bar{\eta}\bar{\eta}-\bar{\xi}\bar{\xi}$, $\bar{\eta}\bar{\xi}$		$[\underline{70}, 2^+]$
	$\bar{\eta}\times\bar{\xi}$		$[\underline{20}, 1^+]$
4	$(\bar{\eta}\cdot\bar{\eta}+\bar{\xi}\cdot\bar{\xi})^2$		$[\underline{56}, 0^+]$
	$(\bar{\eta}\times\bar{\xi})^2$		$[\underline{56}, 0^+]$

Table II. Quark model configurations and photoexcitation helicity amplitudes off protons.

Resonance	SU(6) x O(3)	SU(3) (Quark Spin)	$A_{1/2}$(GeV$^{-1/2}$)	$A_{3/2}$(GeV$^{-1/2}$)
$P_{11}(1460)$	$[\underset{\sim}{56}, 0^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	-0.03	
$D_{13}(1512)$	$[\underset{\sim}{70}, 1^-]_{N=1}$	$\underset{\sim}{8}$ (1/2)	-0.04	+0.11
$S_{11}(1550)$	$[\underset{\sim}{70}, 1^-]_{N=1}$	$\underset{\sim}{8}$ (1/2)	+0.16	
$D_{15}(1680)$	$[\underset{\sim}{70}, 1^-]_{N=1}$	$\underset{\sim}{8}$ (3/2)	0	0
$F_{15}(1688)$	$[\underset{\sim}{56}, 2^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	-0.01	+0.07
$S_{11}(1710)$	$[\underset{\sim}{70}, 1^-]_{N=1}$	$\underset{\sim}{8}$ (3/2)	0	
$P_{11}(1750)$	$[\underset{\sim}{56}, 0^+]_{N=4}$	$\underset{\sim}{8}$ (1/2)	0.00	
$P_{11}(1750)$	$[\underset{\sim}{70}, 2^+]_{N=2}$	$\underset{\sim}{8}$ (3/2)	0	
$P_{11}(1750)$	$[\underset{\sim}{70}, 0^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	+0.04	
$P_{11}(1750)$	$[\underset{\sim}{56}, 0^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	-0.06	
$P_{11}(1750)$	$[\underset{\sim}{20}, 1^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	0	
$P_{13}(1860)$	$[\underset{\sim}{56}, 2^+]_{N=2}$	$\underset{\sim}{8}$ (1/2)	+0.12	-0.03

cannot be photoexcited from protons,[15] mixing will account for a reduced amplitude for $S_{11}(1550)$; similarly, this would allow for the occurrence of some $S_{11}(1710)$, explaining the forward dip observed in the region dominated by $P_{11}(1750)$. Alternatively, the tail of $S_{11}(1550)$ may account for it.

(2) The absence of the state $D_{15}(1680)$ and of anything but an admixture of $S_{11}(1710)$ is accounted for; this was originally pointed out by Moorhouse.[15]

(3) $F_{15}(1688)$ is quark model allowed; its absence is due to the values of SU(3) Clebsch-Gordan coefficients.[19]

(4) $P_{11}(1460)$ and $P_{11}(1750)$ can belong to $[\underline{56}, 0^+]$ or $[\underline{70}, 0^+]$. In shell-model terms, these states are[18]

$$[\underline{56}, 0^+] = (\tfrac{2}{3})^{1/2}(1s)^2(2s) + (\tfrac{1}{3})^{1/2}(1s)(1p)^2,$$

$$[\underline{70}, 0^+] = (\tfrac{1}{3})^{1/2}(1s)^2(2s) + (\tfrac{2}{3})^{1/2}(1s)(1p)^2.$$

The low-mass state $P_{11}(1460)$ will be more naturally associated with the $[\underline{56}, 0^+]$ because of its higher symmetry. Since this state contains mainly the configuration $(1s)^2(2s)$, this assignment fits in with its experimentally observed strong diffractive production in pp collisions.[20,21] (We are assuming that the diffractive process would favor a mere change in radial nodes to an orbital excitation.)

What other representations can $P_{11}(1750)$ belong to? Is it possible that this state corresponds to a second radial excitation of the nucleon? A look at Table II shows that its assignment to a $[\underline{56}, 0^+]$ multiplet at the $N=4$ level will lead to an almost vanishing photoproduction amplitude. This is in obvious disagreement with the notion that this state causes the second maximum in the η-photoproduction amplitude.

The question may arise as to whether this enhancement around mass values of 1700-1800 MeV may be due to the allowed state $P_{13}(1860)$, thus voiding our arguments on $P_{11}(1750)$. However, all octet members of the $[\underline{56}, 2^+]$ are expected to have the same F/D ratio, so that we expect the $P_{13}(1860)$ not to couple to $N\eta$, just as the $F_{15}(1688)$ does not. Also, the helicity amplitudes for this state as resulting from our model (cf. Table II) produce a forward peak instead of the dip observed,[12] so that we rule out this possibility.

Once we accept the $P_{11}(1460)$ as occupying the $[\underline{56}, 0^+]$ at the $N=2$ level, the remaining possibilities for the $P_{11}(1750)$ at this level are the representations $[\underline{70}, 0^+]$, $[\underline{70}, 2^+]$, and $[\underline{20}, 1^+]$. Out of these, the vertex $\gamma p N^*$ $[\underline{70}, 2^+]$ is forbidden by Moorhouse's argument[15]; the $[\underline{20}, 1^+]$ cannot be photoexcited due to the total antisymmetry of its space and SU(6) wave functions.

We are therefore led to the conclusion that the $P_{11}(1750)$ belongs to the $[\underline{70}, 0^+]$, $N=2$ multiplet. This is the only assignment which can give a quantitative understanding of the features observed in η photoproduction, in the framework of this model which works remarkably well in classifying all established nuclear resonances.

The configuration $[\underline{70}, 0^+]$ contains some radially excited part, and hence should be diffraction-produced in pp collisions, although less prominently than $P_{11}(1460)$. However, this argument applies to the space part of the wave function alone; an extended formulation of diffraction production by Carlitz, Frautschi, and Zweig[22] postulates that the SU(6) labels be conserved in such processes, in which case $P_{11}(1750)$ should not be diffraction-produced if our assignment is correct.

The experimental situation with respect to diffraction production of $P_{11}(1750)$ is not clear at this time; this state lies in a mass region clustering around the prominent $F_{15}(1688)$, and present experimental evidence is insufficient to isolate the individual states.

Nevertheless, we stress that future, more accurate experiments will decide whether our assignments are correct. There may be no diffraction production of $P_{11}(1750)$ at all, or some production less prominent than that of $P_{11}(1460)$, proceeding via an SU(6)-breaking mechanism. Either possibility is in contrast to Carlitz, Frautschi, and Zweig[22] who assume the $P_{11}(1750)$ to be a $[\underline{56}, 0^+]$, and therefore would have to allow for plentiful diffractive production. This possibility appears to be ruled out by experiment even at this time.

We mention in conclusion that the existence of a $[\underline{70}, 0^+]$ as an exchange-degenerate multiplet to the $N=0$ $[\underline{56}, 0^+]$ is predicted in an unbroken duality picture. The seeming lack of observed states belonging to this representation (as well as of exotic mesons) has been explained in terms of duality-breaking mechanisms.[23]

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