MAGNETIC TRANSMISSION RESONANCE IN FERROMAGNETIC GADOLINIUM AND NICKEL*

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A theory of the ferromagnetic transmission resonance is presented for the case where the static magnetic field is perpendicular to the sample surface. The theory is compared with experiments performed on ferromagnetic gadolinium and nickel. The agreement between theory and experiment is satisfactory. Since the transmission resonance is a bulk effect it will allow the study of the bulk spin relaxation time and exchange integral as functions of temperature.

Following the observation,[1] identification,[2,3] and accurate description[4] of magnetic transmission resonance in Gd, due to microwave skin-depth modulation (enhancement) under resonance conditions, similar transmission signals were observed in ferromagnetic Gd,[4] Fe, Ni, Co and Connetic AA.[5] The Gd samples were 60 μm thick and 99.9% pure. The Fe, Ni, Co samples were about 10 μm thick and of much higher purities. The Connetic AA samples had intermediate thicknesses. The experiments were done at a frequency of 9.2 GHz. The apparatus was similar to that used for spin transmission experiments in Pauli paramagnetic metals[6] and utilized a pair of cavities, which when assembled have their H fields crossed at 90°. As in the case of paramagnetic Gd we have observed ferromagnetic transmission signals for the above metals in two geometries: the static magnetic field being either parallel or perpendicular to the sample surface. Since the transmission resonance effect is a bulk effect, it occurs for both geometries at the field $B_{\text{int}} = \omega_{\text{rf}}/\gamma$ and for the perpendicular geometry appears at applied fields lower than the absorption resonance does. The transmission in paramagnetic Gd is described very accurately through the solution of the Maxwell and Bloch-Bloembergen[7] equations with boundary conditions and no consideration given to the waves reflected back to the bulk from the second surface of the sample.

Attempts to describe the observed [4,5] ferromagnetic transmission resonances through the same equations with an exchange term added to the magnetic field[8,9] and boundary conditions enforced on the magnetization[8,10,11] failed originally. We have, however, recently realized that the waves reflected from the second sample face back to the bulk cannot be neglected in the ferromagnetic case, at least for the sample thicknesses we used, since the ferromagnetic magnetization is very large compared with the paramagnetic one. Following these ideas, we developed theories for both the perpendicular and parallel static-field geometries. In this paper we present the results for the first case in connection with Gd and Ni. The parallel case results are similar to the ones discussed below but more involved.

The general equations of the problem are

$$\nabla^2\vec{H}_t = \frac{4\pi\sigma}{c^2}\left[\frac{\partial\vec{H}_t}{\partial t} + 4\pi\frac{\partial\vec{M}_t}{\partial t}\right],$$

$$\partial M_t/\partial t = r[\vec{M}\times\vec{H}_{\text{int}}]_t - \vec{M}_t/\tau, \tag{1}$$

and H_t, M_t, are the transverse H and M fields, σ is the conductivity of the metal, H is the applied static field, H_{int} is the total field inside the metal, τ is the transverse relaxation time, M is the saturation magnetization, and A is the exchange stiffness constant. In the following equations δ is the skin depth given by $\delta^2 = c^2/2\pi\sigma\omega$.

It is well known that the exchange-field contribution is important in ferromagnets because of the strong coupling between spins. Thus, changes in the magnetization at any point inside the metal will influence the internal field "seen" by the spins throughout the bulk of the metal.

When the static magnetic field is perpendicular to the sample surface it is easier to work with the positive and negative helicity fields represented by lower case letters (e.g., $h^p = H_x + iH_y$, $e^n = E_x - iE_y$). Since the mathematical treatment of the two waves is the same we will do the analysis for the positive helicity wave, dropping the superscripts for the time being.

If $H_t, M_t \propto \exp[i(kz-\omega t)]$ we obtain the following equation for k:

$$k^4+\left[\frac{M}{A}\left(H-4\pi M-\frac{\omega}{\gamma}-\frac{i}{\gamma\tau}\right)-\frac{2i}{\delta^2}\right]k^2+\frac{2M}{i\delta^2 A}\left(H-\frac{\omega}{\gamma}-\frac{i}{\gamma\tau}\right)=0. \quad (3)$$

As a result of (3) the incoming wave will, inside the metal, split into four waves described by the propagation vectors $\pm k_1$ and $\pm k_2$, solutions of (3). This is due to the inclusion of the exchange field which is proportional to k^2.

We define the six positive helicity amplitudes involved in this problem as $h_j^{\pm}$ where $j=0, 1, 2$. The + (−) sign represents waves propagating in the positive (negative) z direction. The surface of the sample in which the absorption resonance is excited is taken to be at $z=0$. The second surface of the sample is at $z=L$. The index $j=0$ represents the incoming and reflected waves in the excitation cavity. The $j=1, 2$ indices represent the k_1, k_2 waves inside the metal. The total transmitted wave in the receiving cavity is h_t. By applying the boundary conditions on the e and h fields at the two sample surfaces and imposing the Kittel boundary conditions,[10] $m^- + m^+|_{z=0,L}=0$, we obtain the following six equations:

$$h_1^+ + h_1^- + h_2^+ + h_2^- = h_0^+ + h_0^-,$$
$$k_1(h_1^+ - h_1^-) + k_2(h_2^+ - h_2^-) = ak(h_0^+ - h_0^-),$$
$$\alpha_1(h_1^+ + h_1^-) + \alpha_2(h_2^+ + h_2^-) = 0,$$
$$h_1^+ e^{ik_1L} + h_1^- e^{-ik_1L} + h_2^+ e^{ik_2L} + h_2^- e^{-ik_2L} = h_{tr} e^{ikL},$$
$$k_1(h_1^+ e^{ik_1L} - h_1^- e^{-ik_1L}) + k_2(h_2^+ e^{ik_2L} - h_2^- e^{-ik_2L}) = akh_{tr} e^{ikL},$$
$$\alpha_1(h_1^+ e^{ik_1L} + h_1^- e^{-ik_1L}) + \alpha_2(h_2^+ e^{ik_2L} + h_2^- e^{-ik_2L}) = 0, \quad (4)$$

where we have used $m_j = \alpha_j h_j$, $\alpha_j = 1 + ik_j^2\delta^2/2$ $(j=1, 2)$, $a = 4\pi\sigma/\omega$.

Solving the system of Eqs. (4) for h_{tr} we obtain, for the positive helicity field,

$$h_{tr}^p = -\frac{2akh_0^p e^{-ikL}(x_1+x_2)}{(x_1'+x_2'-ak)^2-(x_1+x_2)^2}, \quad (5)$$

where

$$x_{1,2} = \frac{k_{1,2}}{(1-\alpha_{1,2}/\alpha_{2,1})i\sin k_{1,2}L} \quad (6)$$

and

$$x_{1,2}' = \frac{k_{1,2}}{(1-\alpha_{1,2}/\alpha_{2,1})i\tan k_{1,2}L} \quad (7)$$

as defined by Phillips[12] who has solved this problem independently and applied the results to Permalloy resonance at 4.2°K. The expression for h_{tr}^n is the same as (5) with h_0^p replaced by h_0^n and the solutions to the negative helicity equation for k,

$$k^4+\left[\frac{M}{A}\left(H-4\pi M+\frac{\omega}{\gamma}+\frac{i}{\gamma\tau}\right)-\frac{2i}{\delta^2}\right]k^2+\frac{2M}{i\delta^2 A}\left(H+\frac{\omega}{\gamma}+\frac{i}{\gamma\tau}\right)=0, \quad (8)$$

substituted into (6) and (7).

Since the inclusion of the exchange field generates extra waves inside the metal the determination of all the field amplitudes uniquely would be impossible without the use of an extra boundary condition. The physical basis of the Kittel boundary condition[10] is that the lower local symmetry of a spin at the surface as compared with one inside the metal causes a higher anisotropy energy at the surface. This in turn causes the pinning of the surface spins. Surface pinning could also be caused by a layer of antiferromagnetic oxide or other surface impurities.

Since our apparatus is phase sensitive, detects amplitudes, and utilizes crossed cavities, the theoretical signal must be

$$\text{Signal} = \text{Im}[(h_{tr}^p + h_{tr}^n)e^{i\varphi}], \quad (9)$$

where φ is any phase shift introduced by the experimental apparatus. In Fig. 1, (9) is plotted

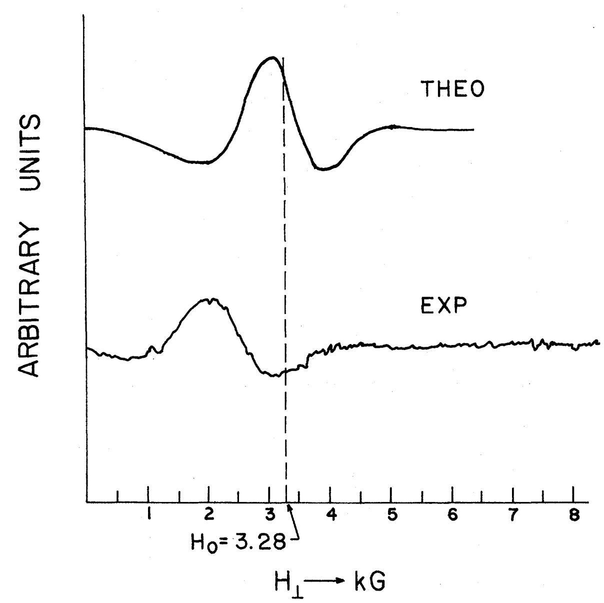

FIG. 1. Theoretical and experimental results for Gd at $T=287$°K. For the parameter values used (see text) the calculated signal strength is about an order of magnitude larger than the experimental one.

with (3) through (8) taken into account, and compared with the experimental result for Gd at $T = 287°K$. (The Gd Curie point is $T_C = 289°K$.) The parameter values used are $M = 925$ G, $\delta = 4$ μm, $\gamma = 1.77\times10^7$ Hz/G, $\tau = 1\times10^{-10}$ sec, $A = 1\times10^{-6}$ erg/cm, $\varphi = 0$ rad.

In Fig. 2 the signal expression is plotted for $M = 485$ G, $\delta = 1$ μm, $\gamma = 1.94\times10^7$ Hz/G, $\tau = 2\times10^{-10}$ sec, $A = 0.75\times10^{-6}$ erg/cm, and $\varphi = \pi$ rad, and compared with the experimental result for Ni at $T = 300°K$. The Ni Curie point is $T_C = 631°K$. The value for A is as quoted by Martin.[13]

As an indication of the relative magnitudes of k_1L and k_2L we found for the positive helicity wave at $H = 0$, $k_1L = 7.53 + 6.59i$, $k_2L = 4.24\times10^2 + 2.23\times10^4 i$ for Gd and $k_1L = 7.38 + 6.96i$, $k_2L = 43 + 3.03\times10^3 i$ for Ni.

The theoretical results in Figs. 1 and 2 should, at this point, be compared only qualitatively with the experimental ones. The value $A = 1\times10^{-6}$ erg/cm used for Gd is essentially arbitrary. We are unaware of any established value for A_{Gd}. To obtain the accurate values for the parameters of these metals we have to fit the experimental results with the theory through the use of an accurate fitting scheme, as was done in Ref. 4 for paramagnetic Gd. For this it may be necessary to utilize Landau-Lifshitz damping and/or the boundary condition of Ref. 11. We intend to investigate all of these factors in the future when we plan to pursue the accurate fitting of experimental results with the theory at various temperatures.

The strengths of the calculated signal amplitudes for both Ni and Gd are about an order of magnitude higher than the observed experimental strengths. The experimental transmitted power is of the order of 10^{-14} W.

Because the k_1 wave is damped much faster than the k_2 wave (see values for k_1L and k_2L above) a theory which treats the k_1 and k_2 waves in the metal as independent after they are formed gives results virtually identical to the theory described above as long as only one wave (uniform precession mode) is excited in the metal. By treating the waves independently we mean that the $M_t = 0$ boundary condition is applied at $z = 0$ only and then the waves are treated as two independent waves in the usual series approach to transmission through a barrier.

By the systematic study of this bulk resonance effect as a function of temperature, fitting the results with experiment accurately,[4] we can obtain τ and A as functions of temperature. A is related to J by $A = ZS_0^2a^2J/6\Omega_0$,[10,14] where Z is the number of nearest neighbors, S_0 is the spin of the electrons responsible for ferromagnetism, a is the lattice constant, Ω_0 is the atomic volume, and J is the exchange integral. By determining A as a function of temperature from the experimental data and using the appropriate values for Z, S_0, a, and Ω_0 we can then find J as a function of temperature.

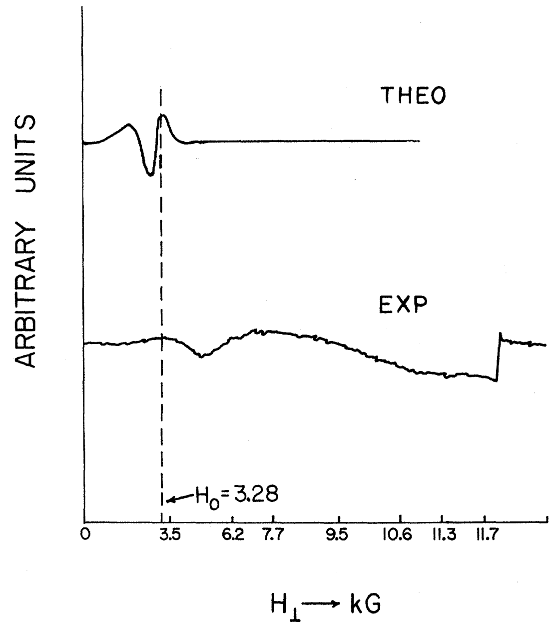

FIG. 2. Theoretical and experimental results for Ni at $T \cong 300°K$. For the parameter values used (see text) the calculated signal strength is about an order of magnitude larger than the experimental one.

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DEUTERON-DEUTERON ELASTIC SCATTERING AT 2.2 AND 7.9 GeV/c: EXPERIMENTAL STUDY

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Differential scattering data have been obtained in bubble-chamber exposures and are compared with theoretical predictions using the Glauber multiple-scattering theory. The effect of the quadrupole deformation of the deuteron and the influence of the various orders of multiple scattering are put in evidence. Agreement between theoretical predictions and experiment is very good at the higher energy. At the lower energy, agreement is good only for momentum transfers below 0.2 (GeV/c)^2. Possible reasons for the disagreement above 0.2 (GeV/c)^2 are explored.

Recent experiments[1] studying the elastic scattering of particles on deuterons have shown the success of Glauber's multiple-scattering theory when the quadrupole deformation of the deuteron is included. However, certain features of multiple-scattering theories can be explored only when both colliding particles are composite. Deuteron-deuteron scattering is the simplest example of such a process. Furthermore, the study of composite nuclear systems of familiar particles may help in formulating theoretical models which treat hadron-hadron scattering as collisions between composite systems of unknown subunits.[2]

We have performed a series of experiments on d-d elastic scattering from 0.68 to 7.9 GeV/c laboratory momenta. The first experiment[3] was a general survey of d-d elastic cross sections from 0.68 to 2.12 GeV/c (referred to in this paper as Experiment I). The main conclusions from Experiment I were as follows: (a) The total cross section defects provide an insensitive way of studying the effects of multiple scatterings within the deuteron. (b) The forward peak in the elastic differential cross section is well predicted with the Glauber formalism[4] including only single- and double-scattering terms. (c) At large momentum transfer, the flat region of the differential cross section demonstrates qualitatively the importance of the simultaneous multiple-scattering processes. (d) The best agreement between theory and experiment is obtained at the highest beam momentum.

The two experiments described in this paper provide data which extend and make more precise the conclusions reached in Experiment I. In particular, differential cross sections at momentum transfers larger than 0.05 (GeV/c)^2 are presented with much larger statistics and up to appreciably higher energies. These data allow us to make a quantitative comparison between theory and experiment at large t values and in the "dip" region where interference between single and double scattering and the quadrupole deformation of the deuteron are important.

The experiment at 2.2 GeV/c was performed at the Princeton-Pennsylvania Accelerator (PPA). The PPA 15-in. rapid-cycling bubble chamber was exposed to an electrostatically separated secondary deuteron beam with a momentum bite of 0.5%. The contamination of protons in the beam was 4% as indicated by a time-of-flight spectrum. The flash was triggered only on pic-