d_∞ in solids is noticeably larger than in gases. There is some evidence that d_∞ in a carbon foil is 10 times as much as in argon gas for Br ions at 140 MeV.[5]

(4) In dilute gaseous targets, charge-changing cross sections can be derived from the measurement of nonequilibrium charge distributions. A similar technique is applicable to solids only if one also includes the complex excitation cross sections and Auger probabilities.

(5) We conjecture that inside a solid the equilibrium value of $n\epsilon$ will be reached when the Auger process which results in de-excitation and electron loss balances collision excitation and loss. In that case, the mean time for de-excitation by the Auger effect is of the order of

$$\Delta t \approx n/(\sigma^* g V), \tag{5}$$

where σ^*, g, and V are an effective excitation cross section, the density of the solid, and the ion velocity, respectively. In the above example for iodine ions passing through carbon, we estimate $\sigma^* \approx 4\times10^{-16}$ cm^2/atom and Eq. (5) leads to $\Delta t \approx 10^{-15}$ sec, a not unreasonable lifetime for the Auger effect. This mean time is then also the time for the first electrons to de-excite after the ions leave the solid.

Possibilities for experimental tests of the present model are evident. We also note that the properties of the emerging ions—the many states of excitation and angular momentum—have implications for several fields such as beam foil spectroscopy or perturbed angular correlations of nuclear states following recoil from solids into vacuum.

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DETECTION OF A VORTEX-FREE REGION IN ROTATING LIQUID HELIUM II†

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We observe a nonlinear dependence of the number of vortex lines in a rotating container on the rotation frequency. The data are consistent with a model in which there are about two missing rows of vortices near the walls.

The question of the distribution of quantized vortex lines in rotating superfluid helium has received considerable attention, both theoretical[1,2] and experimental,[3-7] in the past decade. Most experimental studies have been carried out either in situations in which the distribution is indistinguishable from a uniform one, or at the other extreme, near the threshold for production of vortices. In the latter case the distribution may be determined more by the dynamics of vortex nucleation than by considerations of thermodynamic equilibrium.

We will present here the results of an experiment concerning an intermediate region—one far enough from threshold so that nucleation should not interfere with the achievement of thermodynamic equilibrium, yet close enough so that certain nonuniformities in the distribution predicted by the equilibrium theory become apparent.

In the absence of boundaries the free energy of rotating He II will be minimized by a uniform distribution of vortex lines parallel to the rotation axis. The density of vortex lines n_0 is related to the angular frequency Ω by

$$n_0 = 2\Omega/\kappa, \tag{1}$$

where κ is the quantum of circulation (h/m_{He}). When the effect of boundaries is included this theoretical vortex configuration is modified slightly.[1,2] As a first approximation the minimum free energy should be achieved by a distribution like that indicated schematically in Fig. 1(a). Vortices are present with their normal density n_0 everywhere except in a vortex-free region next to the boundaries whose width

$$(R-R_A) = \beta n_0^{-1/2} \tag{2}$$

is proportional to the mean vortex spacing. The predicted value of the constant β is about 1.4, which corresponds roughly to one missing row of vortices near the walls. The only previous attempt[7] to detect this effect experimentally has been interpreted as indicating the existence of a vortex-free region more than an order of magnitude larger than predicted.

When these boundary effects are included, the total number of vortices N in a rotating container is no longer a linear function of Ω but instead has the form $N \propto \Omega(1-\alpha\Omega^{-1/2})$, where α is a constant. Our experiment is designed to look for this low-frequency departure from linearity, by a technique which utilizes the trapping of negative ions by vortex lines. Electrons in liquid helium form small bubbles which are attracted to vortex lines by the Bernoulli force and captured. The trapping of free charges propagating perpendicular to the vortex lines can be adequately described in terms of a capture diameter or cross section.[8] Once trapped, the charges are free to move along the line and discharge at the ends. At the temperatures of concern here they have a negligible probability of escape radially from the line.

Our apparatus is illustrated schematically in Fig. 1(b). The cell is in the form of an enclosed annulus formed by a sequence of gold-plated electrodes whose potentials could be independently adjusted. These electrodes are spaced 0.005 in. apart by means of insulating rings [shown as black squares in Fig. 1(b)]. Note that these spacers are recessed 0.040 in. to avoid the possibility of space charge accumulating on them. The entire apparatus and Dewar are mounted on a large rotating table which has been described previously.[6] At room temperature the cell was measured

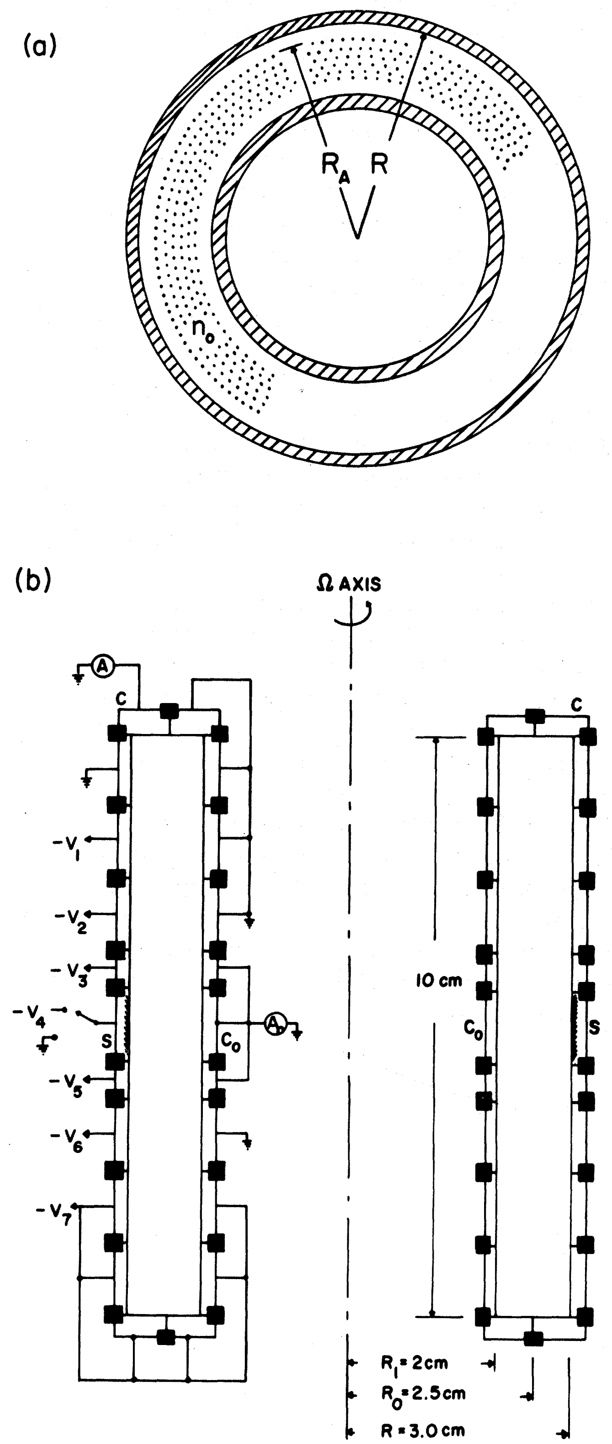

FIG. 1. (a) Schematic of model vortex distribution viewed along rotation axis. The dots represent a region of uniform vortex density bounded on the outside by a circle of radius R_A. (b) Schematic of experimental cell. The electrode potentials for the data shown in Figs. 2 and 3 were as follows: $V_1 = 20$ V, $V_2 = 30$ V, $V_3 = 40$ V, $V_4 = 42.5$ V, $V_5 = 45$ V, $V_6 = 55$ V, and $V_7 = 65$ V.

to be concentric with the table rotation axis to within 0.005 in. and to have no measurable tilt, but this measurement could not be checked at low temperatures.

A 10-mCi Po^{210} α source is plated on the inside of electrode S. A radial electric field draws a negative ion current I_0 from S to a collector C_0 which forms part of the inner cylinder. Some of this transverse current is trapped on each vortex line and, under the influence of space-charge fields, diffusion, and any vertical component of the applied field, will reach a second collector C at the top of the annulus. This current I is measured by a Cary vibrating-reed electrometer. Under the assumption that the capture cross section σ is independent of the position of a vortex line,[9] the ratio I/I_0 will be proportional to the number of lines ending on C. Our data consist of measurements of this ratio as a function of Ω. The procedure followed was to adjust Ω to a new value, wait 15 minutes to ensure equilibrium, and then by digital integration techniques average the current over a 45-min period to produce one data point. During this time the currents were turned on and off by alternately switching S between V_4 and ground over a 5-min cycle, and the difference signals were measured. This very low-frequency manual "lock-in" procedure effectively eliminated any electronic drift.

Representative data are shown in Fig. 2. These data are a combination of the results of three separate runs taken over a period of two weeks. Data were taken both by increasing and decreasing Ω in steps, and also by returning to $\Omega = 0$ between each measurement. At this temperature, 1.43°K, there was little hysteresis. However, at 1.25°K there was considerable hysteresis which prevented meaningful measurements at lower temperatures. In this case the transverse current I_0 was about 4×10^{-12} A and the reproducibility of the point at $\Omega = 0.05$ rad/sec reflects a noise in I of less than 1×10^{-16} A. The additional noise at large Ω is caused by fluctuations in I and not instrumental effects. Note that $\Omega = 0.05$ rad/sec corresponds to a mean vortex spacing of 1 mm. The angular frequency was stable to within ±0.003 rad/sec when averaged over 2-sec intervals. Temperature was regulated electronically to within 0.001°K.

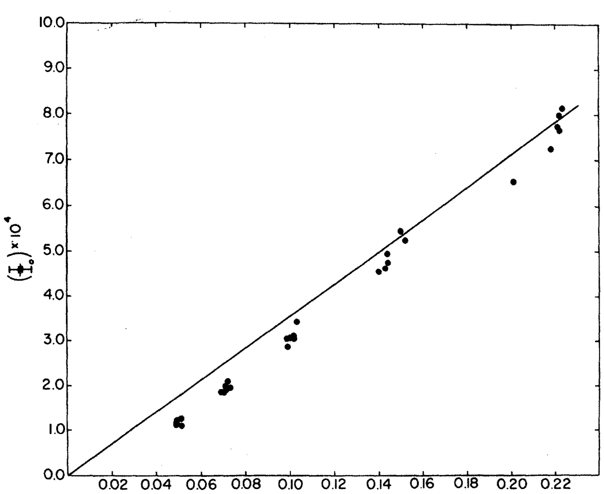

FIG. 2. (I/I_0) vs Ω. $T = 1.43$°K. The only significance of the solid line is to indicate the nonlinear behavior.

It is clear from the figure that the fraction of the transverse current collected is not linear in Ω but drops consistently below a straight line at low frequencies. This behavior is more clearly illustrated in Fig. 3, where we have plotted $(I/I_0)/\Omega$ against $\Omega^{-1/2}$. A linear relation would give a horizontal line on this graph. The vortex-free strip model predicts that for our geometry

$$(I/I_0)/\Omega = A(1-B\Omega^{-1/2}), \qquad (3)$$

where

$$A \equiv \sigma(2/\kappa)(R-R_0),$$

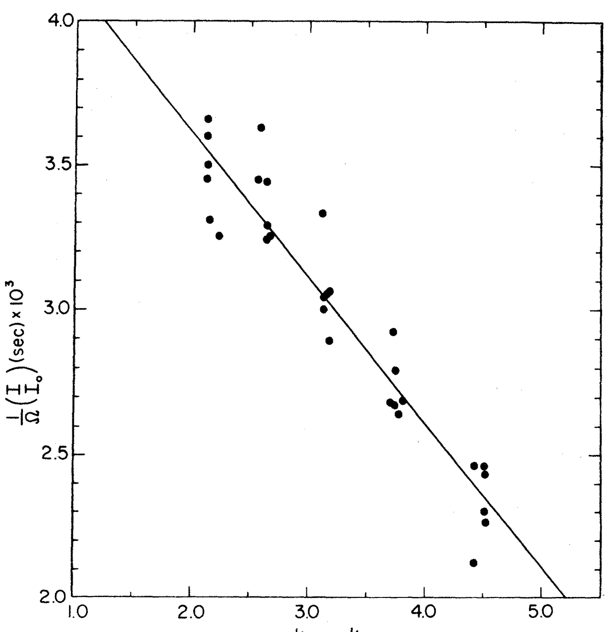

FIG. 3. $(I/I_0)/\Omega$ vs $\Omega^{-1/2}$. The straight line has been fitted by least squares to the data. Values of β and σ obtained by Eq. (3) are, in this case, $\beta = 2.5$, $\sigma = 4.5 \times 10^{-6}$ cm.

and

$$B \equiv \beta(R-R_0)^{-1}(2/\kappa)^{-1/2},$$

and R and R_0 are the outer radius of the annulus and the inner radius of C, respectively. This predicted decrease, linear in $\Omega^{-1/2}$, is consistent with the data. The straight line in Fig. 3 has been fitted by least squares to the data, and the value of the parameter β so determined is 2.5. On different runs we have measured values of β which range from 2.3 to 3.1 with an average value of 2.7. This means that we measure about two missing rows near the boundary rather than the predicted value of about one missing row. There may be a slight systematic dependence of the measured value of β on the field configuration, but it is difficult to distinguish it from the experimental scatter. If present, it is too small to affect the conclusions presented here.

The values of the cross section determined are in reasonably good agreement with measurements of σ made by Douglass[5] and are about a factor of 2 smaller than those obtained by Tanner.[6,10] They show the expected inverse relationship with the mean transverse electric field.

While we obtain a value of β much closer to the calculated value than did Tsakadze,[7] we regard our measurement as still significantly larger than the theoretical prediction. This difference is too large to be accounted for either by uncertainties in the vortex core parameters or by the approximations made in the calculation. Certainly one possible explanation could be that the assumption of thermodynamic equilibrium is inappropriate in the present situation. However, we would like to propose a second possible reason for the discrepancy. We suggest that the liquid may well have the equilibrium distribution of vortices, but that the lifetime of vortices in the outermost row is too short for them to be detected in this experiment. There may be a detailed balance between vortices entering the outermost row from the walls and others leaving it, so that the row is filled on the average, yet an individual vortex line is not present for a sufficient length of time to trap charge and deliver it to the top collector. If this lifetime is much shorter than a minute we are not likely to detect such a vortex in this experiment. Consequently, our measurements would indicate that the outermost row was absent.

In this model we must conclude that our measured value of β is an upper bound on the true value, and that the discrepancy is a qualitative measure of the amount of motion taking place in the outer rows of the vortex array. A related conclusion is that any experiments designed to determine the detailed vortex lattice structure and which require long-term stability of the lattice are unlikely to succeed.

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