ing from exchange of an exotic meson (or its Regge trajectory). However in the case of Reaction (1), present knowledge is not sufficient to distinguish between these two exchange mechanisms.

"Single-exchange forbidden" reactions, like (1), were also observed in $\bar{p}p$ and meson-nucleon collisions. The reaction $\bar{p}p \to \bar{\Sigma}^+\Sigma^-$ was observed between 2 and 7 GeV/c.[8] However its cross section is smaller than (1) and it drops faster with energy, like $p_{1\text{ab}}^{-6.5\pm2.0}$.[9] Substantial forward peaks were observed in the reactions $\pi^+n \to \pi^-\Delta^{++}$ and $\pi^-p \to K^+Y^*(1385)$ between 2 and 4 GeV/c.[4] However it is not clear that all these reactions can be explained in the same way as (1). For instance, the exotic $b\bar{b}$ meson of Rosner[6] is not coupled to mesons and cannot explain the meson-nucleon reactions.

More experiments on forbidden reactions above 4 GeV/c and direct search for exotic mesons are required in order to solve the basic questions of the existence of exotic states and their importance in high-energy reactions.

We would like to thank Brookhaven National Laboratory for enabling us to obtain the pd exposure at 6.98 GeV/c. We also acknowledge the contribution of Dr. O. Benary to the pd experiment.

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PROPERTIES OF THE $N^*(1730)$ †

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The $N^*(1730)$ with a mass of 1730 ± 15 MeV and a width of 130 ± 30 MeV is produced in the reaction $\pi^-p\to\pi^-N^*(1730)^+$ at 6 GeV/c. The principal decay mode of this N^* is $N\pi\pi$ and not $\Delta\pi$. The $N\pi$ decay rate is found to be much smaller, and the ΛK decay rate is found to be much larger than the accepted rates for the well-known $N_{1/2}^*(1680)$ states ($D_{5/2}$ and $F_{5/2}$), thus establishing that the object that we observe is not the $N_{1/2}^*(1680)$.

Recent production experiments have indicated that there may exist a nuclear isobar decaying into $N\pi\pi$ and ΛK with a mass of ~1710 MeV, somewhat above that of the well-known $N_{1/2}^*(1680)$.[1-3] Arguments that this object differs from the $N_{1/2}^*(1680)$ rest primarily on the small mass difference, since no single production experiment has observed all possible decay modes. Formation experiments indicate one or more possible resonances decaying into $N\pi$ and/or ΛK in the $N^*(1680)$ region with a branching ratio $N\pi/N\pi\pi$ of approximately 1.[4]

The data presented here come from an exposure of the Brookhaven National Laboratory (BNL) 80-in. hydrogen bubble chamber to a 6 GeV/c $\pi^\pm$ beam.

The reactions of interest are

$$\pi^-p\to\pi^-X^+$$
$$\quad\hookrightarrow N\pi\pi$$
$$\quad\hookrightarrow N\pi$$
$$\quad\hookrightarrow N\eta$$
$$\quad\hookrightarrow \Lambda K,$$

$$\pi^+p\to\pi^+X^+$$
$$\quad\hookrightarrow n\pi^+.$$

Our sample consists of ~70 000 two-prong, 30 000 four-prong, 4000 six-prong, 1000 eight-prong, and 20 000 strange particle events. These events were measured on the BNL flying-spot digitizer.

For the four-, six-, and eight-prong samples, only events with one non-minimum–ionized positive track were measured. This selection gives a sample of identifiable proton events with proton laboratory momenta less than 1.5 GeV/c, and does not affect our subsequent analysis. For the two-prong π^+p samples, only events with nonproton tracks were analyzed.

The $p\pi^+\pi^-$ mass spectrum from the $p\pi^+\pi^-\pi^-$ final state is shown in Fig. 1(a), where the more peripheral of the two possible $p\pi^+\pi^-$ combinations for each event is plotted. A clear enhancement of approximately 6 standard deviations is seen in the $p\pi^+\pi^-$ spectrum at 1730 MeV with a smaller enhancement at ~1470 MeV. Fitting an s-wave Breit-Wigner resonance and estimated background to the data as shown yields a mass m of 1730 ± 15 MeV and width Γ of 130 ± 30 MeV for the 1730 enhancement. The uncertainty in the estimate of different background levels is reflected in the large error in width; however, it does not affect the central mass value of this resonance. The fitted mass value is inconsistent with that of the $N_{1/2}^*(1680)$.[4] To determine whether the $N^*(1730)$ decays into $\Delta(1238)\pi$, we made a direct fit to the $p\pi^+\pi^-$ Dalitz-plot (not shown) density using the maximum-likelihood technique. A standard Breit-Wigner resonant shape was assumed for the $\Delta(1236)$. The results are displayed in Fig. 2(a) which shows that there is no significant $\Delta(1236)^{++}\pi^-$ contribution to the $N^*(1730)^+$ while an enhancement is clearly present in the uncorrelated $p\pi^+\pi^-$ system, Fig. 2(b). We have also looked for a $p\rho^0$ decay mode and found none. A smooth curve calculated from the parameters obtained from the fit in Fig. 1(a) is seen to give a good fit to the enhancement in Fig. 2(b). The

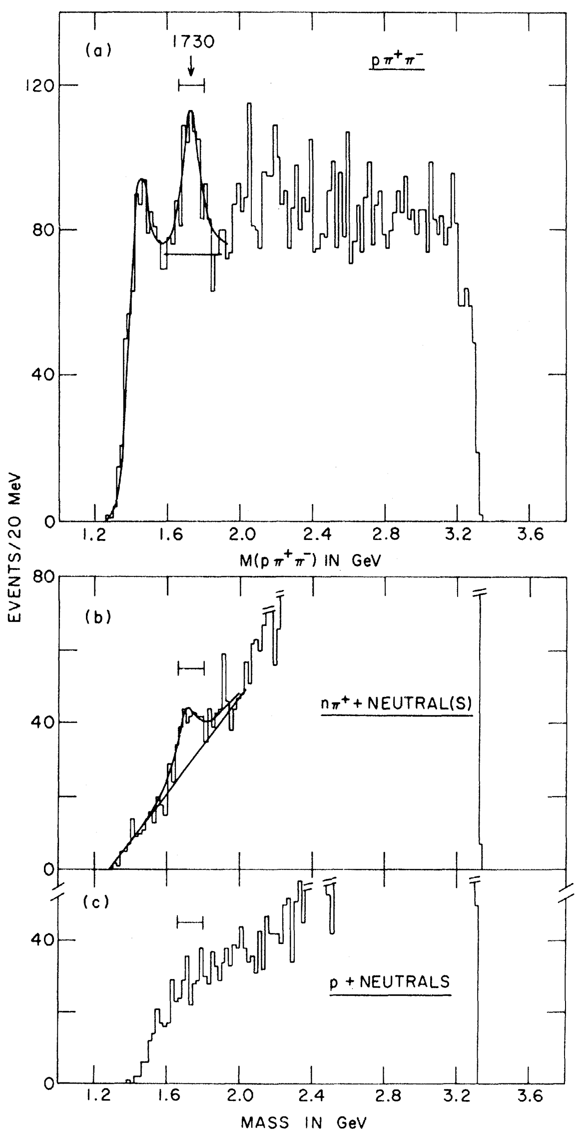

FIG. 1. Mass spectrum, X^+, recoiling against π^- as indicated from the reactions $\pi^-p \to \pi^-X^+$. See text for details.

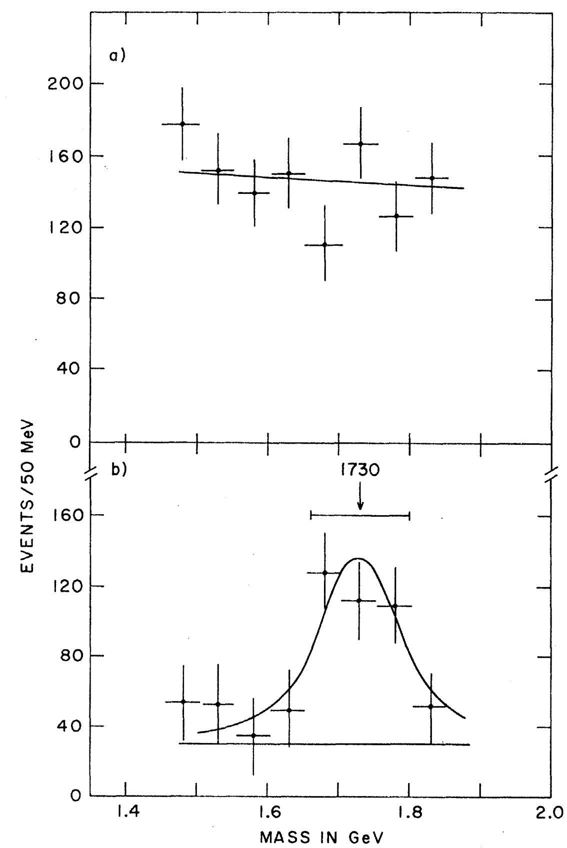

FIG. 2. (a) The $\Delta^{++}\pi^-$ component of the $p\pi^+\pi^-$ mass spectrum. (b) The non-$\Delta^{++}\pi$ component of the $p\pi^+\pi^-$ mass spectrum.

number of events under the curve in Fig. 2(b) is consistent with the number under the curve in Fig. 1(a) and indicates that the 1730-MeV enhancement decays entirely into an uncorrelated $p\pi^+\pi^-$ system, a result in agreement with Ref. 2 but in disagreement with Ref. 3. Due to this uncorrelated three-body decay, it is not possible to make predictions for the decay rates into $n\pi^+\pi^0$ and $p\pi^0\pi^0$. However, an attempt is made to obtain the "$n\pi^+\pi^0$" mass spectrum from the reactions $\pi^-p\to\pi^-[\pi^+n+\text{neutral(s)}]$ where the mass of n+neutral(s) is greater than the neutron mass,[5] as shown in Fig. 1(b), and the "$p\pi^0\pi^0$" mass spectrum from the reactions $\pi^-p\to\pi^-(p+\text{neutrals})$ where the mass of neutrals is greater than one π^0 mass,[5] as shown in Fig. 1(c). A significant 1730-MeV enhancement is present in "$n\pi^+\pi^0$" but not "$p\pi^0\pi^0$". The "$n\pi^+\pi^0$" enhancement in Fig. 1(b) is believed to be due to $n\pi^+\pi^0$ and not to $n\pi^+$ plus two or more π^0's, since a search for the analogous decays $p\pi^+\pi^-\pi^0$ and $p\pi^+\pi^-\pi^+\pi^-$ (not shown) indicates no $N^*(1730)$ signal. An s-wave Breit-Wigner resonance with a fixed width of 130 MeV and a smooth background fitted to the "$n\pi^+\pi^0$" enhancement gives a central mass of 1710 ± 25 MeV in good agreement with the better determined $p\pi^+\pi^-$ state. The lack of any marked 1730-MeV enhancement in "$p\pi^0\pi^0$" supports our earlier conclusion that $N^*(1730)$ does not decay into $\Delta\pi$, since in this case equal enhancements should appear in "$n\pi^+\pi^0$" and "$p\pi^0\pi^0$."

We have also examined the slope, b, of the production differential cross sections $d\sigma/dt=ae^{bt}$ for $p\pi^+\pi^-$ and "$n\pi^+\pi^0$" in the $N^*(1730)$ mass region for $|t|\leq0.5$ GeV^2 and obtained $b(p\pi^+\pi^-)=4\pm1$ and $b(\text{"}n\pi^+\pi^0\text{"})=5\pm1.5$ GeV^{-2}, respectively. This further substantiates the similarity between these two decay modes of the same resonance. In this respect, it is interesting to note that the value of b for the enhancement at ~1470 MeV obtained from this experiment [Fig. 1(a)] is 8 ± 1 GeV^{-2}.

We have estimated that there are about 140 ± 40 events from "$n\pi^+\pi^0$" and 20 ± 20 events from "$p\pi^0\pi^0$" that contribute to the $N^*(1730)^+$ corresponding to 37 ± 9 μb in cross section. This plus the contribution from $p\pi^+\pi^-$ (345 ± 60 events corresponding to 41 ± 7 μb) gives a cross section of 78 ± 12 μb for the production of the $N^*(1730)^+\to N\pi\pi$ in the reaction $\pi^-p\to\pi^-N^*(1730)^+$ at 6 GeV/c. The quoted errors are statistical and do not include the systematic errors. The uncertainties in absolute cross sections can be as large as 50%. This, however, does not affect the ratios of cross sections among the samples examined here.

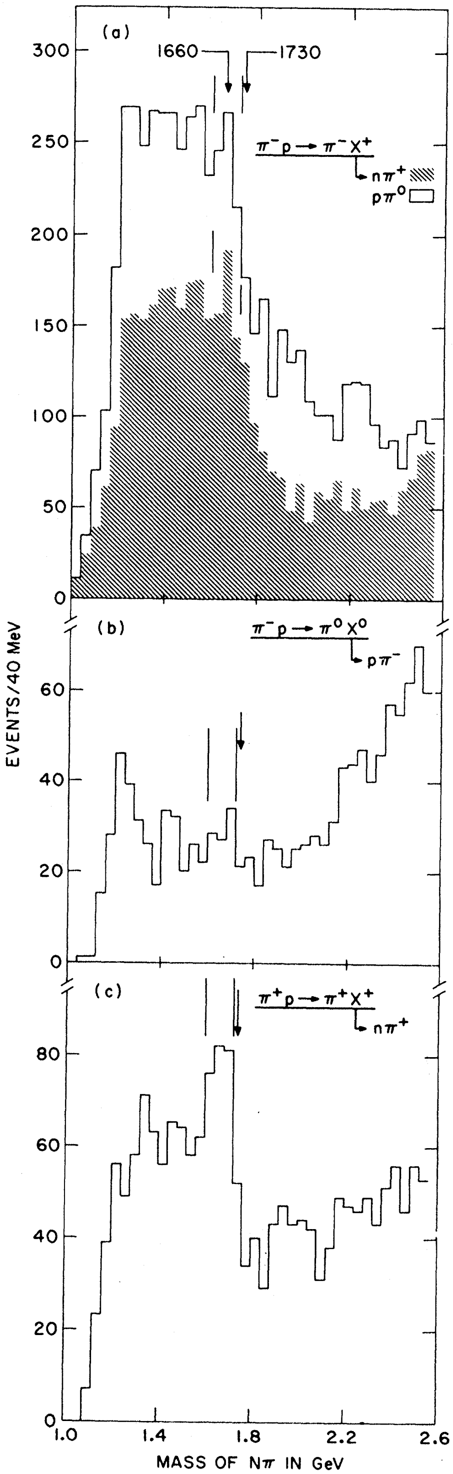

FIG. 3. $N\pi$ mass spectra as shown.

The $p\pi^0$ and $n\pi^+$ mass spectra[6] from the reactions $\pi^-p\to\pi^-\pi^0p$ and $\pi^-\pi^+n$ are illustrated in

Fig. 3(a). The low $N\pi$ mass enhancement extending from 1.2 to 1.75 GeV complicates the estimate of $N^*(1730)\to N\pi$ contribution. Nevertheless, we note that there is no marked enhancement centered at the 1730 MeV. We do, however, see some excess of events in the $n\pi^+$ mass spectrum centered at the 1660 MeV mass region [Fig. 3(a)] indicating maybe some weak signal of the $N_{1/2}^*(1680)$ decaying predominantly into $n\pi^+$ (shaded) and not $p\pi^0$ because the isospin of the $N_{1/2}^*(1680)$ is $\frac{1}{2}$. We have fitted the mass spectrum [Fig. 3(a)] with various reasonable assumed backgrounds in order to examine the possibility that this 1660 MeV enhancement could be shifted to 1730 MeV. The fits we obtained give the mass value of 1670 ± 15 MeV, a value significantly different from 1730 ± 15 MeV. We have also studied the systematic mass shifts of πN and $\pi\pi p$ from the two-prong samples ($\pi\pi N$) and four-prong samples ($\pi\pi\pi p$), and find the possible systematic shift is too small in comparison with the statistical error to be significant. We therefore consider the $n\pi^+$ enhancement at 1670 ± 15 MeV is not consistent with the mass value of 1730 ± 15 MeV. Furthermore, we have observed the $N_{1/2}^*(1680)$ in the reactions $\pi^-p\to\pi^0N_{1/2}^*(1680)^0\to\pi^0(\pi^-p)^0$ and $\pi^+p\to\pi^+N_{1/2}^*(1680)^+\to\pi^+(\pi^+n)^+$ as shown in Figs. 3(b) and 3(c) where the position as well as signal of the $N_{1/2}^*(1680)$ are better defined than those shown in Fig. 3(a). From the charge independence we expect the production of $N_{1/2}^*(1680)^+$ from the reaction $\pi^-p\to\pi^-N_{1/2}^*(1680)^+\to\pi^-(\pi N)^+$ to be at least 40 events [in Fig. 3(a)] via the triangle inequality relationship as follows:

$$[\sigma(\pi^-p\to\pi^-N_{1/2}^*(1680)^+)]^{1/2}\geq[2\sigma(\pi^-p\to\pi^0N_{1/2}^*(1680)^0)]^{1/2}-[\sigma(\pi^+p\to\pi^+N_{1/2}^*(1680)^+)]^{1/2}.$$

This result gives us further confidence that $n\pi^+$ enhancement at 1660 MeV in the reaction $\pi^-p\to\pi^-\pi^+n$ is due to the expected production of $N_{1/2}^*(1680)$ and not $N^*(1730)\to N\pi$. Finally, we give an estimate of 2 ± 10 μb for any $N^*(1730)^+\to(N\pi)^+$ contribution from the reactions $\pi^-p\to\pi^-N^*(1730)\to\pi^-(N\pi)^+$.

The $p\eta(550)$ mass spectrum (not shown) from $\pi^-p\to\pi^-p\eta\to\pi^-p\pi^+\pi^-(\pi^0)$ or (γ) does not seem to exhibit any structure in the 1730-MeV region, and an estimate of 1.5 ± 1.5 μb may be made. A clear enhancement of $\sim25\pm5$ events is observed in the ΛK^+ channel (Fig. 4) with mass and width similar to the 1730-MeV enhancement observed in the $p\pi^+\pi^-$ decay mode. This corresponds to a branching fraction into ΛK^+ of $\sim2\%$ and is clearly incompatible with the accepted value[4] of $<0.1\%$ for $N_{1/2}^*(1680)$. In addition, the slope $b(\Lambda K^+)$ of the production differential cross section of ΛK^+ channel is 6 ± 2 GeV^{-2} for $|t|\leq0.5$ GeV^2. This is again consistent with the $p\pi^+\pi^-$ enhancement in the 1730-MeV mass region. If we assume that these effects in ΛK^+ and $p\pi^+\pi^-$ are due to the same resonant state, then the isospin must be $\frac{1}{2}$. If we assume all effects observed here in the 1730 mass region to be one object, then we can summarize the two-body decay rates of the $N^*(1730)$ as follows:

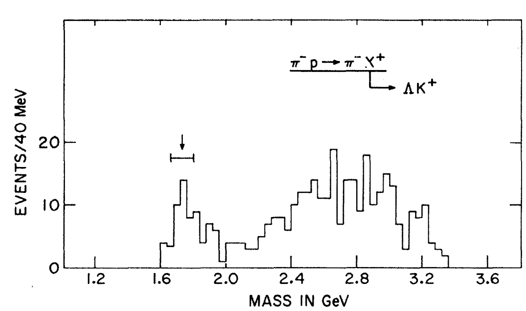

FIG. 4. The ΛK^+ mass spectrum where Λ decay is visible in the chamber.

$\pi^-p\to\pi^-N^*(1730)$

$\to(N\pi)^+$, 2.0 ± 10.0 μb;

$\to p\eta$, 1.5 ± 1.5 μb;

$\to\Lambda K^+$, 2.0 ± 0.5 μb.

In the context of SU(3) symmetry, we use the three experimental decay rates of the $N^*(1730)$ into baryon-meson systems ($N\pi$, $N\eta$, ΛK) and compare with SU(3) predictions. Because of small experimental rates with large errors into two-body modes for the $N^*(1730)$, our results do not enable us to distinguish between the $\underline{8}$, $\underline{10}$, and $\underline{27}$ representations. It is, however, interesting to note that the $D/(D+F)$ ratio for an octet assignment of this N^* is 0.3 ± 0.2.

In summary, we have established that the $N^*(1730)\to N\pi\pi$ and the well-known $N_{1/2}^*(1680)$ are two different states, based on the difference in mass as well as decay rates into the $N\pi$ and ΛK systems. Further we have shown that $\Delta(1236)\pi$ does not contribute to the $N\pi\pi$ decay mode of $N_{1/2}^*(1730)$.

We wish to thank Dr. Ralph Shutt for his support and encouragement throughout this experiment, and Dr. Uri Karshon for his participation in the early stages of this experiment. The successful conclusion of this experiment involved with data of this magnitude is not possible without the tremendous effort of the BNL Bubble Chamber Group data reduction staff under the direction of Dr. Paul V. C. Hough.

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[4]For an extensive summary of references see, Rev. Mod. Phys. 42, 87 (1970).

[5]The (π^+n + neutrals) spectrum consists of events having a mass of n + neutral(s) > 1.090 GeV. The (p + neutrals) spectrum contains events with mass of neutrals ≥ 0.5 GeV.

[6]The $n\pi^+$ events are selected with $0.790 \leq MM \leq 1.090$ GeV while $p\pi^0$ events are required to have $-0.3 \leq MM \leq 0.4$ GeV, χ^2 probability greater than 10 %, and no fit to the elastic hypothesis.

CONSTRUCTION OF PHYSICAL DUAL-RESONANCE MODELS*

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We construct and study six- and eight-charged-pion amplitudes. Factorization properties of the parent π trajectory show the π and A_1 to be simple and the $\pi(2^-)$ to be a double state reminiscent of the A_2 doubling.

The structure of the N-point dual amplitudes is currently the subject of much study. Fubini and Veneziano[1] and others[2] have shown that it is likely that there is a large degeneracy of states in order to maintain duality. One major drawback of the model amplitudes considered is that they lack several basic ingredients that physical amplitudes must possess. Hence, the investigations are performed with the hope that the properties established on the basis of these models will survive the modifications required by realistic amplitudes. As shown below, they do not, and radical changes occur when the intercepts are allowed to be positive. As a consequence, the particle spectrum is changed and in the case under study the degeneracy of the leading trajectory is affected, providing for doubling of the spin-2 and higher states, in a way reminiscent of the A_2 splitting.

First we report on a general method to construct N-point functions for arbitrary trajectories. For reasons stated below we will specialize in the six- and eight-point functions. We have not solved all the standing problems of the N-point functions but we have been able to make a choice that bypasses all difficulties.[3] Our amplitude is an exact representation of the $(\frac{1}{2}N)\pi^+$, $(\frac{1}{2}N)\pi^-$ amplitude when the three-body channels are dominated by the pion trajectory. Our isospin and parity are well defined in all channels and, as long as the contributions of other trajectories are additive (as they are generally believed to be), our solution is the most general one.

(a) All the relevant singularities, (b) Regge behavior, (c) bootstrap condition, and (d) absence of ghosts, are conditions explicitly obeyed by our formulas. We have no analytic expression but a construction procedure only. We have studied in detail the six-point function and some aspects of the eight-point one. One of the most interesting questions is the level structure. Both π and A_1 are simple states. However, the 2^- on the leading trajectory is doubled. These conclusions cannot be affected by the higher-order functions. We have not solved the factorization problem in general. The number of states depends on the number of external legs as opposed to the case in Ref. 1. Nevertheless, for fixed J, there is a finite number of diagrams that contribute to the level structure. Since we have computed only the six-point function we completely determined the degeneracy of the three lowest ones on the leading trajectory. No statement can be made on