Measurement of the Muonium Hfs Splitting and of the Muon Moment by "Double Resonance," and a New Value of α*

R. De Voe,† P. M. McIntyre, A. Magnon,‡ D. Y. Stowell, R. A. Swanson,§ and V. L. Telegdi
Enrico Fermi Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 10 August 1970)

We have determined the hyperfine interval $\Delta\nu$ and the muon moment $\mu_\mu = g_\mu'\mu_B/2$ from the frequencies of the two alternative Zeeman transitions in muonium, working at that "magic" field where $\partial\nu_1/\partial B = \partial\nu_2/\partial B = 0$. Extrapolating determinations of $\Delta\nu$ in Kr and Ar to zero pressure, we obtain consistent results, yielding jointly $\Delta\nu(0) = 4463.3022(89)$ MHz. Assuming that $g_j(M)$ is unshifted by collisions, we obtain $g_e/g_\mu' = 206.765\,09(80)$, which corresponds to $f_\mu/f_p = 3.183\,373(13)$. The combination of these results gives $\alpha^{-1} - 137 = 0.036\,54(30)$ (2.1 ppm in α^{-2}), in good agreement with the recommended value of 0.036 02(21).

Knowing both the muonium ground-state hyperfine interval $\Delta\nu(\mu e)$ and the muon magnetic moment $\mu_\mu \equiv g_\mu'\mu_B/2$ to high accuracy (say, <5 ppm) one can—aside from checking the quantum electrodynamics (QED) corrections to $\Delta\nu(\mu e)$—do two interesting things: (a) determine, with competitive precision, the fine-structure constant α, from an independent new source, and (b) set limits on the polarizability of the proton from the observed ratio $\Delta\nu(pe)/\Delta\nu(\mu e)$. To date this program could unfortunately not be realized in practice. While $\Delta\nu(\mu e)$ has become known with increasing accuracy over the past decade (from 13 ppm in 1964[1] to 2.0 ppm now), there has been no corresponding progress in the measurement of μ_μ. The most recent published determination[2] of μ_μ has an accuracy of 9 ppm, based on a comparison of the muon and proton precession frequencies in water.[3] This determination, as well as any other based on muon precession in matter, furthermore requires a systematic correction, presumably of the same magnitude,[4] to allow for differences in magnetic shielding.

We describe here a determination of μ_μ and $\Delta\nu$ from the frequencies of the Zeeman (F, M_F) transitions $(1,1) \to (1,0)$ and $(1,-1) \to (0,0)$ in the region of intermediate coupling.[5] The Breit-Rabi formula yields for these

$$\nu_j = (\Delta\nu/2)[1 + (-1)^j D(x, G)], \quad j = 1, 2, \tag{1}$$

where

$$D = (1+x^2)^{1/2} - x(1-2G),$$

with $G = g_\mu'(M)/[g_\mu'(M) - g_j(M)]$, $g_\mu'(M) = (1-\alpha^2/3)g_\mu'$, and $x = B\mu_B[g_\mu'(M) - g_j(M)]/h\Delta\nu$. We choose to work at that "magic" field $x_0(G)$ where $\partial D/\partial x = 0$, i.e., where both ν_1 and ν_2 become, to first order, field independent. The advantages of this choice have already been discussed[6] in connection with a measurement of ν_1. At x_0 ν_1 and ν_2 depend obviously, aside from proportionality of $\Delta\nu$, on G alone, as do their sum and difference. Explicitly, one has

$$\nu_+ = \nu_1 + \nu_2 = \Delta\nu, \tag{2a}$$

$$\nu_-(x_0) = \nu_2 - \nu_1 = 2\Delta\nu[G(1-G)]^{1/2}, \tag{2b}$$

$$\nu_-/\nu_+ = 2[G(1-G)]^{1/2}. \tag{2c}$$

Note that the ratio of direct interest for determining g_μ', (2c), is not affected by the pressure dependence of $\Delta\nu$.

While it would be adequate to determine ν_1 and ν_2 in independent runs (at x_0), there are statistical and systematic advantages to measuring ν_+ and ν_- directly by "double resonances," as we shall now discuss. Inasmuch as stopping (polarized) muons can capture electrons of either spin direction, muonium is formed 50-50 in one of the upper and lower Zeeman levels, for example, $F = 1$, $M_F = 1$; $F = 0$, $M_F = 0$. Thus inducing ν_1 and ν_2 transitions is completely equivalent. By having two microwave fields of the appropriate frequencies $\nu_{1,2}$ simultaneously present, one can hence induce a resonant spin-flip transition for each (μe) atom formed. By keeping ν_+ (ν_-) constant, one can thus sweep out ν_- (ν_+); the resultant "double-resonance" curves will have twice the height and width that a single resonance in ν_1 (or ν_2) would exhibit, provided one chooses the two rf power levels so as to insure equal signals in the two single (ν_1, ν_2) resonances. The importance of this proviso and of the initial choice of the frequency that is kept constant will be discussed later.

Experimental arrangement.—The apparatus used was substantially that of Ref. 6, except for the following essential modifications: (a) The field homogeneity was greatly improved (to <0.1%

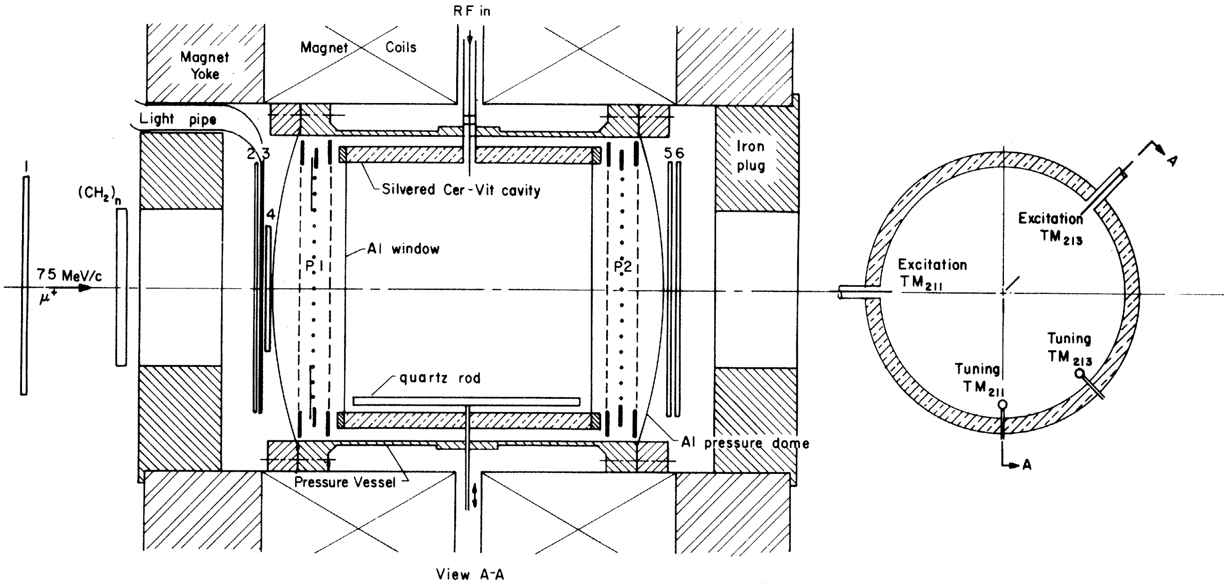

FIG. 1. Experimental setup to observe "double resonances" in muonium. The two iron plugs provide a better homogeneity of the axial static field. The rf modes have been rotated at 45° to ensure decoupling.

over the relevant volume) by reducing the end apertures of the magnet with iron plugs (see Fig. 1). (b) The microwave cavity (made of CerVit) was dimensioned for simultaneous excitation of the TM_{211} ($\nu_1 \simeq 1920$ MHz) and TM_{213} ($\nu_2 \simeq 2540$ MHz) modes; an azimuthal maximum of one of these corresponds to a minimum of the other, so that both modes can be tuned independently with two suitably placed quartz rods. Microwave power was provided by two klystrons, each driven by a thermostatted, crystal-controlled oscillator. During each run, the rf powers were kept stable to ±1% and the frequencies to ±0.01 ppm; the choice of decoupled modes enabled one to measure their Q's independently.

Experimental procedure and results.—First, single resonances in ν_1 and ν_2 as in Ref. 6 were taken, with rf power levels so chosen as to produce essentially identical signals; minor power adjustments were then made to make the observed curves equal. Next, double resonances were performed under the same conditions, varying ν_1 and ν_2 so as to keep either their sum or their difference constant (at "best estimate" values based on existing knowledge of $\Delta\nu$, g_μ', and $\partial\Delta\nu/\partial p$). Figure 2 shows a typical ν_+ resonance so obtained. Inasmuch as the field-dependent term $D(x)$ affects ν_1 and ν_2 with opposite signs, it cancels in their sum, and one performs in measuring ν_+ essentially a direct field-independent determination of $\Delta\nu$, as in zero field. We therefore reproduce in Fig. 2 for comparison an actual "zero-field" resonance.[7]

The quadratic and higher field corrections add however for ν_-, and it is hence for this resonance particularly important to work at the proper field. Fortunately, the double resonance in ν_- can itself be exploited to establish the correct "magic" field. One simply measures $\nu_-(x)$ in the neighborhood of the "best estimate" for x_0, and fits a parabola (actually a cubic) of a priori known curvature to the experimental points. The

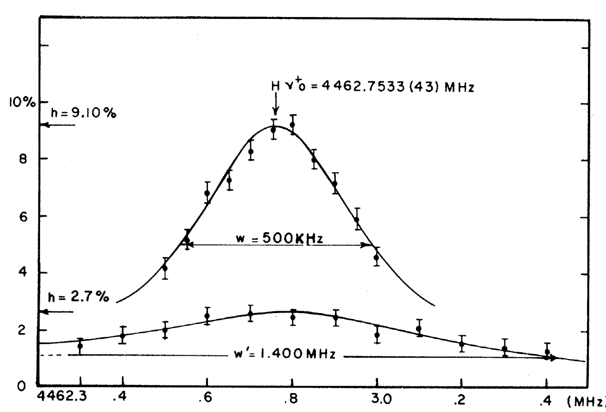

FIG. 2. The upper curve is one of the double resonances for the sum of frequencies at 11.830×10^3 Torr of krypton. The transitions in (F, M_F) are $(1,1) \leftrightarrow (1,0)$ and $(1,-1) \leftrightarrow (0,0)$. Error bars are counting statistics (1 standard deviation). The lower curve (Ref. 7) is a resonance for the same frequency obtained at 0.010 G in 32.800×10^3 Torr of krypton. It should be noticed that for a given counting rate, the error on the fitted center is for a Lorentzian line proportional to W/h.

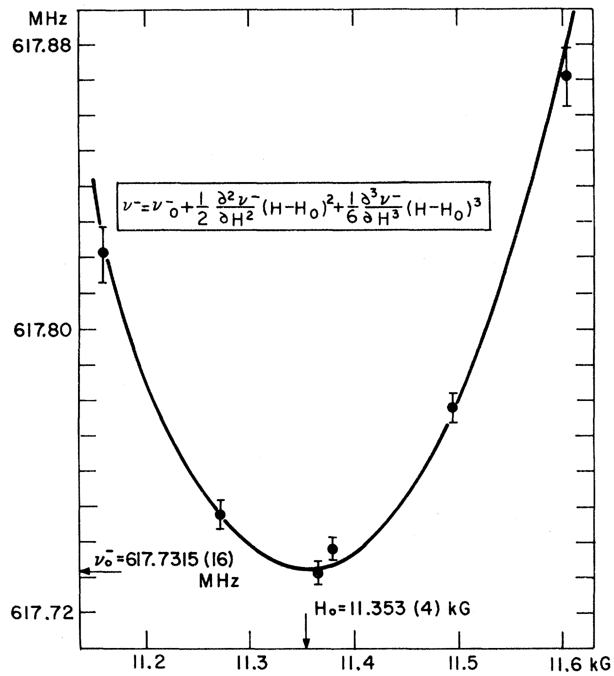

FIG. 3. Difference between frequencies versus static field. Errors (1 standard deviation) include counting statistics and field uncertainties.

parameters of this fit are essentially decoupled, so that one obtains $\nu_-(x_0)$ without significant loss in statistical power. Note that this procedure, illustrated in Fig. 3, automatically yields the correct effective average field, properly weighted with the muon stop distribution (which is constant in time in virtue of the excellent stabilization of the beam transport), positron detection efficiency, etc.

The main series of measurements was done in krypton at fairly high pressure (230 lb/in.[2] at 30°C), interspersing ν_- and ν_+ runs at the same gas density. Additional measurements of ν_+ were performed in Kr and Ar at low pressure for extrapolation purposes. The results are as follows:

$$\nu_+(\text{Kr, } 11\,830 \text{ Torr}) = 4462.7504(22) \text{ MHz}, \quad (3a)$$

$$\nu_-(\text{Kr, } 11\,830 \text{ Torr}) = 617.7299(12) \text{ MHz}, \quad (3b)$$

$$\nu_+(\text{Kr, } 2598 \text{ Torr}) = 4463.1824(50) \text{ MHz}, \quad (3c)$$

$$\nu_+(\text{Ar, } 3030 \text{ Torr}) = 4463.220(22) \text{ MHz}, \quad (3d)$$

where the pressures used as arguments indicate the "equivalent densities"[8] in Torr at 0°C, while the errors are standard deviations allowing for systematic uncertainties.

Extrapolating (3a) with (3c) to zero density one obtains, for Kr,

$$\Delta\nu(0) = 4463.3040(100) \text{ MHz} \quad (4a)$$

corresponding to a fractional pressure shift (FPS) $-(1/\Delta\nu)(\partial\Delta\nu/\partial p) = 10.50(32)\times 10^{-9}$ Torr^{-1}. This FPS agrees with that found for H in Kr [$-10.4(2)\times10^{-9}$ Torr^{-1}][9]; the extrapolation of (3c) with the latter shift yields, for Kr,

$$\Delta\nu(0) = 4463.3030(62) \text{ MHz} \quad (4b)$$

which is of course consistent with (4a). Similarly, (3d) extrapolates, using an FPS of 5.44(45) $\times 10^{-9}$ Torr^{-1},[6] to Ar

$$\Delta\nu(0) = 4463.293(23) \text{ MHz} \quad (4c)$$

which incidentally confirms an earlier result.[6] Thus the Kr and Ar values for $\Delta\nu(0)$ appear consistent with each other, and we take as the final value

$$\Delta\nu(0) = 4463.3022(89) \text{ MHz (2.0 ppm).} \quad (4d)$$

Using Eq. (2c) and assuming that $g_J(M)$ has its "vacuum value" $g_e(1-\alpha^2/3)$, one obtains from (3a) and (3b)

$$g_e/g_\mu' = 206.765\,09(80) \text{ (4 ppm)} \quad (5)$$

which is equivalent to

$$f_\mu/f_p = 3.183\,373(13). \quad (5')$$

This latter ratio may be compared with $f_\mu/f_p = 3.183\,36$ (3), based on a direct measurement[2] of f_μ. The agreement with (5′) suggests that no correction of the magnitude advocated by Ruderman[4] appears necessary; this agreement is, however, statistically not compelling.

Using the theoretical expression[1] for $\Delta\nu$, one obtains with (5) and (4d)

$$1/\alpha = 137.036\,54(30) \text{ (2.0 ppm)} \quad (6)$$

which agrees well with the currently "recommended"[10] value, $1/\alpha = 137.036\,02(21)$. Conversely, this agreement can be interpreted as a quantitative experimental verification of the QED corrections[11] entering $\Delta\nu$.

The ratio of the observed hydrogen and muonium hf intervals was formerly used[6] to obtain μ_μ/μ_p on the assumption that the proton polarizability $\delta_N^{(2)}$ is negligible. Now that this moment ratio is measured [Eq. (5)], one can, using (4d) and $\Delta\nu(pe) = 1420.405\,75$,[12] derive the value

$$\delta_N^{(2)} = -9.6 \pm 4 \text{ ppm}, \quad (7)$$

Systematic errors.—There are two working hypotheses underlying our method of measuring ν_- (or ν_+): (a) that the frequency not being determined, i.e., ν_+ (ν_-), is maintained at its correct (and unknown!) value $\nu_+{}^0$ (or $\nu_-{}^0$); (b) that the signals contributed are identical. Obviously both hypotheses are only approximations to experimental reality, and we have to discuss the systematic errors induced by them. The observed signal will have the general form

$$L(\nu_+) = L(\nu_+) = L_1[(\nu_+ - \nu_+{}^0 + \delta\nu_-)^2] + L_2[(\nu_+ - \nu_+{}^0 - \delta\nu_-)^2],$$

where $\delta\nu_- \equiv \nu_-{}^0 - \nu_-$ (i.e., the departure of ν_- from its correct value). Thus the center of a symmetric (say Lorentzian) curve fitted to the data will in general be shifted with respect to $\nu_+{}^0$, precisely by a systematic uncertainty Δ_+. This uncertainty, which is proportional to $\Delta\nu_-$, depends on the differences between L_1 and L_2 (as measured, see above); a numerical study gives

$$\Delta_+ \simeq \delta\nu_-/6.$$

Thus, e.g., in the measurement yielding (3a) the uncertainty $\delta\nu_- \doteq 6$ kHz in the set value of ν_-(= 617.732 MHz) contributed $\Delta_+ = 1$ kHz to the quoted error. The situation for ν_- is entirely analogous, except that to obtain (3b) ν_+ was fixed at its best estimate at this time yielding $\Delta_- = 0.85$ kHz.

Another source of potential systematic error is our neglect of the so-called g_j shift, i.e., of the fact that not only $\Delta\nu$ but also g_j can be affected by collisions with the host gas atoms.[13] For Rb in He, where this phenomenon has been measured,[14] the effect is small. It is not clear how one should extrapolate to H in Kr, a case for which no direct measurement exists. Considering the theory of effect, in particular Eq. (15) of Herman,[15] one would tentatively conclude that the g_j shift in our case should be negligible.

Like our previous work on muonium,[6] this experiment was made possible through the continued generosity of many individuals and companies. We are particularly indebted to Dr. E. L. Ginzton (Varian Associates) for a 4K3SK klystron and two precision oscillators, and to Mr. J. Duncan (Owens-Illinois) without whose enthusiastic cooperation our CerVit cavity could not have been obtained. We are grateful to M. Neumann, R. Norton, and T. A. Nunamaker of this Institute for their unfailing technical assistance, and wish to thank Dr. H. G. Robinson (Duke), Dr. E. Ensberg (Yale), and Dr. R. M. Herman (Pennsylvania State) for several illuminating discussions.

Note added in proof.—Since this Letter was submitted, several relevant new facts have become known, viz.:

(a) The work of Ref. 3 has been published.[16] Its result, $f_\mu/f_p = 3.183\,347(9)$, agrees with (5′) to 2σ and yields with our $\Delta\nu$ essentially the "recommended" α.

(b) Fulton, Owen, and Repko[17] have computed a new QED correction term to $\Delta\nu(\mu e)$ which raises this frequency by 5.4 ppm. This entails an increase of α^{-1} by 2.7 ppm for given values of μ_μ and $\Delta\nu$!

(c) Using semiempirical extrapolations, Dr. T. Herman has now suggested that $g_j(M)$ should suffer a pressure shift of −11 ppm under our experimental conditions. Assuming this g_j shift and using the corrected $\Delta\nu$ formula of (b), we find $f_\mu/f_\mu = 3.183\,37(13)$ in perfect agreement with (a), and $\alpha^{-1} - 137 = 0.0317(30)$, where the errors contain no allowance for uncertainties in the g_j shift.

*Research supported in part by National Science Foundation Grant No. Gp6135.

†National Science Foundation trainee, 1969–1970.

‡Visitor from Centre à l'Energie Atomique, Saclay, France.

§On sabbatical leave from the University of California at San Diego, La Jolla, Calif. 92037.

[1]V. W. Hughes, Annu. Rev. Nucl. Sci. 16, 445 (1966).

[2]D. P. Hutchinson, F. L. Larsen, N. C. Schoen, D. I. Sober, and A. S. Kanofsky, Phys. Rev. Lett. 24, 1254 (1970).

[3]More accurate precision measurements of f_μ are discussed by J. F. Hague *et al.*, Bull. Amer. Phys. Soc. 15, 608 (1970), but detailed results are as yet unpublished. (But see *Note added in proof.*)

[4]M. Ruderman, Phys. Rev. Lett. 17, 794 (1966).

[5]There is considerable conceptual similarity between this work and the g_e/g_p determination of T. Myint *et al.*, Phys. Rev. Lett. 17, 405 (1966). Our notation (M) for *bound*-state g factors is analogous to their (H).

[6]R. D. Ehrlich, H. Hofer, A. Magnon, D. Y. Stowell, R. A. Swanson, and V. L. Telegdi, Phys. Rev. Lett. 23, 513 (1969).

[7]P. A. Thompson, T. T. Amato, P. Crane, V. W. Hughes, R. M. Mobley, G. zu Putlitz, and J. E. Rothberg, Phys. Rev. Lett. 22, 163 (1969).

[8]Kr densities are expressed in terms of the pressure at 0°C, using the virial data of J. A. Beattie, J. S. Brierley, and R. J. Bariault, J. Chem. Phys. 20, 1615 (1952).

[9]E. S. Ensberg, and C. L. Morgan, Phys. Lett. 28A, 106 (1968).

[10]W. H. Parker, B. N. Taylor, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1959).

[11]S. D. Drell and J. D. Sullivan, Phys. Rev. 154, 1477 (1967).

[12]S. B. Crampton, D. Kleppner, and N. Ramsey, Phys. Rev. Lett. 11, 338 (1963).

[13]This possible effect was first mentioned to us by Dr. C. Bouchiat.

[14]G. S. Heyne and H. G. Robinson, private communication.

[15]R. M. Herman, Phys. Rev. 136A, 1576 (1964).

[16]J. F. Hague *et al.*, Phys. Rev. Lett. 25, 628 (1970).

[17]T. Fulton, D. A. Owen, and W. W. Repko, private communication.

Decisive Tests of High-Energy Models[*]

Edmond L. Berger and Geoffrey Fox[†]

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 21 September 1970)

Regge-pole and strong-absorption (diffractive) models are based upon fundamentally different physical postulates. Although both reproduce available $d\sigma/dt$ and polarization data, profound differences are evident in basic amplitude structure. We show that measurements of the spin-rotation parameters R and A, for any high-energy exchange process, will determine structure of amplitudes and thus provide unambiguous tests of essential assumptions in the models. We examine explicitly the experimentally feasible reactions $\pi N \to K(\Lambda,\Sigma)$, $\bar{K}N \to \pi(\Lambda,\Sigma)$, and $\gamma p \to K(\Lambda,\Sigma)$, as well as multiparticle processes.

In this Letter, we stress the importance of new measurements designed to determine individual helicity amplitudes. We propose a set of experiments which will decisively test models of strong interactions. These experiments are both feasible and directly interpretable.

There are profound differences between current models[1] which are not merely the result of alternative parametrizations. For example, dip-bump phenomena in $d\sigma/dt$ are interpreted in completely different ways in traditional Regge-pole theory and in the strong-cut Regge-absorption model (SCRAM).[2,3] Although both models reproduce available data on differential cross sections and polarization (P), the basic structure of helicity amplitudes differs greatly. More precise data on $d\sigma/dt$ and P will serve primarily to determine better the parameters within models. However, progress in testing underlying assumptions and in distinguishing among models demands a more complete set of experiments. In meson-nucleon scattering, measurements of the spin-rotation parameters R and A are essential.[4] It is also important to realize that structure in the R and A distributions, themselves, are unambiguously characteristic of the models. Indeed, by a simple glance at the experimental distributions for associated production, without detailed fits, one will be able to determine immediately whether assumptions underlying SCRAM are correct. As far as we are aware, this has not been pointed out before. It certainly gives great impetus to work with polarized particles.

In this note, we concentrate on reactions with hyperons in the final state; examples are $\bar{K}p \to \pi\Sigma$, $\gamma p \to K\Lambda$, $\pi p \to K\pi\Sigma$. If these are done with a polarized target (and/or with polarized photons), the weak decay of the hyperon can be used to determine the crucial spin-rotation parameters. In paragraphs which follow, we first summarize distinctive features of models which should be tested; secondly, we discuss relevant experimental observables for meson-baryon scattering; and finally, we treat photoproduction and multiparticle processes.

Models[1] are characterized by quite different positions and interpretation of zeros of amplitudes. The zeros of Regge-pole theory are the wrong-signature nonsense zeros. In the exchange degeneracy form of the Regge-pole theory, these occur when the signature factor $(1 \pm e^{-i\pi\alpha})$ vanishes—i.e., at values of t such that the trajectory function $\alpha(t) = 0, -2, \cdots$ (odd signature), or $-1, -3, \cdots$ (even signature). We emphasize that (i) the location of a given zero is the same in all spin amplitudes; (ii) the location is determined by the trajectory function and signature of the exchanged particle, and thus it will occur at different values of t for processes dominated by different trajectories; (iii) the real part of the amplitude has double zeros at $1 \pm \cos\pi\alpha = 0$, whereas the imaginary part has single zeros at $\sin\pi\alpha = 0$. By contrast, (i) the zeros of SCRAM[2]