As is discussed in detail by Yosida and Watabe[5] and also seen in the Elliott and Wedgwood formalism, the splitting of the band does not imply the stability of a spiral structure immediately. Instead, the ferromagnetic spin-wave spectrum $J(0)-J(\vec{q})$ normally remains positive, demonstrating the ferromagnetic stability. If, however, the Fermi level intersects the lower part of the upper band, a Kohn anomaly hump[6] appears in the $J(0)-J(\vec{q})$ curve, yielding negative excitation energies and the ferromagnetic instability. Then the spiral structure, which has the maximum $J(\vec{q})$ at $\vec{q}=\vec{Q}$, will appear. According to Elliott and Wedgwood, as $\mathcal{J}(E)S$ increases the value of Q decreases but stays finite until Q becomes equal to Q_c (>0). Beyond Q_c, the $J(0)-J(\vec{q})$ curve is no longer negative and the spiral structure disappears abruptly, exhibiting the first-order transition to the ferromagnetic state. This discontinuous change in Q is a consequence of the analytical property of the $J(0)-J(\vec{q})$ curve, which contains two distinct minima, one at $\vec{Q}$ and the other at $\vec{Q}=0$ under the s-d interaction. According to the present theory as soon as the Fermi level intersects the upper band, $\mathcal{J}(E)$ involved becomes inversely proportional to $\ln T$ and increases as T decreases. Thus the Elliott and Wedgwood results, together with our T-dependent $\mathcal{J}(E)$, explain the temperature behavior of the pitch parameter $\vec{Q}$ and reproduce the observed ferromagnetic transition temperatures for Tb, Dy, and Er, ranging from 220 to 20°K, by the use of a single parameter, suggesting that the present T dependence is quantitatively correct. At higher temperatures where a spiral structure is observed, the upper band should intersect the Fermi level yielding a Kohn anomaly hump in the $J(\vec{q})$ around $\vec{q}=\vec{Q}$; but at lower temperatures, where the crystal is ferromagnetic, the hump is suppressed and disappears because $\mathcal{J}(E)$ increases. This behavior of $J(\vec{q})$ is also parallel to the observed $J(0)-J(\vec{q})$ deduced from the magnon dispersion curve by Møller, Houmann, and Mackintosh.[2] Elliott and Wedgwood have introduced the temperature dependence of $\mathcal{J}S$ through the magnetization $M(T)=\langle S\rangle$ alone, but its contribution is negligible around the ferromagnetic transition temperature T_c, thus failing to predict the observed change in $J(0)-J(\vec{q})$, since $M(T)$ has nearly reached its saturation value S.

Finally we note that the present theory predicts the ferromagnetic state at lower temperatures. The spin-up and spin-down bands will then split, suppressing the spin-flip interaction as well as the Kondo phenomena in these ordered systems.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]J. Kondo, Progr. Theor. Phys. 32, 37 (1964); also see J. Kondo, in *Solid State Physics*, edited by F. Seitz, D. Turnbull, and H. Ehrenreich (Academic, New York, 1969), Vol. 23, pp. 183-281.

[2]H. B. Møller, J. C. G. Houmann, and A. R. Mackintosh, Phys. Rev. Lett. 19, 312 (1967).

[3]R. J. Elliott and F. A. Wedgwood, Proc. Phys. Soc., London 81, 846 (1963), and 84, 63 (1964).

[4]Y. Nagaoka, Phys. Rev. 138, A1112 (1965).

[5]K. Yosida and A. Watabe, Progr. Theor. Phys. 28, 361 (1962).

[6]W. Kohn, Phys. Rev. Lett. 2, 383 (1959).

Direct Radiative Capture of ^{3}He by Tritons and t + ^{3}He Cluster States in ^{6}Li

A. M. Young, S. L. Blatt, and R. G. Seyler
Department of Physics, The Ohio State University, Columbus, Ohio 43210*
(Received 10 October 1970)

A direct-capture model is used to calculate the cross section for the capture of ^{3}He by tritons. Comparisons with data to 20 MeV show that the energy dependence of the ground-state capture cross section is well accounted for and that there exists substantial $t+{}^3$He clustering in the first three ^{6}Li states.

Considerable attention has been devoted to the description of low-lying states of the ^{6}Li nucleus using a two-body cluster model.[1-3] Such descriptions have favored $\alpha+d$ cluster configurations with $t+{}^3$He clustering assigned a nonexistent or, at best, a minor role. The neglect of $t+{}^3$He clustering in the early models may have been partly due to a lack of experiments sensitive to the $t+{}^3$He channel from which quantitative estimates could be made regarding the relative importance of $t+{}^3$He and $\alpha+d$ clustering in the ^{6}Li nucleus. More recently, significant experimen-

tal evidence for $t+{}^3\mathrm{He}$ clustering in the low-lying ${}^6\mathrm{Li}$ states has been obtained from investigations on the reactions $\mathrm{T}({}^3\mathrm{He},\gamma){}^6\mathrm{Li}$, ${}^6\mathrm{Li}(\gamma,t){}^3\mathrm{He}$, ${}^{11}\mathrm{B}({}^3\mathrm{He},{}^6\mathrm{Li}){}^8\mathrm{Be}$, ${}^7\mathrm{Li}({}^3\mathrm{He},\alpha){}^6\mathrm{Li}$, and ${}^6\mathrm{Li}(p,p{}^3\mathrm{He})\mathrm{T}$, as discussed in recent publications.[4-8]

In the present work, attention is focused on the reaction $\mathrm{T}({}^3\mathrm{He},\gamma){}^6\mathrm{Li}$ and its inverse. As discussed in Ref. 4, the experimental results on these reactions are controversial. There is no agreement among the sets of cross-section data for ${}^6\mathrm{Li}(\gamma,t){}^3\mathrm{He}$; however, the existing capture measurements[4,9-11] are reasonably consistent with one another. As pointed out by Blatt et al.,[4] the reaction $\mathrm{T}({}^3\mathrm{He},\gamma){}^6\mathrm{Li}$ may proceed through a direct-capture mechanism: The angular distributions for transitions to the ground and first two excited states are satisfactorily accounted for by direct capture and the excitation curves for these transitions vary smoothly with energy with an apparent lack of resonant behavior. Furthermore, theoretical attempts[12] to describe this reaction from the standpoint of compound nuclear states do not account satisfactorily for the experimental observations. In the present work, results of a direct-capture calculation are presented in an attempt to clarify the mechanism through which $\mathrm{T}({}^3\mathrm{He},\gamma){}^6\mathrm{Li}$ proceeds as well as to obtain quantitative estimates on the importance of $t+{}^3\mathrm{He}$ clustering in the low-lying ${}^6\mathrm{Li}$ states.

The direct-capture cross section is calculated in first-order perturbation theory by considering matrix elements of the electromagnetic-multipole operators between initial scattering states and final bound states of the incident and target nuclei. In the notation of Tombrello and Parker and of Bailey, Griffiths, and Donnelly,[13] the differential cross section for direct capture is given by

$$\frac{d\sigma}{d\Omega}=\frac{K}{hV_i(2J_{^3\mathrm{He}}+1)(2J_{\mathrm{T}}+1)}\sum_{\substack{m_i m_f\\ P=\pm1}}|\langle f,m_f|H_{\mathrm{int}}{}^{P}|i,m_i\rangle|^2. \tag{1}$$

For emission of $E1$ radiation, the interaction Hamiltonian is

$$H_{\mathrm{int}}{}^{P}=-i\left(\frac{4\pi}{3}\right)^{1/2}Ke\mu\left(\frac{Z_{^3\mathrm{He}}}{M_{^3\mathrm{He}}}-\frac{Z_{\mathrm{T}}}{M_{\mathrm{T}}}\right)\sum_M D_{MP}{}^{1\dagger}(\varphi_\gamma,\theta_\gamma,0)\mathcal{O}_{E1}(r)Y_1{}^{M*}(\theta,\varphi), \tag{2}$$

where $\mathcal{O}_{E1}$ is the radial part of the electric-dipole operator, which is approximated by

$$\mathcal{O}_{E1}=(3/2\rho^3)[\rho\cos\rho-(1-\rho^2)\sin\rho]r. \tag{3}$$

The initial- and final-state wave functions are written as

$$|i,m_i\rangle=\sum_{l=0}^{\infty}[4\pi(2l+1)]^{1/2}i^l\exp[i(\omega_l+\delta_{lS})](R_{lS_i}/kr)Y_l{}^0\chi_S{}^0, \tag{4}$$

and

$$|f,m_f\rangle=\theta_f[U_{L_f}{}^{(f)}/r]\sum_{m_L,m_S}\langle L_fS_fm_L,m_S|J_fm_f\rangle Y_{L_f}{}^{m_L}\chi_{S_f}{}^{m_S}. \tag{5}$$

Here, θ_f is the fractional parentage coefficient of the $t+{}^3\mathrm{He}$ cluster in the total wave function for state f, ω_l is the usual Coulomb phase, and δ_{lS} is the $t+{}^3\mathrm{He}$ scattering phase shift for the partial wave of orbital angular momentum l and channel spin S.

The radial wave functions for both the initial scattering states R_{lS} and final bound states U_{L_f} were generated using a computer code written by Thompson, who, with Tang,[14] was successful in using resonating-group theory to describe the scattering of tritons by ${}^3\mathrm{He}$ over a wide energy range. The calculations were performed on the Ohio State University 360/75 computer. The function R_{lS} was normalized to behave asymptotically as the Coulomb wave function $F_l\cos\delta_{lS}+G_l\sin\delta_{lS}$ for each energy at which the capture cross sections were calculated.

The lowest states of ${}^6\mathrm{Li}$ can be described in the L-S coupling scheme as ${}^{13}S_1$, ${}^{13}D_3$, and ${}^{31}S_0$, for the ground and first two excited states, respectively. Assuming the ground state to be a pure S state, only the ${}^{33}P$ partial wave can contribute to an $E1$ transition to that state thereby yielding an energy-independent angular dependence of $\sin^2\theta$ for that transition. Contributions from incident ${}^{33}F$ partial waves to $E1$ transitions to the first excited state have been neglected. However, there is evidence[11] that at higher bombarding energies the F waves may indeed contribute to that transition. For each of the three bound states the cor-

responding final-state wave functions $U_{L_f}^{(f)}$ were obtained by searching for stationary solutions of the resonating-group problem and requiring that $U_{L_f}^{(f)}$ asymptotically behave as the Whittaker function $W_{\eta,l}(\rho)$, and they were normalized so that

$$\int_0^\infty U_{L_f}^{(f)2} dr = 1.$$

These wave functions would appear to be appropriate for the $t + {}^3\mathrm{He}$ components of the experimentally observed final states. (Where the calculated bound-state energies differ from the experimental energies, the experimental values were used in the direct capture computation.) The radial wave functions R_{lS} and $U_{L_f}^{(f)}$ so obtained were used to compute, as a function of bombarding energy, the capture cross sections for transitions γ_0, γ_1, and γ_2 to the ground state and first two excited states of ^{6}Li, respectively.

The calculated total cross section for γ_0 is shown in Fig. 1, along with the Ohio State/Stony Brook[4] capture data. The calculated cross section agrees quite well with experiment. The normalization of the theoretical total cross section to that measured experimentally determines the value of the fractional parentage coefficient (spectroscopic factor or reduced width) θ_0^2 for the ^{6}Li ground state. It should be emphasized that θ_0^2 is the only adjustable parameter in the capture calculation and that the wave functions employed are entirely consistent with results of $t + {}^3\mathrm{He}$ scattering as determined by Thompson and Tang.[14] A least-squares fit of the theoretical cross section to the experimental points yielded a value of 0.69 for θ_0^2.

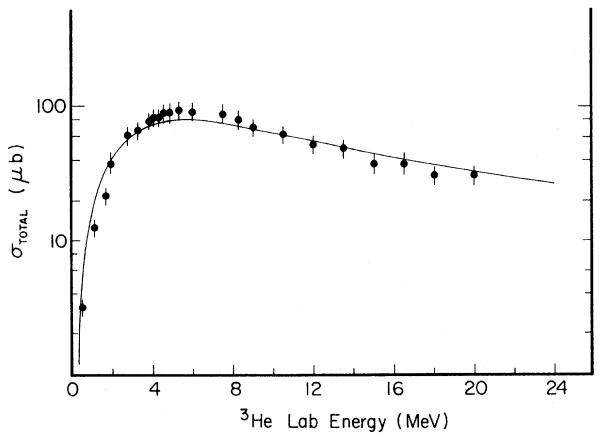

FIG. 1. Results of the direct-capture calculation for the energy dependence of the radiation to the ground state of ^{6}Li (solid curve). The theoretical cross section is normalized to the data of Ref. 4, yielding a value $\theta_0^2 = 0.69$.

The value of θ_0^2 reflects the probability that the ground state of ^{6}Li has the $t + {}^3\mathrm{He}$ cluster configuration (or parentage). However, caution must be exercised in making this interpretation of the magnitude of θ_0^2 since other cluster substructures such as $\alpha + d$ are also present in the ^{6}Li ground state and their wave functions are not necessarily orthogonal to that of the $t + {}^3\mathrm{He}$ component. This nonorthogonality among the wave functions for the various cluster substructures arises from nuclear exchange forces and from antisymmetrization exchange effects. Since we do not take these other channels into account, the magnitude we obtain for θ_0^2 should probably be considered as an upper limit to the $t + {}^3\mathrm{He}$ strength in the ^{6}Li ground state. However, such a large value of θ_0^2 obtained in a single-channel calculation does indicate appreciable clustering for this component; one would not expect reasonable agreement between this theory and experiment if the ground state of ^{6}Li were predominantly of some other cluster configuration such as $\alpha + d$.

Capture cross sections for transitions to the first and second excited states were also computed and compared with that obtained for the ground-state transition. We define a gamma ray energy-independent "reduced capture-cross-section ratio" $[\gamma_n/\gamma_m] = (\sigma_{\gamma_n}/E_{\gamma_m}{}^3)(\sigma_{\gamma_m}/E_{\gamma_m}{}^3)^{-1}$, and find that

$$\left[\frac{\gamma_1}{\gamma_0}\right] = \frac{14}{15}\left(\frac{\theta_1}{\theta_0}\right)^2 \left\{\frac{\int_0^\infty U_2{}^{(1)} \Theta_{E1} R_{11} dr}{\int_0^\infty U_0{}^{(0)} \Theta_{E1} R_{11} dr}\right\}^2, \tag{6}$$

$$\left[\frac{\gamma_2}{\gamma_0}\right] = \frac{1}{3}\left(\frac{\theta_2}{\theta_0}\right)^2 \left\{\frac{\int_0^\infty U_0{}^{(2)} \Theta_{E1} R_{10} dr}{\int_0^\infty U_0{}^{(0)} \Theta_{E1} R_{11} dr}\right\}^2. \tag{7}$$

The calculated reduced capture-cross-section ratios were adjusted to the experimental points by a least-squares fit (see Fig. 2), determining the values for θ_1^2 and θ_2^2 to be, respectively, 0.55 and 0.48, when θ_0^2 is taken to be 0.69, as determined above.

Although values of θ^2 are not obtained by Young, Forsyth, and Marion[6] from investigating ^{11}B(^{3}He, ^{6}Li)^{8}Be, appreciable evidence for $t + {}^3\mathrm{He}$ clustering was seen for the ground and first excited states of ^{6}Li with relatively little for the second excited state. Studying ^{7}Li(^{3}He, α)^{6}Li, Orihara et al.[7] find significant $t + {}^3\mathrm{He}$ clustering in the ground state and second excited state of ^{6}Li. Their extracted reduced widths are expressed as a product of the reduced width for a ^{3}He cluster to be in a given ^{6}Li state and the reduced width for an α cluster to be in the ^{7}Li ground state, and therefore cannot be directly compared

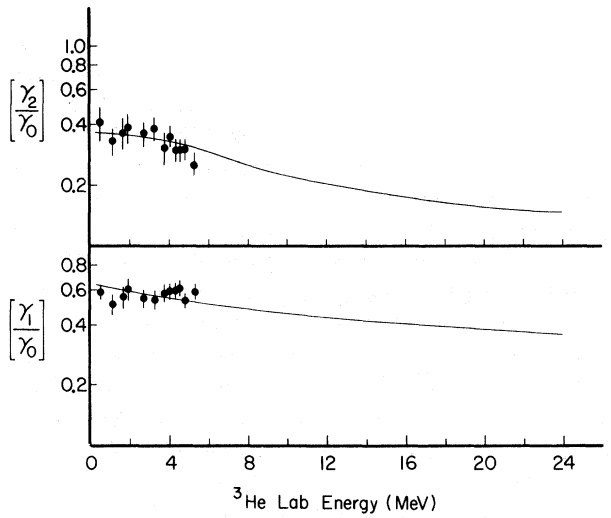

FIG. 2. The "reduced capture-cross-section ratio" (defined in the text) for γ_1/γ_0 and γ_2/γ_0, as calculated (solid curves). Comparisons are with data of Ref. 4.

with our results. Orihara et al. conclude that the amount of $t+{}^3$He clustering is about equal for the ground and second excited states. Kurdyumov, Neudatchin, and Smirnov[15] consider both the $\alpha+d$ and $t+{}^3$He channels in their analysis of the data of Bachelier et al.[8] on the reaction ^{6}Li$(p,p\,{}^3$He$)$T, and obtain a value of about 0.5 for $\theta_0{}^2$ which agrees quite well with the present results. Recently, Clement and Wittern[16] have also analyzed ^{6}Li$(\gamma,t)^3$He as a direct reaction; a plane wave was employed for the $t+{}^3$He scattering state and a less realistic bound-state wave function was assumed. Their results have roughly the same shape but only half the magnitude of our results.

The success of the present calculation indicates that the amount of $t+{}^3$He clustering in the ground as well as the first and second excited states is more substantial than has previously been appreciated. The large widths for $t+{}^3$He clustering exhibited in the various ^{6}Li states considered in the present work and the significant evidence, from other reactions,[1-3] for $\alpha+d$ clustering in these same states points out the desirability of a coupled-channel calculation to generate more realistic wave functions to be employed in the direct-capture calculation. By performing this type of calculation better quantitative estimates of the relative importance of $t+{}^3$He and α-d clustering in ^{6}Li might be obtained.

The authors wish to thank Dr. D. R. Thompson for kindly providing the $t+{}^3$He resonating-group program used to generate radial wave functions for the present calculation.

---

*Work supported in part by the National Science Foundation.

[1]G. C. Phillips and T. A. Tombrello, Nucl. Phys. 19, 555 (1960).

[2]V. G. Neudatchin and Yu. F. Smirnov, in *Progress in Nuclear Physics*, edited by D. M. Brink and J. H. Mulvey (Pergamon, New York, 1969), Vol. 10, p. 273 ff.

[3]K. Wildermuth and W. McClure, in *Springer Tracts in Modern Physics, Ergebnisse der exakten Naturwissenschaften*, edited by G. Höhler (Springer, Berlin, 1966), Vol. 41.

[4]T$({}^3$He$,\gamma)^6$Li: S. L. Blatt, A. M. Young, S. C. Ling, K. J. Moon, and C. D. Porterfield, Phys. Rev. 176, 1147 (1968); A. M. Young, S. L. Blatt, J. F. Amann, and E. M. Diener, Bull. Amer. Phys. Soc. 15, 125 (1970).

[5]^{6}Li$(\gamma,t)^3$He: N. K. Sherman, J. E. E. Baglin, and R. O. Owens, Phys. Rev. 169, 771 (1968).

[6]^{11}B$({}^3$He$,{}^6$Li$)^8$Be: F. C. Young, P. D. Forsyth, and J. B. Marion, Nucl. Phys. A191, 209 (1967).

[7]^{7}Li$({}^3$He$,\alpha)^6$Li: H. Orihara, M. Baba, M. Akyama, S. Iwasaki, T. Nakagana, H. Veno, and M. Watanabe, J. Phys. Soc. Jap. 29, 533 (1970).

[8]^{6}Li$(p,p\,{}^3$He$)$T: D. Bachelier, M. Bernas, C. Detraz, P. Radvanyi, and M. Roy, Phys. Lett. 26B, 283 (1968).

[9]D. Kohler and S. M. Austin, Bull. Amer. Phys. Soc. 8, 290 (1963).

[10]F. Nusslin, H. Werner, and J. Zimmerer, Z. Naturforsch. 21a, 1195 (1966).

[11]E. Ventura, C. C. Chang, and W. E. Meyerhof, private communication.

[12]I. V. Kurdyumov, Yu. F. Smirnov, K. V. Shitikova, and S. Kh. El Samarai, Phys. Lett. 31B, 163 (1970).

[13]T. A. Tombrello and P. D. Parker, Phys. Rev. 131, 2582 (1963); G. M. Bailey, G. M. Griffiths, and T. W. Donnelly, Nucl. Phys. A94, 502 (1967).

[14]D. R. Thompson and Y. C. Tang, Nucl. Phys. A106, 591 (1968), and Phys. Rev. Lett. 19, 87 (1967).

[15]I. V. Kurdyumov, V. G. Neudatchin, and Yu. F. Smirnov, Phys. Lett. 31B, 426 (1970).

[16]D. Clement and H. Wittern, in *Proceedings of the International Conference on Clustering Phenomena in Nuclei, Bochum, Federal Republic of Germany, 1969* (International Atomic Energy Agency, Vienna, Austria, 1969).