$2\kappa T$) rather than the triangular lattice value quoted in the text.

[18]In the correlated regime, it is expected that the transition between the high- and low-temperature regime of the square lattice Hall mobility (cf. Ref. 17) will occur at $\kappa T \approx \epsilon_5 - \epsilon_4 \approx \epsilon_2/10$. In the high-temperature correlated regime, $E_{\rm Hall} \approx (\epsilon_5-\epsilon_2)-(\epsilon_3-\epsilon_2) \approx \epsilon_2/4$ while $E_{\rm drift} \approx \epsilon_3 - \epsilon_2 \approx \epsilon_2/3$, thereby yielding an even smaller exponential factor $[\exp(\epsilon_2/12\kappa T)]$ than the estimate for a triangular lattice given in the text.

Electron Relaxation Rates in Bismuth at Microwave and Far-Infrared Frequencies

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Temperature- and frequency-dependent relaxation rates in bismuth have been measured by studies of the magnetic-field-dependent reflectivity of microwave and far-infrared radiation in single-crystal samples. Retardation effects have been taken into account in the analysis of the cyclotron resonance line shapes. The temperature and frequency dependence of the relaxation rates are analyzed in terms of the theory of electron-electron scattering and the experiment provides strong support for electron-electron dominated scattering in bismuth. The experimental results are inconsistent with an electron-phonon dominated scattering mechanism.

There has been considerable interest recently in the frequency and temperature dependence of electron effective masses (m^*) and relaxation times (τ) in metals. These effects are expected from the electron-phonon interaction and electron-electron scattering, and their investigation can lead to a better understanding of electron interactions in metals. The frequency and temperature dependence of m^* and τ are determined by the line-shape analysis of measurements such as cyclotron resonance, and it is important to note that accuracy of the line-shape theory is crucial to these experiments.

The first measurements of frequency dependent relaxation times have been reported recently by Goy and Weisbuch[1] for lead and Edel'man and Cheremisin[2] for bismuth. In this Letter new measurements are reported of relaxation rates in bismuth over a larger frequency and temperature range than previously investigated. Moreover, in the analysis of cyclotron-resonance measurements, important corrections to the line-shape theory due to retardation effects have been considered. The relaxation rates obtained here, which are not in agreement with the measurements of Edel'man and Cheremisin, are interpreted in terms of electron-electron scattering in bismuth.

The bismuth samples were single-crystal disks (20-mm diam and 2-mm thickness) grown in graphite boats in vacuum from 99.9999% pure starting material, seeded so that the trigonal axis was normal to the disk faces, then chemically lapped and polished. X-ray diffraction measurements were made to check the crystal orientation and the crystal quality.

At microwave frequencies of 36, 28, and 8 GHz the magnetic-field-dependent surface resistance R_s arising from Azbel'-Kaner cyclotron resonance was studied by use of a microwave reflection spectrometer. With the magnetic field H parallel to the binary axis of the bismuth crystals the cyclotron resonance series corresponding to the mass $m^* = 0.0093 m_0$ was observed. To obtain information on electron relaxation rates from these data it was first necessary to take retardation effects[3,4] into account in the cyclotron-resonance line-shape theory. The Azbel'-Kaner theory only applies if the time dependence of the rf field can be ignored while the electrons are passing through the rf skin layer. For circular orbits this leads to the condition $\omega^2\delta/\omega_c V_{\rm F} \ll 1$ for the applicability of Azbel'-Kaner theory. Here δ is the rf skin depth, $V_{\rm F}$ is the Fermi velocity, ω_c is the cyclotron frequency, and ω the rf frequency. For $\omega_c\tau < 10$ the surface impedance $Z(H)$ incorporating retardation effects can be written approximately as[5]

$$Z(H) - Z(0) \simeq \tfrac{1}{3} Z(0) \exp\left[-2\pi\left(i\frac{\omega}{\omega_c} + \frac{1}{\omega_c\tau}\right)\right] \times \exp\left(\frac{\tilde{\omega}^2\delta}{\omega_c V_{\rm F}}\right), \qquad (1)$$

where $\tilde{\omega} = \omega - i/\tau$. The retardation factor in Eq. (1) with complex δ ($\delta = \delta_1 - i\delta_2$) produces line-shape effects indistinguishable from frequency-dependent m^* and $1/\tau$. Such retardation effects were not considered in previous measurements of frequency-dependent masses and relaxation times.

A method commonly used to determine relaxation rates from cyclotron resonance is based on the exponential decrease of the amplitude of the subharmonic resonances.[6] The amplitude of dR_s/dH for the nth subharmonic resonance including retardation effects can be written as

$$dR_s/dH \propto n^2 \exp(-2\pi n/\omega\tau_a), \tag{2}$$

where we define an apparent relaxation rate τ_a^{-1} as

$$\tau_a^{-1} = \tau^{-1}\left(1 - \frac{\omega\delta_2}{\pi V_F}\right) + \frac{\omega^2\delta_1}{2\pi V_F}. \tag{3}$$

For bismuth at 36 GHz, $\omega\delta_2/\pi V_F \simeq 0.04$ and $\omega^2\delta_1/2\pi V_F \simeq 8\times10^9$ sec^{-1} whereas τ^{-1} for our samples was of order 10^9 sec^{-1}. Thus, the retardation effect dominates the amplitude decay of cyclotron subharmonics at this frequency. Since $\delta \propto \omega^{-1/3}$ the retardation effect produces an $\omega^{5/3}$ frequency dependent term in the apparent relaxation rate. For bismuth, therefore, the dominating presence of retardation precludes a direct measurement of the frequency dependence of τ^{-1} by the amplitude method. Only the value of τ^{-1} at zero frequency can be directly determined, and this is obtained by measuring τ_a^{-1} as a function of frequency and extrapolating τ_a^{-1} to zero frequency on a $\omega^{5/3}$ plot. In this way we have determined $\tau^{-1}(\omega, T)$ at $\omega = 0$ over the temperature range 2-10°K and found that τ^{-1} satisfies the relation $\tau^{-1}(0, T) = \tau_0^{-1} + BT^2$, where $\tau_0^{-1} = (0.32 \pm 0.06) \times 10^{10}$ sec^{-1} and $B = (2.02 \pm 0.07)\times 10^8$ sec^{-1} °K^{-2}.

Information on the frequency dependence of τ^{-1} in bismuth was found from magnetoreflectance measurements at 891 GHz. These measurements were made with a highly stable (3 parts in 10^4 short-term power noise) cw-mode HCN laser source tuned to the 337-μm line. In this frequency range magnetoplasma effects produce large magnetic-field-dependent changes in the reflectivity of bismuth.[7] With the applied magnetic field parallel to the bisectrix axis of the samples the reflectivity at near normal incidence was measured. In this geometry there is a dielectric anomaly at an applied field such that $\omega_c \simeq \omega/\sqrt{2}$ (where $\omega_c = eH/m^*c$ for $m^*/m_0 = 0.0081$). In a field interval inversely proportional to $\omega\tau$ at the dielectric anomaly the reflectivity changes by typically 50%. The relaxation time was determined by comparison of the observed line shape with line shapes calculated from the local magnetoconductivity tensor of bismuth[7] with $\omega\tau$ as the only adjustable parameter. Experimental and theoretical line shapes are compared in Fig. 1. The results of the temperature dependence of τ^{-1}, shown in Fig. 2, can be written as $\tau^{-1}(T=0) = (1.97 \pm 0.07)\times10^{10}$ sec^{-1} and $\tau^{-1}(T) - \tau^{-1}(T=0)$

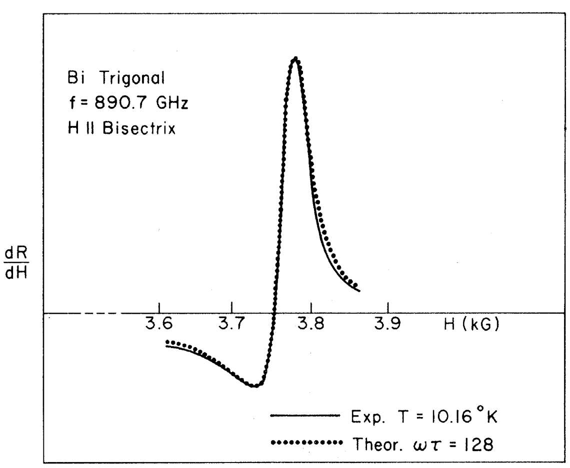

FIG. 1. Derivative of the reflectivity (R) of bismuth as a function of applied magnetic field. The far-infrared radiation is incident nearly normal and the H field is parallel to the sample surface. The theoretical curve is based on a local model with $\omega\tau$ the only adjustable parameter.

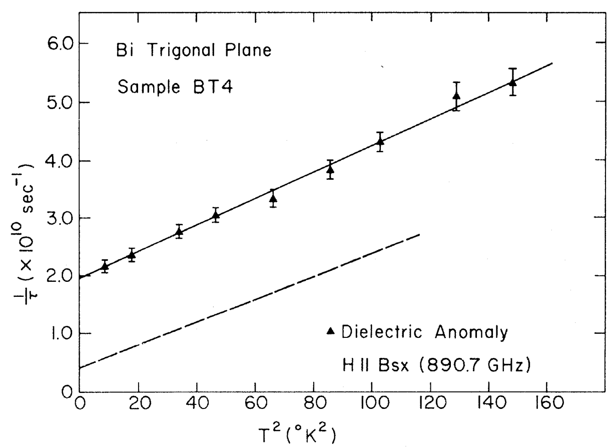

FIG. 2. Temperature dependence of the electron relaxation rate τ^{-1} as measured by line-shape analysis of the dielectric anomaly. The data have been corrected for magnetic field inhomogeneity; nonlocal corrections have not been included. The dashed line represents the microwave results for τ^{-1} corrected for retardation effects as described in the text.

$=[(2.30\pm 0.08)\times 10^8\ \mathrm{sec}^{-1}\ {}^\circ\mathrm{K}^{-2}]T^2$. Throughout this work temperatures were measured by means of a calibrated carbon resistor mounted on the back of the samples.

To evaluate properly the significance of the high-frequency measurements of τ^{-1} it is first necessary to consider effects other than electron scattering that could broaden the dielectric anomaly. Inhomogeneity of the magnetic field was taken into account and the estimated correction to τ^{-1} was ~10% at the greatest. Nonlocal effects on the line shape have also been considered. Our use of a local conductivity is justified in the limit $(\omega\delta/V_F)^2 \gg 1$. In zero magnetic field $(\omega\delta/V_F)^2 \sim 36$ for bismuth at 891 GHz. At the dielectric anomaly, however, the skin depth increases by approximately a factor of $(\omega\tau)^{1/2}$ and the locality is improved.[8] Thus nonlocal corrections become more important at higher temperatures where $\omega\tau$ is smaller and are thought to account for most of the ~15% increase observed in the T^2 term in the relaxation rate at 891 GHz. If this conjecture holds, a corresponding correction is also expected for the relaxation rate at zero temperature.[8] Other line-broadening effects such as beam spread (~2°) are not considered significant.

We conclude that our measurements of τ^{-1} indicate that the frequency and temperature-dependent relaxation rate in bismuth is approximately of the form

$$\tau^{-1}=\tau_0^{-1}+\tau^{-1}(\omega)+BT^2. \quad (4)$$

Experimentally $\tau^{-1}(\omega)=(1.65\pm 0.13)\times 10^{10}\ \mathrm{sec}^{-1}$ at 891 GHz, where the correction for the magnetic-field inhomogeneity has been considered. All measurements at microwave and far-infrared frequencies were repeated with a bismuth sample of considerably higher impurity concentration ($\tau_0^{-1}\simeq 2\times 10^{10}\ \mathrm{sec}^{-1}$). The results for $\tau^{-1}(\omega)$ agreed within the quoted experimental uncertainty.

Our experimental measurements differ significantly from the results recently reported by Edel'man and Cheremisin. These authors observe a large frequency dependence of the T^2 term in τ^{-1}. In addition the magnitude of their frequency-dependent term $[\tau^{-1}(\omega)]$ is nearly two orders of magnitude greater than our results indicate. There is reason to believe that these discrepancies are attributable to the analysis of their cyclotron resonance data in terms of the Chambers line-shape theory.[9] In particular, we question applicability of Chambers's theory to bismuth since resonant electrons are expected to make a large contribution to the surface impedance. Furthermore, the condition $\delta/R_c \ll 1$, assumed in the Azbel'-Kaner as well as the Chambers theory, becomes less well satisfied at higher frequencies. Thus at 76 GHz (the highest frequency used by Edel'man and Cheremisin), $\delta/R_c \sim \frac{1}{2}$ at the cyclotron fundamental.

A T^2 dependence of the relaxation rates in bismuth has been observed in a wide variety of experiments persisting to temperatures as high as 20°K.[10-12] There is disagreement on the source of this quadratic temperature dependence which has been variously interpreted as arising from electron-electron scattering[11,13] and electron-phonon scattering.[10,12] Because the bismuth Fermi-surface dimensions are a small fraction of a reciprocal lattice vector, the effective Debye temperature for intravalley electron-phonon scattering is only about 10°K. It is therefore difficult to understand a T^2-dependent scattering rate on this basis over the observed 20° temperature range.[12,13] The results of this paper also rule out an interpretation based on phonon-assisted intervalley scattering. Because the phonons involved in this process have frequencies comparable with the laser frequency, significantly different temperature dependencies of the relaxation rates at microwave and far-infrared frequencies are expected.[14]

A T^2 scattering rate is a well-known consequence of electron-electron scattering in a degenerate electron gas. It is therefore of interest to discuss the relaxation rates observed here in terms of electron-electron scattering. The scattering rate of a state k due to this mechanism is given by[15]

$$\tau_k^{-1}=\Lambda_k[(\pi kT)^2+(E_k-E_F)^2], \quad (5)$$

where E_F is the Fermi energy.

The form of this scattering law suggests an ω^2 dependence of the relaxation rate and a relation between the temperature and frequency dependences of τ^{-1}. The anisotropy of the bismuth Fermi surface and the presence of a large magnetic field in the experiment ($\hbar\omega_c/E_F \sim 0.15$) complicates a calculation of the conductivity incorporating electron-electron scattering. Thus, a model calculation has been made for the case of an isotropic Fermi surface at $H=0$ with a scattering rate given by Eq. (5). The conductivity was calculated by standard response theory as presented, for example, by Nam.[16] For this model with $\omega\tau_k \gg 1$ (for $|E_k-E_F|\sim\hbar\omega$) a frequency-dependent relaxation time $\tau(\omega)$ can be defined so that the conductivity $\sigma(\omega)$ can be expressed as

$$\sigma(\omega) = \sigma(0)/[1 + i\omega\tau(\omega)] \quad (6)$$

$1/\tau(\omega)$ is found to be of the form of Eq. (4) with $\tau^{-1}(\omega)$ given approximately by

$$\tau^{-1}(\omega) \simeq \tfrac{1}{4}B(\hbar\omega/\pi k)^2 \sim 0.94\times 10^{10}\ \mathrm{sec}^{-1} \quad (7)$$

at 891 GHz . The observed enhancement of the relaxation rate at 891 GHz (with consideration of nonlocal effects) is about 60% greater than predicted by Eq. (7). The significance of the discrepancy is questionable because of the crudeness of the model calculation. However, such a deviation is of interest as it would suggest a phonon-mediated (BCS) electron-electron coupling in which Λ_k is energy dependent.

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Parallel Pumping in Ferromagnetic Resonance Transmission?*

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A distinctive and strong microwave magnetic transmission resonance mode is reported in ferromagnetic Gd, Fe, and Ni when the static magnetic field is parallel both to the sample foil surface and to the microwave magnetic field. Some evidence exists to show that this may involve sensitive detection of two-magnon processes as in parallel pumping.

Microwave magnetic-resonance transmission in paramagnetic Gd was first observed[1] under the speculation that the phenomenon was caused by spin-wave transmission. Subsequent measurements and interpretation[2-5] revealed that the transmission signal from an excitation cavity to a receiver cavity through a metal sample foil was created by the enhancement of the skin depth caused by resonance of the constituent magnetization of the metal in a manner similar to, but