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g_A and g_V from Mirror Nuclei

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We have analyzed the superallowed β transitions of mirror nuclei and calculated the Gamow-Teller matrix elements in order to determine g_A and g_V. Agreement with the more recent measurements is obtained.

The determination of g_A and g_V (axial vector and vector coupling constants) is of great importance in the weak-interaction theory. The measurement of g_A and/or g_V comes out essentially from experiments on nuclear β decay, μ decay, and μ capture. In spite of the fact that several measurements have already been done in the past years, stability in the results has not yet been achieved. In particular, for a long time, the value -1.18 has been accepted for the ratio g_A/g_V.[1] Some recent measurements[2,3] suggest that the absolute value of this ratio has to be considerably increased. The aim of this Letter is to give further evidence for an increase of the ratio $|g_A/g_V|$ of about 6% with respect to the value 1.18.

We will briefly recall the most classical measurements of g_A/g_V: (a) neutron mean life and ft measurements of the superallowed $0^+ \rightarrow 0^+$ Fermi transitions (Sosnovsky et al.[4] gives the value -1.18, and recently Christensen et al.[2] give -1.23); (b) electron asymmetry from polarzed neutrons (Burgy et al.[5] and Clark et al.[6] give -1.25, Christensen et al. -1.26); (c) electron-neutrino correlations in neutron decay (Vladimirski[7] gives -1.33); (d) total rates in nuclear μ capture (several experiments are in course at CERN[8]; there are no definitive data).

As far as methods (a), (b), and (c) are concerned we refer directly to the quoted references for the discussion of the difficulties, errors, and related problems. We will briefly comment on method (d). In principle, the nuclear μ-capture rates are very sensitive to the ratio $g_A^{(\mu)}/g_V^{(\mu)}$, and assuming the (μ, e) universality we expect $g_A^{(\mu)}/g_V^{(\mu)} = g_A^{(\beta)}/g_V^{(\beta)}$. In general, the nuclear structure complicates the problem. However, in Ref. 9 it has been pointed out that in order to fit the experimental data on total capture rates, taking into account the gross feature of the nuclear structure [SU(4) breaking], a ratio $|g_A/g_V| \simeq 1.28$ is necessary.

A further possible evaluation of $|g_A/g_V|$ may be given in studying the superallowed, mirror β transitions of isodoublets. In the following we analyze this possibility in detail.

For mirror nuclei, assuming T (isospin) as a good quantum number, and in the nonrelativistic approximation, we have

$$(ft)^{-1} = g_V^2 + g_A^2 \frac{J+1}{J} |\langle \textstyle\sum_i \sigma_i \tau_{i\pm} \rangle|^2. \tag{1}$$

Therefore, if we know ft and $\langle \sum \sigma_i \tau_{i\pm} \rangle$, it is simple to determine g_A^2 and g_V^2. The method is not new[10] and it has been used long ago by Trigg[10] and Kofoed-Hansen and Christensen[11] who estimated $|g_A/g_V| = 1.10$ and 1.11, respectively. This value was considered very rough.[2] In fact, the experimental ft values were crudely known, and $\langle \sum \sigma_i \tau_{i\pm} \rangle$ was extracted with an interpolation formula from the magnetic moments.[10] It has also been argued[12] that bound nucleons may have a "renormalized" coupling constant and this may explain the very low value deduced from mirror nuclei. More recently Wallace and Welch,[13] from their ft measurements obtained $|g_A/g_V| \simeq 1.2$. At present the situation allows a more careful analysis. Indeed: (i) Several new-magnetic moments are known[14]; (ii) it is possible to estimate $\langle\sum\sigma_i\tau_{i3}\rangle$ with a good approximation[15]; (iii) some ft values are measured with good accuracy.[16]

The main problem is obviously the calculation of the Gamow-Teller matrix elements. Our starting point is the formulas[15]

$$\langle\textstyle\sum\sigma_i\rangle = 2[(2\mu_0 - J)/(g_p + g_n - 1)], \tag{2}$$

$$(-)^N\langle\textstyle\sum\sigma_i\tau_{i3}\rangle = 2[(2\mu_3 - J)/(g_p - g_n - 1)], \tag{3}$$

where $\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_3$ with $\vec{\mu}_0$ an isoscalar and $\vec{\mu}_3$ an isovector. In the limit in which $\vec{\mu}$ is a one-body operator (2) is exact and (3) is exact to second order in the configuration mixing. The possible two-body exchange terms (isovector) modify the relation (3) in the sense that in this case $\langle\sum\sigma_i\tau_{i3}\rangle$, as defined by (3), contains these exchange effects.[17]

Furthermore, the systematics given in Ref. 15 point out that $\langle\sum\sigma_i(A)\rangle$ and $(-)^N\langle\sum\sigma_i\tau_{i3}(A)\rangle$ (A is the nucleon number and N is the neutron number of the nucleus considered) vary with A in a similar way along a shell. In particular, except for the case $A = 21$, in all cases in which both the magnetic moments of an isodoublet are measured, we have

$$(-)^N\langle\textstyle\sum\sigma_i\tau_{i3}(A)\rangle - \langle\sum\sigma_i(A)\rangle \simeq +0.10\text{-}0.20. \tag{4}$$

We interpret the right-hand side of (4), apart from small isovector corrections, as mainly due to the exchange effects, and consequently, we assume

$$\langle\textstyle\sum\sigma_i\rangle \simeq [(-)^N\langle\sum\sigma_i\tau_{i3}\rangle]_{\text{no exch}}, \tag{5}$$

where the subscript means that exchange effects are not included. With the assumption (5) we deduce the important relations

$$(-)^N\langle\textstyle\sum\sigma_i\tau_{i3}\rangle = 2\mu_n(A)/g_n, \tag{6}$$

$$(-)^N\langle\textstyle\sum\sigma_i\tau_{i3}\rangle = 2[\mu_p(A) - J]/g_p, \tag{7}$$

where $\mu_n(A)$ $[\mu_p(A)]$ is the magnetic moment of the odd-neutron (odd-proton) nucleus of the isodoublet A. We have obviously (6) + (7) = (3).

In order to test the consistency of our hypothesis (5) we may calculate $\langle\sum\sigma_i\tau_{i3}\rangle$ separately from (6) and (7) in all the possible cases ($A = 3$, 11, 13, 15, 17, 19, 21, and 35). The splitting we obtain is very small (of the order of 0.02 or smaller). Furthermore, excluding the case $A = 21$ (with a splitting of 0.04), $\langle\sum\sigma_i\tau_{i3}\rangle$ calculated from Eq. (6) is systematically higher than that calculated from (7).

As far as the case $A = 21$ is concerned, we have from our formulas (2) and (3)

$$(-)^N\langle\textstyle\sum\sigma_i\tau_{i3}\rangle < \langle\sum\sigma_i\rangle$$

and

$$\langle\textstyle\sum\sigma_i\tau_{i3}\rangle_{\text{from (6)}} < \langle\sum\sigma_i\tau_{i3}\rangle_{\text{from (7)}}.$$

This is not surprising. In fact, in this region a first-order configuration mixing of the type described in Ref. 15 is a bad approximation and deformations play an important role. Evidently, $\langle\sum\sigma_i\tau_{i3}\rangle$ is underestimated with our formulas. We assume that the same is true for $A = 23$ because of the similarity of these nuclei. For all the other cases in which only one of the magnetic moments of the isodoublet is measured we extrapolate our systematics [i.e., relations (4), (6), and (7)].

We consider now the Gamow-Teller matrix elements. In a first approximation we have

$$|\langle\textstyle\sum_i\sigma_i\tau_{i3}\rangle| = |\langle\sum_i\sigma_i\tau_{i\pm}\rangle|.$$

The first term, evaluated from relation (3), includes, as observed, the corrections which contribute to the isovector magnetic moment (exchange effects). The right-hand side comes mainly from the axial part of the weak current. The exchange corrections for this matrix element are in general very different from those that contribute to $\langle\sum\sigma_i\tau_{i3}\rangle$.[18] Recently, in the framework of the partial conservation of axial-vector current, these corrections have been calculated[19] and are in general very small and positive, or even negative. So one should really write

$$|\langle\textstyle\sum_i\sigma_i\tau_{i\pm}\rangle| < |\langle\sum_i\sigma_i\tau_{i3}\rangle|.$$

But according to our systematics on magnetic moments we know that $\langle\sum\sigma_i\rangle$ gives us, with a good approximation, the value of $\langle\sum_i\sigma_i\tau_{i3}\rangle$ free from exchange effects.

In order to evaluate $\langle\sum\sigma_i\tau_{i\pm}\rangle$, which is not so modified by the related exchange effects, we use the relation

$$|\langle\textstyle\sum_i\sigma_i\rangle| \lesssim |\langle\sum_i\sigma_i\tau_{i\pm}\rangle| < |\langle\sum\sigma_i\tau_{i3}\rangle|,$$

the only exception being $A = 21$, 23, in which we

Table I. Experimental values of $(ft)^{-1}$ taken from Refs. 1 and 16; spin-isospin and spin distributions calculated from Eqs. (2), (3), (6), and (7); assumed Gamow-Teller matrix elements. Errors are noted in parentheses.

A	$(ft)^{-1}\times 10^4\,\mathrm{sec}^{-1}$	$\lvert\langle\sum\sigma_i\tau_i^3\rangle\rvert$	$\lvert\langle\sum\sigma_i\rangle\rvert$	$\lvert\langle\sum\sigma_i\tau_i^\pm\rangle\rvert$	$\frac{J+1}{J}\lvert\langle\sum\sigma_i\tau_i^\pm\rangle\rvert^2$
3	8.63(±.07)	1.1	0.92	0.95(±.02)	2.70
7	4.35(.15)	0.77		0.70(.05)	0.81
11	2.48(.10)	0.53	0.42	0.45(.03)	0.34
13	2.12(.05)	0.36	0.31	0.31(.03)	0.29
15	2.23(.02)	0.36	0.16	0.29(.03)	0.25
17	4.20(.05)	0.97	0.87	0.90(.03)	1.13
19	5.20(.20)	0.94	0.87		
21	2.86(.20)	0.37	0.58	0.52(.05)	0.45
23	2.23(.10)	0.31		0.45(.05)	0.34
25	2.34(.20)	0.45		0.38(.03)	0.20
27	2.22(.05)	0.50		0.43(.03)	0.26
29	2.11(.10)	0.30		0.29(.05)	0.26
31	2.07(.10)	0.28		0.28(.05)	0.23
33	1.67(.10)	0.33		0.23(.05)	0.11
35	1.76(.01)	0.33	0.13	0.21(.03)	0.07
37	2.35(.25)	0.56		0.50(.03)	0.40
39	2.40(.20)	0.48		0.41(.05)	0.28
41	3.60(.15)	0.83		0.76(.05)	0.74

have

$$|\langle\sum\sigma_i\tau_{i3}\rangle|_{\mathrm{from}\,(3)} < |\langle\sum\sigma_i\rangle| \simeq |\langle\sum\sigma_i\tau_{i\pm}\rangle|.$$

The numerical results on $\langle\sum\sigma_i\rangle$, $\langle\sum\sigma_i\tau_{i3}\rangle$, and the assumed[20] values for $\langle\sum\sigma_i\tau_{i\pm}\rangle$ are listed in Table I, with the experimental data on $(ft)^{-1}$ values.

Results.—We have performed some fits of the straight line given by Eq. (1) in the variables $(ft)^{-1}$ and $[(J+1)/J)]|\langle\sum\sigma_i\tau_i\rangle|^2$ utilizing the MINUIT program of CERN, and assuming for the variables the errors quoted in Table I. The mean values of the variables are reported in Fig. 1. In all the fits we have disregarded the case $A=19$, since the data are outside the systematics. Probably the given $(ft)^{-1}$ value is a lower limit of the true value, or our formulas are inadequate in this case.

In a first fit we have utilized the data in Table I. We obtain[21]

$$g_A^2=(2.61\pm0.15)\times10^{-4}\ \mathrm{sec}^{-1},$$

$$g_V^2=(1.54\pm0.08)\times10^{-4}\ \mathrm{sec}^{-1},$$

and

$$1.23\leqslant g_A/g_V\leqslant 1.37\quad(\chi^2=4.1).$$

With the same data, but fixing $g_V^2=1.59\times10^{-4}$ sec^{-1}, we obtain

$$g_A^2=(2.57\pm0.13)\times10^{-4}\ \mathrm{sec}^{-1},$$

$$1.24\leqslant g_A/g_V\leqslant 1.30\quad(\chi^2=4.6).$$

Finally, the previous two fits have been repeated, but excluding $A=3$. We obtain

$$g_A^2=(2.60\pm0.3)\times10^{-4}\ \mathrm{sec}^{-1},$$

$$g_V^2=(1.55\pm0.09)\times10^{-4}\ \mathrm{sec}^{-1},$$

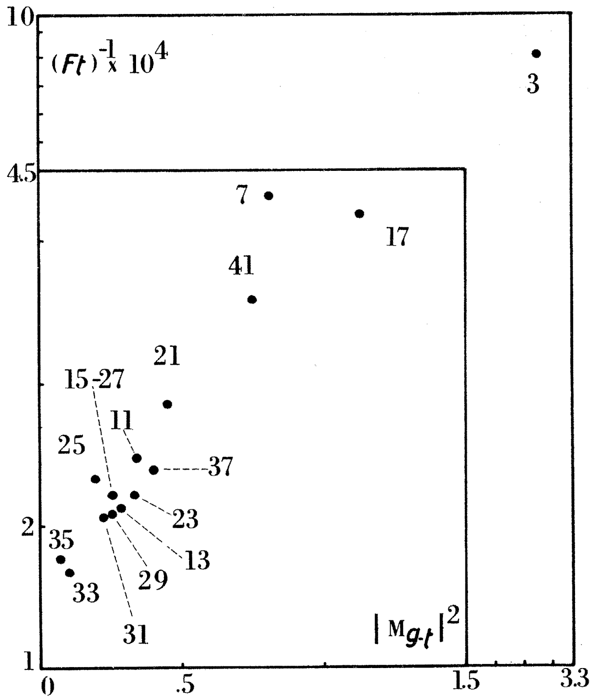

FIG. 1. Experimental values of $(ft)^{-1}$ versus values of the Gamow-Teller matrix element deduced from experimental nuclear magnetic moments.

and

$$1.19 \leq g_A/g_V \leq 1.41 \quad (\chi^2 = 4.5);$$

and fixing $g_V^2 = 1.59 \times 10^{-4}\ \mathrm{sec}^{-1}$,

$$g_A^2 = 2.53 \pm 0.19$$

$$1.22 \leq g_A/g_V \leq 1.35 \quad (\chi^2 = 4.5).$$

Concluding remarks.—(a) The mirror-nuclei analysis supports the recent larger value of the ratio g_A/g_V; the discrepancy of previous results[10,11] is removed. We note that a reasonable value on g_A/g_V may be determined without the data on $A = 3$. Obviously the errors are greater. (b) Further measurements of magnetic moments would be very useful in order to evaluate $\langle \sum \sigma_i \rangle$ and consequently $\langle \sum \sigma_i \tau_{i\pm} \rangle$, and confirm or properly modify our extrapolations. (c) Refinements on ft measurements would allow a significant reduction of the errors in computing $|g_A/g_V|$ with this method.

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When $\langle \sum \sigma_i \rangle$ is not known, we utilize the data on the nuclei in the same shell, or we assume $\langle \sum \sigma_i \tau_{i\pm} \rangle \simeq \langle \sum \sigma_i \tau_{i3} \rangle$. So we feel that $|\langle \sum \sigma_i \tau_{i\pm} \rangle|$ may be even smaller than we assume.

[21] We remember that we assume errors in both the coordinates of the data. The errors in g_A^2 and g_V^2 are the symmetrized standard errors of the MINUIT program.