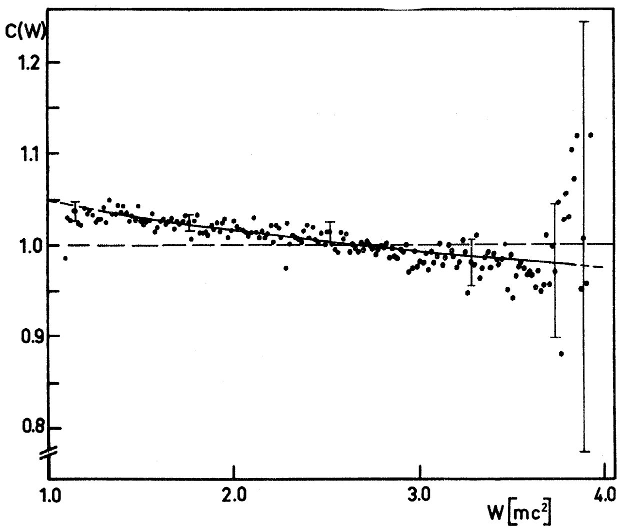

FIG. 1. Spectral shape factor of the $^{206}Tl \rightarrow {}^{206}Pb$ β transition. The statistical accuracy is below 1 % up to $3mc^2$. The curve is obtained by fitting the data in the energy range between 1.38 and $3.87mc^2$.

are taken from the tables of Landolt-Börnstein.[7] We see that only small deviations from the statistical form occur, just as expected from theory. Thus no destructive interference of nuclear matrix elements will play an important role in this decay.

Besides the spectral shape of the ^{206}Tl decay, its half-life was also measured. The result was $\tau_{1/2} = 4.27 \pm 0.05$ min.

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[1]D. A. Howe and L. M. Langer, Phys. Rev. 124, 519 (1961).

[2]W. Bühring, Nucl. Phys. 49, 190 (1963).

[3]J. Damgaard, R. Broglia, and C. Riedel, Nucl. Phys. A 135, 310 (1969).

[4]J. M. Trischuk and E. Kankeleit, Nucl. Phys. A 90, 33 (1967).

[5]H. Fabricius, K. Freitag, and S. Göring, Nucl. Instrum. Methods 38, 64 (1965); H. Fabricius, K. Freitag, S. Göring, and H. J. Langmann, Kernforschunes-zentrum Karlsruhe Report No. KFK 511, 1966 (ORNL Report No. ORNL TR 1738) (unpublished).

[6]D. Fothmann, W. Wiesner, R. Löhken, and H. Rebel, Z. Phys. 225, 164 (1969).

[7]H. Behrens, in *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, edited by H. Schopper (Springer, Berlin, 1969), New Series, Group I, Vol. 4.

Exchange Effects in the Nucleon-Nucleus Optical Potential

L. W. Owen* and G. R. Satchler
Oak Ridge National Laboratory,† Oak Ridge, Tennessee 37830
(Received 10 October 1970)

A simple model of the nucleon-nucleus optical potential is used to explore the effects of exchange. The resulting potential is close to those found from analysis of actual scattering. Exchange effects are important.

A natural interpretation of the optical potential for nucleon-nucleus scattering is that it represents the effective interaction v between the projectile and each target nucleon averaged over the distribution of target nucleons. This point of view has been exploited recently by Greenlees et al.[1] who take as a model for the potential

$$U_d(r) = \int \rho(r')v(\vec{r}-\vec{r}\,')d\vec{r}\,', \tag{1}$$

where ρ is the target density distribution. This model neglects exchange. The dominant contributions from exchange are expected to be the knock-on terms.[2] These give a nonlocal contribution with a kernel of the form

$$U_e(\vec{r}, \vec{r}\,') = \rho(\vec{r}, \vec{r}\,')v(\vec{r}-\vec{r}\,'), \tag{2}$$

where $\rho(\vec{r}, \vec{r}\,')$ is the single-nucleon density matrix for the target ground state. We report here explicit calculations of the effects of this exchange term on the scattering. We neglect other exchange terms of the "core exchange" type[2]; these vanish identically if the scattering wave function is orthogonal to all the occupied orbitals in the target, as in Hartree-Fock theory.

The independent-particle model was used to generate the density matrices ρ, using eigenfunctions in a Woods-Saxon (WS) well. (Hence our potential has the same form as a Hartree-Fock potential, although we have not imposed self-consistency in detail.) The WS parameters were chosen to reproduce the binding energies of the least bound nucleons and to give $\langle r^2 \rangle$ for the proton distributions in agreement with those found from electron scattering. Oscillator functions were also used for ^{16}O and ^{40}Ca. The effective interaction v in even states was taken to be the long-range part of the Hamada-Johnston potential, plus the closure approximation to the second-order tensor terms.[3] (A separation distance of $d = 1.05$ fm and a mean excitation energy of 240 MeV were used.) The ^3P interaction was taken to be zero; that in ^1P states was either assumed to be zero (force "E") or taken as a repulsive Gaussian [$v = 120\exp(-0.78r^2)$ MeV] adjusted to fit the ^1P phase shifts (force "$E+O$"). A phenomenological, local, imaginary term of the usual kind was added to the resulting optical potentials. (Subsidiary calculations showed that this did not affect our results.) Potentials were obtained for ^{16}O, ^{40}Ca, ^{48}Ca, ^{60}Ni, ^{90}Zr, ^{120}Sn, and ^{208}Pb.

The scattering from these nonlocal potentials was then calculated exactly.[4] [Previous calculations[5] with a similar model used a local energy approximation to reduce the exchange potential (2) to an energy-dependent local potential.] In order to interpret the results, this scattering was then treated as "data" and subjected to a conventional optical-model analysis using WS potentials $U_{\rm WS}$. The results for scattering from a given element at energies E from 10 to 60 MeV were fitted simultaneously,[6] allowing only the WS well depth to vary linearly with E. The results shown in Fig. 1 for 30-MeV protons on Sn are typical. In each case a good fit to the scattering could be obtained, so that it is meaningful to regard the corresponding WS potential as a local equivalent to the nonlocal model potential $U_d + U_e$. The negative of the potentials U_d and $U_{\rm WS}$ are shown in Fig. 2. It is clear that the exchange term U_e has large effects at this energy.

The WS potential is defined by

$$U_{\rm WS} = -V(e^x+1)^{-1}, \quad x = (r - r_0 A^{1/3})/a,$$

with $V = V_0 - \alpha E$. The proton parameters obtained for each element are given in Table I. Also given are the J/A, the volume integrals per nucleon, where

$$J = -\int U(r) d^3r,$$

and the mean square radii of the various potentials. These are the two quantities which are determined with greatest accuracy.[1] The WS parameters are surprisingly close to those found from analyses of actual scattering data.[7] There

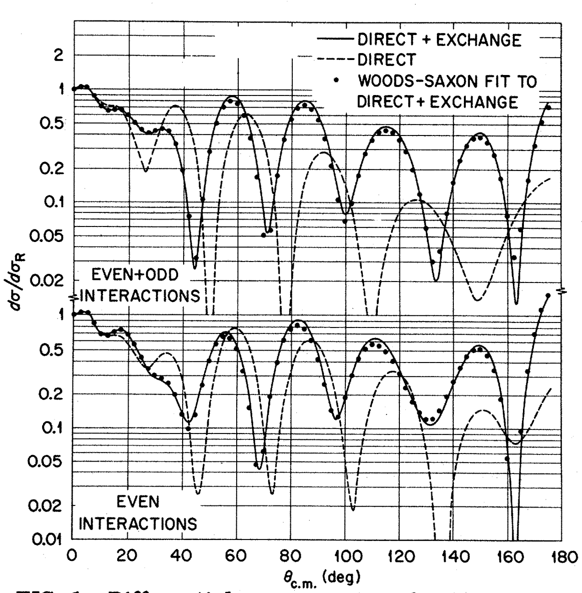

FIG. 1. Differential cross sections for 30-MeV protons on ^{120}Sn.

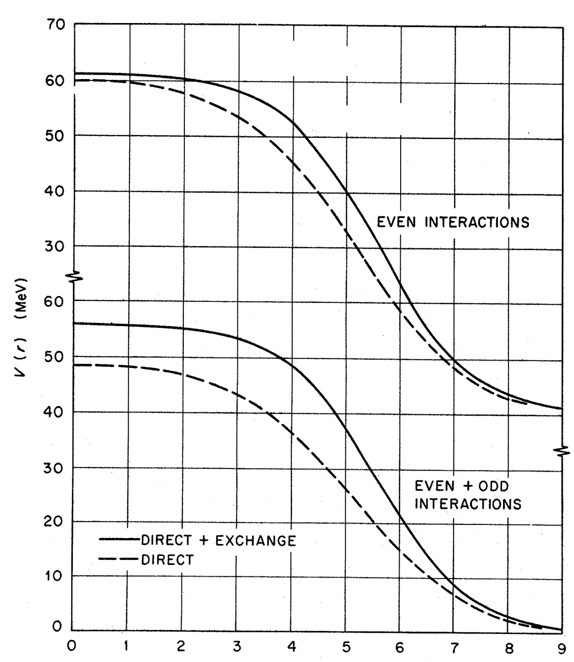

FIG. 2. The potentials U_d and $U_{\rm WS}$ corresponding to Fig. 1.

Table I. Parameters of the equivalent local Woods-Saxon potentials U_{WS} for proton scattering.

Target	Force	V_0 MeV	α	r_0 F	a F	J_d/A MeV F^3	J_{WS}/A[a] MeV F^3	$\Delta J/J_d$[a] %	$\langle r^2\rangle_d$ F^2	$\langle r^2\rangle_{WS}$ F^2	$\Delta\langle r^2\rangle/\langle r^2\rangle_d$ %
^{16}O	E	61.8	0.241	1.039	0.690	371	432	16	13.3	10.6	-21
^{16}O	E+O	59.0	0.301	1.048	0.663	311	392	26	14.2	10.2	-28
^{40}Ca	E	61.1	0.223	1.109	0.782	371	441	19	18.6	17.0	- 9
^{40}Ca	E+O	56.8	0.267	1.125	0.754	311	401	29	19.5	16.7	-15
^{48}Ca[b]	E+O	62.2	0.243	1.094	0.695	326	392	20	19.5	16.1	-17
^{48}Ca[c]	E+O	55.0	0.246	1.173	0.703	326	408	25	21.1	17.6	-16
^{60}Ni	E+O	60.4	0.229	1.107	0.784	317	403	27	22.1	19.7	-11
^{90}Zr	E+O	64.9	0.233	1.100	0.759	321	399	24	24.9	22.5	-10
^{120}Sn	E	67.1	0.194	1.132	0.861	397	459	16	29.4	28.8	- 2
^{120}Sn	E+O	62.7	0.226	1.136	0.841	326	419	28	30.3	28.5	- 6
^{208}Pb	E	69.1	0.176	1.146	0.842	404	463	15	38.1	37.3	- 2
^{208}Pb	E+O	63.4	0.202	1.154	0.838	330	424	28	38.9	37.7	- 3

[a]For 30-MeV protons.
[b]$\langle r^2\rangle$ for protons and neutrons taken equal.
[c]$\langle r^2\rangle$ for neutrons greater than for protons by 2.3 F^2.

is a tendency for the surface diffuseness to increase as A increases, whereas the empirical potentials show the opposite tendency. The energy dependence (which in our model comes entirely from the exchange terms) is close to the empirical value $\alpha \approx 0.22$ found[7] for protons of 30 to 60 MeV. The empirical value increases at lower energies, i.e., the energy dependence is not linear. In principle this is true for our model potential also, but the fitting procedure we used could not establish a nonlinear variation. (It does appear in the local approximation of Slanina and McManus,[5] however.) Including a repulsive odd-state force reduces the direct and enhances the exchange contributions, and results in slightly larger values of α. There is also a tendency for our values of α to decrease with increasing A.

We see that inclusion of exchange for 30-MeV protons increases J/A by about 15% for the E force, and by 25-30% for the $E+O$ force. At 10 MeV, this increase may be as much as 40%. (In ^{48}Ca this effect is found to depend also upon the distribution of neutrons relative to the pro tons.) The empirical values[1] of J/A for 30-MeV protons on nuclei heavier than Ca are all approximately 400 MeV F^3, close to but slightly less than the values obtained with the $E+O$ force in our model.

The effect of exchange is a reduction in the mean-square radius $\langle r^2\rangle$ of the potential of as much as 28% for 30-MeV protons on ^{16}O, falling to just a few per cent for the heaviest nuclei. (The approximate treatment of exchange by Slanina and McManus[5] lead to a slight increase in $\langle r^2\rangle$.) This comes about (cf. Fig. 2) because, although exchange increases the effective radius parameter r_0, it also reduces the surface diffuseness of the potential. As expected, the effect is larger for the $E+O$ force than for the E force. Inspection of the kernel $U_e(r,r')$ itself shows that it maintains its strength to larger radii than does $U_d(r)$. This can be related to the appearance of high multipoles of the interaction v in the construction of U_e, whereas only the monopole term enters U_d. Changes in shape of the effective potential such as those shown in Fig. 2 indicate it is improper to draw conclusions from empirical potentials about nuclear density distributions without considering exchange.

Similar calculations were made for neutron scattering, with similar results. The energy dependence parameter α was found to have similar values to those for protons (Table I), but with a somewhat greater tendency to decrease with increasing A. Neutron scattering was also studied at lower energies, between 1 and 10 MeV. The well-known optical model ambiguities prevented us from seeing any simple systematic changes in U_{WS} in this energy region except that the values of J_{WS} were consistently a few per cent larger than would be obtained by linear extrapolation from the higher energies.

Phenomenological analyses of proton scattering usually assume a Coulomb correction[8] to the potential depth V of the form $cZ/A^{1/3}$, with $c \approx 0.4$ MeV (although it has been contended[9] that it is not necessary). Such a term should appear in our U_{WS} since the model potential is nonlocal. The model potentials are identical for neutrons and protons on the $N=Z$ nuclei ^{16}O and ^{40}Ca since the neutron and proton densities were taken to be equal in these nuclei. However, different neutron and proton well depths were definitely required for the local equivalent U_{WS}, with $V_p - V_n = 0.44Z/A^{1/3}$ MeV when the $E+O$ force is used. When the calculations were repeated using densities constructed from oscillator single-nucleon functions, a coefficient of $c=(0.40\pm0.01)$ MeV was obtained with either E or $E+O$ force. In the simplest picture,[8] c is proportional to α and $c=0.4$ corresponds to $\alpha=0.3$. There is no such strict correlation between the c and α values in our results. However, it is clear that a term of magnitude similar to that used phenomenologically is also required here to correlate neutron and proton scattering from self-conjugate nuclei.

There are general arguments[8] why the optical potential should depend upon the asymmetry $\epsilon = (N-Z)/A$ as a result of the presence of a charge-exchange component in the nucleon-nucleon force. The empirical values[1] of J/A for proton scattering, however, do not show clear evidence of such a dependence. For the direct parts of our model potentials we have $J_d = J_0 \pm J_1\epsilon$ with the upper sign for protons and the lower for neutrons and with $J_1/J_0 = 0.41$ (E force), 0.30 ($E+O$ force). There are two ways to look for this dependence in the local equivalent U_{WS}: by comparing neutron and proton scattering on the same target, and by studying the variation of J_{WS} itself with ϵ. The former was also studied by comparing the scattering of neutrons and "uncharged" protons, so as to remove uncertainties due to the Coulomb correction term just discussed. Neutron scattering from the model potentials yields fairly consistent results with $J_1/J_0 \approx 0.55$ (E), 0.40 ($E+O$); thus, exchange has enhanced the asymmetry dependence by some 30%. The results for protons show more scatter and are difficult to interpret. Subtracting a Coulomb correction of the type just discussed, or using "uncharged" protons, gives J/A values which are constant to within a few per cent. However, the variation in J_d/A itself from ^{40}Ca to Pb is also only 6-8%, so further study is needed to see what effect exchange has on the asymmetry dependence of proton scattering.

Another consequence of the nonlocality due to exchange is the Perey effect[10]; the wave functions ψ_{NL} associated with the nonlocal potential are reduced in magnitude in the nuclear interior relative to those, ψ_L, for the equivalent local potential U_{WS}. A local energy approximation predicts

$$|\psi_{NL}(r)|/|\psi_L(r)| = [1-\partial U_{WS}(r)/\partial E]^{1/2}. \qquad (3)$$

We observe this effect. The magnitude of the damping is given roughly by Eq. (3) but it varies with r much less smoothly. It is often assumed that this damping can be deduced from the nonlocality range ($\beta = 0.85$ F) of the potential used by Perey and Buck[11] to fit neutron scattering. That potential corresponded to an energy dependence with $\alpha \approx 0.33$ which, except for ^{16}O, is noticeably larger than the values for our model potentials. Hence it is not surprising that we obtain a weaker damping which corresponds more closely to $\beta \approx 0.6$ F.

In conclusion, we see that in the present model exchange terms can produce important effects on elastic scattering and hence on the equivalent local potentials used to describe it. We should also remark on some of the uncertainties associated with the model. The direct potential U_d is not sensitive to the details of the density distribution, but the exchange potential U_e may be more strongly affected by correlations we have ignored. Further, U_e depends somewhat on the distribution of the neutron excess when $N>Z$, about which we know little. The effective nucleon-nucleon interaction is not known in detail (e.g., in odd states, which especially affect the exchange properties). We have not included any density dependence; this could modify the surface properties. Some additional energy dependence can be expected; for example, the separation distance d is momentum dependent.[3,5] Further, the target polarization which gives rise to the imaginary part of the optical potential also contributes to the real potential; these effects are currently being investigated.

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*Former Oak Ridge Graduate Fellow from the University of Tennessee under appointment from Oak Ridge Associated Universities, now at the University of Georgia, Athens, Ga. 30601.

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[1]G. W. Greenlees, V. Hnizdo, O. Karban, J. Lowe, and W. Makofske, Phys. Rev. C 2, 1063 (1970); G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. 171, 1115 (1968).

[2]N. Austern, *Direct Nuclear Reaction Theories* (Interscience, New York, 1970).

[3]G. E. Brown, *Unified Theory of Nuclear Models and Forces* (Wiley, New York, 1967).

[4]M. Reeves and L. W. Owen, J. Comput. Phys. 4, 572 (1969).

[5]D. Slanina and H. McManus, Nucl. Phys. A116, 271 (1968).

[6]The optical model search code GENOA of F. G. Perey was used.

[7]M. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. 156, 1207 (1967); C. B. Fulmer, J. B. Ball, and A. Scott, Phys. Lett. 24B, 505 (1967); G. R. Satchler, Nucl. Phys. A92, 273 (1967); F. D. Becchetti and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[8]G. R. Satchler, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

[9]P. C. Sood, Nucl. Phys. 89, 553 (1966).

[10]N. Austern, Phys. Rev. 137, B752 (1965).

[11]F. G. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

Direct Measurement of n-p and n-d Total Cross Sections from 700 to 2900 MeV/c*

R. E. Mischke
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

and

T. J. Devlin,† W. Johnson, J. Norem, and K. Vosburgh
Physics Department, Rutgers, The State University, New Brunswick, New Jersey 08903

and

W. Schimmerling
Princeton-Pennsylvania Accelerator, Princeton, New Jersey 08540
(Received 22 October 1970)

Neutron-proton and neutron-deuteron total cross sections have been measured directly at the Princeton-Pennsylvania Accelerator using time of flight to determine the incident neutron momentum. The results cover the region from 700 to 2900 MeV/c with a typical accuracy of 0.8% for each of 26 momentum bins. The data are not consistent with the most precise previous measurements in the same momentum range.

We have made direct measurements of neutron-proton and neutron-deuteron total cross sections for incident neutron momenta between 700 and 2900 MeV/c. The statistical accuracy is better than 1% for most of the 26 momentum bins. The total systematic error is believed to be less than 1%. Both the n-p and n-d cross sections are systematically lower by 1.5-2 mb than measurements of p-d and p-n cross sections reported by the Cambridge-Rutherford (CR) group.[1] The CR p-n cross sections were obtained from $\sigma_{pd}-\sigma_{pp}$ with a correction for the Glauber screening effect.[2]

A neutral beam containing a broad spectrum of neutron momenta was produced at 20° by the 3-GeV internal proton beam of the Princeton-Pennsylvania Accelerator. The beam was defined by collimators. A lead filter near the production target attenuated gamma rays, and magnets were used to remove charged particles.

The momentum of each detected neutron was determined by its time of flight (TOF). The proton beam struck the internal target at 67-nsec intervals with bunches less than 1 nsec wide. A Cherenkov counter placed near the internal target gave a signal when each proton bunch struck the target. The neutron detector was located 120 ft away; it provided the other signal for the TOF measurement. The resolution of the system was 2 nsec (full width at half-maximum) corresponding to a momentum resolution of 0.7% at 700 MeV/c and 9.5% at 2900 MeV/c. There was an ambiguity in the TOF since a slow neutron could be overtaken by a fast neutron from the next bunch. This was eliminated by a range requirement in the detector which provided a low-momentum cutoff.

The target system was located 104 ft from the