It is clear that at each tower level, all graphs do not contribute to the final result. To find the subset of those which do is a major problem. Two of us (B.H. and D.K.S.) conjecture that only diagrams which are generalizations of the nested Mandelstam graphs can contribute to the final leading asymptotic form,[5] and a naive calculation along these lines does indeed generate (4). However, work in progress indicates that it is difficult to prove which of the Mandelstam nests contribute at each order, because of the difficulty of making statements about general pinch-ladder structures at the N-tower level.

Two of us (B.H. and D.K.S.) wish to thank Dr. D. Z. Freedman for arousing our interest in this problem and for many helpful discussions about it. We would also like to thank Dr. I. Muzinich for giving us further insight into the problem, and Dr. B. M. McCoy who checked some of our results in low-order cases, using Mellin transforms.

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Second-Class Currents in Weak Interactions*

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A simple way of introducing second-class currents has been suggested by naturally incorporating a tensor density in the theory. A possible explanation for large ξ and λ_+ parameters in K_{l3} decays is also proposed.

In a series of interesting papers,[1] Wilkinson and his collaborator recently suggested that ft values of various nuclear β transitions are consistently different by 10% from those of corresponding mirror nuclei and that we may have to introduce the so-called second-class current[2] in order to explain the difference. Previously, the possible existence of the second-class current has also been advocated by some authors[3] so as to explain certain data on μ-meson capture by the nucleus as well as β decays, although the conclusion appears to be far from definite.[4]

In this paper, we shall assume the existence of second-class current and shall propose a simple model for it. To this end, we first assume the existence of a charged intermediate vector boson. Then the standard weak-interaction Hamiltonian may be expressed[5] by

$$H_1 = g[j_\mu(x) + l_\mu(x)]W_\mu(x) + \text{H.c.}, \tag{1}$$

where $j_\mu(x)$ and $l_\mu(x)$ represent hadronic and leptonic currents, respectively, and $W_\mu(x)$ is the vector-boson field. Now in addition to H_1 given above, we postulate the existence of another interaction involving the first-order derivative of $W_\mu(x)$. The most general form for it is evidently written as

$$H_2 = g\left\{T_{\mu\nu}(x)\left[\frac{\partial}{\partial x_\mu}W_\nu(x) - \frac{\partial}{\partial x_\nu}W_\mu(x)\right] + \theta_{\mu\nu}(x)\left[\frac{\partial}{\partial x_\mu}W_\nu(x) + \frac{\partial}{\partial x_\nu}W_\mu(x)\right] + S(x)\frac{\partial}{\partial x_\mu}W_\mu(x)\right\} + \text{H.c.}, \tag{2}$$

where $T_{\mu\nu}(x)$, $\theta_{\mu\nu}(x)$, and $S(x)$ are antisymmetric tensor, symmetric tensor, and scalar densities, respectively. Moreover, we assume that these new quantities are purely hadronic in origin without containing any leptonic field.

Up to the second order in g, the addition of the new Hamiltonian H_2 is effectively equivalent to replacing the original hadronic current $j_\mu(x)$ of Eq. (1) by

$$j_\mu(x) \to \tilde{j}_\mu(x) = j_\mu(x) + 2\frac{\partial}{\partial x_\nu}[T_{\mu\nu}(x) - \theta_{\mu\nu}(x)] - \frac{\partial}{\partial x_\mu}S(x). \quad (3)$$

Hence, the fundamental current-current nature of effective interaction is still maintained in our theory. Therefore, the usual ratio for $(\pi \to e\nu)/(\pi \to \mu\nu)$ remains, for example, unmodified. Also, due to the derivative character of additional terms in Eq. (3), the new interaction gives only a small modification to the standard theory for reactions involving small energy-momentum transfers such as in β decays. It should be emphasized that we are not introducing ordinary tensor or scalar interactions since the leptonic part consists solely of the standard $V-A$ current.

Our proposal is that the new term may account for the desired second-class current. In order to see it in detail let us consider, for simplicity, the quark model. Then explicit forms for $j_\mu(x)$, $T_{\mu\nu}(x)$, and $S(x)$ may be given by

$$j_\mu(x) = \tfrac{1}{2}\bar{q}(x)\gamma_\mu(1+\gamma_5)Q_V q(x), \quad T_{\mu\nu}(x) = \tfrac{1}{4}\bar{q}(x)[\gamma_\mu, \gamma_\nu](f_T + g_T\gamma_5)Q_T q(x),$$

$$S(x) = \bar{q}(x)(f_S + f_P\gamma_5)Q_S q(x), \quad (4)$$

where Q_V, Q_T, and Q_S are some appropriate 3×3 matrices in the SU(3) space and where f_S, f_T, f_P, and g_T are real or complex coupling constants. In the ordinary Cabibbo theory we have, of course, $Q_V = \cos\theta(\lambda_1 - i\lambda_2) + \sin\theta(\lambda_4 - i\lambda_5)$. We assume analogous forms for Q_T and Q_S with different Cabibbo angles. If we exclude expressions containing derivatives of quark fields, then the simplest choice for $\theta_{\mu\nu}(x)$ is to set $\theta_{\mu\nu}(x) = 0$ identically, which we assume hereafter.

Let us first consider reactions with $\Delta S = 0$ (i.e., no change of strangeness quantum number). In that case, it is obvious that the pseudotensor part of $T_{\mu\nu}(x)$ proportional to g_T gives the desired second-class current. Indeed, the analysis[1] by Wilkinson requires g_T to be of the order of the inverse nucleon mass. However, in order to maintain the experimentally well-satisfied consequences[5] of the weak magnetism from the conserved vector-current hypothesis, we must require $f_T = 0$, or at least a small value for f_T (more accurately $f_T\cos\theta_T$, θ_T being the new Cabibbo angle). Also, present experiments appear to be consistent with $f_P = f_S = 0$, although the conclusion is less definite.[6] As we have remarked already, our new interaction usually gives a small correction to reactions with small Q values. More accurate measurements for $\Sigma^\pm \to \Lambda e^\pm \nu$ decay rates[7] as well as their angular correlation distributions[8] will be a very interesting test of the present theory.

Next, let us turn our attention to $\Delta S = \pm1$ transitions. So far, leptonic decays of hadrons are more or less consistent with the standard Cabibbo theory without the new interaction. However, the experimental error is still large and we do not yet fully understand the various problems encountered in K_{l3} decays. Hence it may be worthwhile to investigate possible consequences of the theory.

First, we notice that the nonexistence theorem[2,5] of the induced pseudotensor terms arising from the axial-vector interaction is no longer valid unless the SU(3) symmetry is exact. Hence, the presence of the new pseudotensor term proportional to g_T in the $\Delta S = \pm1$ transition would be rather difficult to establish experimentally, unless $g_T\sin\theta_T$ is reasonably large in comparison with the induced one which is expected to arise from the violation of the SU(3) symmetry. If this is the case, then its presence may show up[8] in some angular correlation measurement between, say, Λ polarization and neutrino direction for $\Lambda \to pe\bar{\nu}$ decay. Also its existence slightly affects the decay rate and angular decay distributions of the K_{l4} decay. Moreover, if $f_T\sin\theta_T$ or f_S (or both) is nonzero and fairly large, then one can easily explain the experimentally observed large ξ and λ_+ parameters of K_{l3} decays.[9] Actually, one can obtain any value for them by adjusting f_T and f_S suitably. Even if f_S is zero, we can considerably improve the value of the ξ parameter, provided that we have relatively large λ_+ of the order 0.06-0.08, as some recent experiments suggest. Setting

$$\langle \pi(p')|T_{\mu\nu}(0)|K(p)\rangle = (-i)[(p+p')_\mu q_\nu - (p+p')_\nu q_\mu]G(q^2)$$

with $q_\mu = p_\mu - p_\mu'$, we compute

$$\langle \pi(p') | \partial_\nu T_{\mu\nu}(0) | (p) \rangle = \{ q^2 (p+p')_\mu + (m_K{}^2 - m_\pi{}^2) q_\mu \} G(q^2),$$

where $G(q^2)$ is a form factor for the vertex. We see from this expression that the tensor term increases both λ_+ and $-\xi$ in the right direction. Assuming the standard K^*-dominance model for the ordinary vector vertex, we can easily obtain $\lambda_+ = 0.06$ and $\xi = -0.67$. It may be worthwhile to emphasize the fact that such large values for ξ and λ_+ are very difficult,[9,10] if not impossible, to explain by conventional theories. Also, a nonzero value required for $f_T \sin\theta_T$ in order to obtain large λ_+ and ξ may be helpful in understanding preliminary experimental data[8] on $\Lambda \to pe\bar{\nu}$ decay.

We have not discussed possible effects of CP violation which may result if we assume that at least one of g_T, f_T, f_P, and f_S is complex rather than real. This gives a simple way of introducing a CP violation in K_{l3} decays,[11] although the experimental situation on this point is far from being clear.

Finally, the existence of the tensor current $T_{\mu\nu}(x)$ may be welcome from the viewpoint of algebra of currents.[12] It is well known that vector and axial-vector currents together with tensor and scalar densities are closed under equal-time commutation to form the algebra of the U(12) group. So far only vector and axial-vector currents are known to exist in nature. The scalar density may be present also in the strong-interaction Hamiltonian as in the theory of Gell-Mann, Oakes, and Renner.[13] Therefore, the addition of the tensor current into theory fills the algebra of the U(12) group. The possible existence of the symmetric tensor density $\theta_{\mu\nu}(x)$ in Eq. (2) may be interesting since it may correspond to a charged counterpart of the standard energy-stress tensor.

Another amusing way of introducing the tensor current $T_{\mu\nu}(x)$ can be achieved as follows: Suppose that we have[14] a Yang-Mills intermediate vector meson field $W_\mu{}^{(\alpha)}(x)$ $(\alpha = 1, \cdots, 8)$. Setting

$$F_{\mu\nu}{}^{(\alpha)}(x) = \partial_\mu W_\nu{}^{(\alpha)}(x) - \partial_\nu W_\mu{}^{(\alpha)}(x) + g_0 f_{\alpha\beta\gamma} W_\mu{}^{(\beta)}(x) W_\nu{}^{(\gamma)}(x),$$

the free Lagrangian for the W boson is given by

$$L_0 = -\tfrac{1}{4} F_{\mu\nu}{}^{(\alpha)}(x) F_{\mu\nu}{}^{(\alpha)}(x) - \tfrac{1}{2} m^2 W_\mu{}^{(\alpha)}(x) W_\mu{}^{(\alpha)}(x).$$

Now, replace $F_{\mu\nu}{}^{(\alpha)}(x)$ by

$$\tilde{F}_{\mu\nu}{}^{(\alpha)}(x) = F_{\mu\nu}{}^{(\alpha)}(x) + g_{\alpha\beta} T_{\mu\nu}{}^{(\beta)}(x)$$

in the above expression, where $g_{\alpha\beta}$ is a small numerical matrix. Choosing a suitable form for $g_{\alpha\beta}$, this procedure induces the desired tensor interaction of the form of Eq. (2).

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Reversible and Irreversible Transformations in Black-Hole Physics*

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The concepts of irreducible mass and of reversible and irreversible transformations in black holes are introduced, leading to the formula $E^2 = m_{ir}^2 + (L^2/4m_{ir}^2) + p^2$ for a black hole of linear momentum p and angular momentum L.

This note reports five conclusions: (1) The mass energy of a black hole of angular momentum L can be expressed in the form

$$m^2 = m_{ir}^2 + L^2/4m_{ir}^2, \quad (1)$$

where m_{ir} is the irreducible mass [geometrical units: $L(\text{cm}) = (G/c^3)L_{\text{conv}}(\text{g cm}^2/\text{sec})$; $m(\text{cm}) = (G/c^2)M_{\text{conv}}(\text{g})$; $G/c^2 = 0.742\times10^{-28}$ cm/g] of the black hole. (2) Insofar as one looks apart from the atomicity of matter one can approach arbitrarily closely to reversible transformations that augment or deplete the rotational contribution to the square of the mass. (3) The attainable range of reversible transformation extends[1,2] from $L=0$, $m^2 = m_{ir}^2$ to $L = m^2$, $m^2 = 2m_{ir}^2$. (Contrast to the formula for mass energy as it depends upon translation, $E^2 = m^2 + p^2$, where p is unlimited; and with the formula for the squared mass energy of a meson!) (4) An irreversible transformation is characterized (Fig. 1) by an increase in the irreducible mass of the black hole. (5) There exists no process which will decrease the irreducible mass.

Roger Penrose has pointed out[3] a way to extract energy from a black hole endowed with angular momentum. It makes use of the "ergosphere" (Ruffini and Wheeler; cf. Fig. 2, reproduced from their paper[4]), the region between the horizon (surface of black hole; boundary of region from which no particle or radiation can ever escape) and the surface of infinite red shift (coincident with the horizon only for case of the angular-momentum-free Schwarzschild black hole). A particle of energy E_0 is sent from infinity into the ergosphere and decays there into (1) a particle which emerges to infinity with a rest-plus-kinetic energy E_2 greater than E_0, together with (2) a particle ("rocket ejecta") which has an energy E_1, that is negative as measured at infinity ($E_1 = E_0 - E_2$), but positive in the local Lorentz frame, and which is ejected into such a direction that it is captured into the black hole, thereby diminishing its mass. We consider the case where all masses can be regarded as infinitesimal compared with the mass of a black hole.

The energy E, as measured at infinity, of a particle of angular momentum p_φ and rest mass μ, having a turning point at r, is given by the

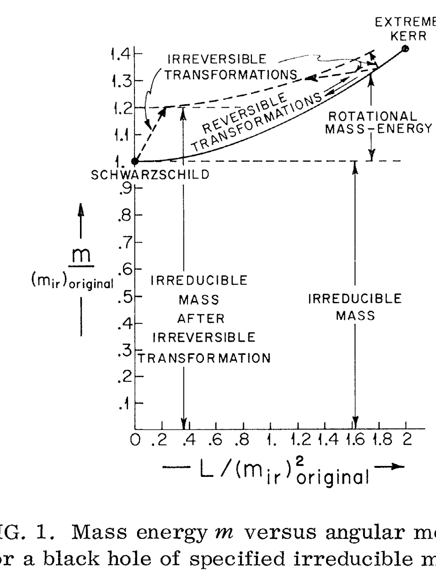

FIG. 1. Mass energy m versus angular momentum L for a black hole of specified irreducible mass m_{ir} illustrating the difference between reversible transformations and irreversible transformations (which increase the irreducible mass).