Tower Exchange in $\lambda\varphi^3$ Theory*

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We study the asymptotic behavior of two-tower exchange with linked ends in φ^3 theory using Feynman parameter techniques. We present a new result for the asymptotic behavior of the Mandelstam graph and note that only this graph contributes for the two-tower case. We generalize the above to the N-tower case and find the eikonal form.

Recently there has been considerable interest in the relativistic analog of the eikonal approximation within the context of various field theories. Since the exchange of elementary quanta does not generate inelastic contributions in this picture, it is natural to study the exchange of more complex structures. One such simple model is the exchange of towers whose ends are linked in all possible ways. Multitower exchange will generate Regge-cut behavior at high energies. Such models have been discussed by Cheng and Wu[1] in quantum electrodynamics and by Chang and Yan[2] in $\lambda\varphi^3$ using momentum-space techniques. Because of the fundamental complexity of towerlike Feynman diagrams, it would seem prudent to investigate in detail the asymptotic energy behavior of individual sets of towers. The two-tower case in $\lambda\varphi^3$ is simple enough to be attacked by Feynman parameter techniques and has the additional virtue of giving the phenomenologically interesting leading contribution to the two-Reggeon cut.

We have done such a detailed study for the two-tower case (see Fig. 1) using methods due Polkinghorne.[3,4] For simplicity in calculation, we retain for each order of the coupling constant only the leading logarithmic s contribution. In Regge language this can be looked on as expanding only to lowest order in the coupling constant, for the residue and trajectory functions.

We write the amplitude described by each configuration in the form

$$M=\kappa\int\frac{d\alpha[C(\alpha)]^n\,\delta(1-\sum_i\alpha_i)}{[g(\alpha)s+d(\alpha,t)+i\epsilon]^{n+2}},\qquad(1)$$

where κ is a constant, α stands for a generic Feynman parameter, and $s=(p_1+p_2)^2$ is the asymptotic variable. In investigating the class of such integrals, one must keep in mind the following observations.

For elementary line exchange, graphs in the same order in the coupling constant all have similar asymptotic behavior and therefore are all equally important. Delicate cancelations occur when the sum over all permutations of the particle lines is performed.

Tower exchange, on the other hand, has the property that only a small number of diagrams, the so-called nested towers shown in Fig. 1, contribute to the asymptotic behavior. It is a well-known result that planar diagrams do not contribute either to the J-plane cuts on the physical sheet or to the leading asymptotic behavior. How-

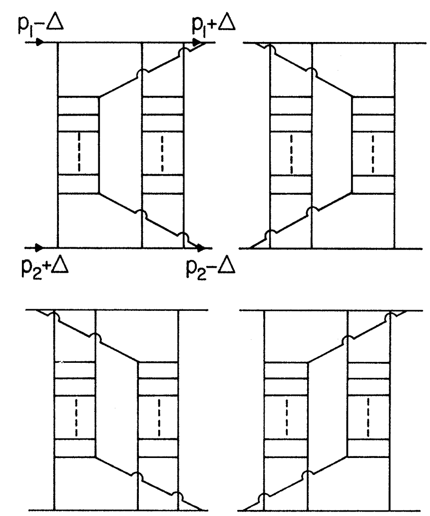

FIG. 1. Four nested two-tower graphs.

ever, in the tower-exchange case, the statement can be extended to say that non-nested, nonplanar graphs also do not contribute to leading order in lns.

The four nested diagrams of Fig. 1 can all be distorted into the canonical Mandelstam form of Fig. 2. Summing over ladder rungs then produces overcounting by a factor of 4. This is handled in a simple way by observing a heretofore overlooked property of the Mandelstam graph. Leading behavior of the Mandelstam graph is associated with a pinch due to the crosses combined with the vanishing of the rung parameters of each ladder. After the pinch conditions have been imposed, the vanishing of either pair of parameters (α_1, α_3) or (α_2, α_4) causes the g function to vanish. In the (α_1, α_3) case, the large momentum P goes along $ABCD$; and in the (α_2, α_4) case, the path $AEFD$. Similar statements hold for the primed parameters at the bottom of the diagram. The asymptotic behavior then comes from four disjoint pieces of parameter space. Returning to Fig. 1, one sees that if all diagrams

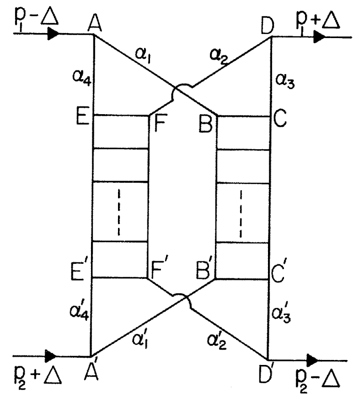

FIG. 2. Two-tower Mandelstam graph.

are retained but only the contributions in each arising from the large-momentum path staying on the edges of the diagrams are counted, then each possible path in the Mandelstam set is counted once. This observation justifies the eikonal picture.

The calculation of the asymptotic behavior of the individual diagrams is lengthy and the details will be given elsewhere.[5,6] After summation over the ladder rungs we find

$$M(s,t)=\frac{i}{2!}\frac{1}{2s}\int\frac{d^2q}{(2\pi)^2}\gamma(\vec{q})\gamma(\vec{q}-\vec{t})s^{\alpha(\vec{q})+\alpha(\vec{q}-\vec{t})}, \tag{2}$$

where

$$\gamma(\vec{q})=\beta\{1+\exp[-i\pi\alpha(\vec{q})]\},\quad \beta=\lambda^2,\quad \alpha(\vec{t})=-1+\frac{\lambda^2}{4\pi}\int\frac{d^2k}{(2\pi)^2}[\vec{k}^2+m^2]^{-1}[(\vec{k}-\vec{t})^2+m^2]^{-1},\quad t=-\vec{t}^{\,2}.$$

Equation (2) is just the result of the naive eikonal model. We emphasize again that the set of graphs that generates this form is very small, consisting of the Mandelstam graph and its associated crossed graphs. The three crossed graphs generate the signature factor of the Reggeon.

It should be noted that Eq. (2) differs from the result given for the Mandelstam graph in Ref. 4 which has been widely quoted in the literature. Other authors apparently missed the fact that once pinch conditions are imposed, two additional scalings along the α lines are possible.

In extending these results to the N-tower case, one must be extremely careful, since the leading contribution to the N-Reggeon cut does not come from the leading asymptotic behavior of the individual diagrams.[7] Two of us (G.M.C. and R.L.S.) have studied these graphs using momentum-space techniques,[6] and we find that if one works to lowest order in the trajectory and residue functions, the Regge-pole amplitude does indeed eikonalize. The N-Reggeon–cut contribution is given by

$$M_n=\frac{i}{n!}\left(\frac{1}{2s}\right)^{n-1}\int\prod_{i=1}^{n-1}\frac{d^2k_i}{(2\pi)^2}\,\gamma(\vec{k}_i)s^{\alpha(\vec{k}_i)}\gamma(\vec{t}-\sum_{j=1}^{n-1}\vec{k}_j)\,s^{\alpha}\,(\vec{t}-\sum_{j=1}^{n-1}\vec{k}_j), \tag{3}$$

which implies the full amplitude

$$M=\sum_{i=1}^{\infty}M_n=2is\int d^2b\,\exp(i\vec{t}\cdot\vec{b})\{\exp[i\delta(\vec{b},s)]-1\}, \tag{4}$$

where

$$\delta(\vec{b},s)=\frac{1}{2s}\int\frac{d^2k}{(2\pi)^2}\exp(-i\vec{k}\cdot\vec{b})\gamma(\vec{k})s^{\alpha(\vec{k})}.$$

Equation 3 is, of course, just the result predicted by the simple eikonal model.

It is clear that at each tower level, all graphs do not contribute to the final result. To find the subset of those which do is a major problem. Two of us (B.H. and D.K.S.) conjecture that only diagrams which are generalizations of the nested Mandelstam graphs can contribute to the final leading asymptotic form,[5] and a naive calculation along these lines does indeed generate (4). However, work in progress indicates that it is difficult to prove which of the Mandelstam nests contribute at each order, because of the difficulty of making statements about general pinch-ladder structures at the N-tower level.

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[7]This does not appear to be the case in quantum electrodynamics.

Second-Class Currents in Weak Interactions*

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A simple way of introducing second-class currents has been suggested by naturally incorporating a tensor density in the theory. A possible explanation for large ξ and λ_+ parameters in K_{l3} decays is also proposed.

In a series of interesting papers,[1] Wilkinson and his collaborator recently suggested that ft values of various nuclear β transitions are consistently different by 10% from those of corresponding mirror nuclei and that we may have to introduce the so-called second-class current[2] in order to explain the difference. Previously, the possible existence of the second-class current has also been advocated by some authors[3] so as to explain certain data on μ-meson capture by the nucleus as well as β decays, although the conclusion appears to be far from definite.[4]

In this paper, we shall assume the existence of second-class current and shall propose a simple model for it. To this end, we first assume the existence of a charged intermediate vector boson. Then the standard weak-interaction Hamiltonian may be expressed[5] by

$$H_1 = g[j_\mu(x) + l_\mu(x)]W_\mu(x) + \text{H.c.}, \tag{1}$$

where $j_\mu(x)$ and $l_\mu(x)$ represent hadronic and leptonic currents, respectively, and $W_\mu(x)$ is the vector-boson field. Now in addition to H_1 given above, we postulate the existence of another interaction involving the first-order derivative of $W_\mu(x)$. The most general form for it is evidently written as

$$H_2 = g\left\{T_{\mu\nu}(x)\left[\frac{\partial}{\partial x_\mu}W_\nu(x) - \frac{\partial}{\partial x_\nu}W_\mu(x)\right] + \theta_{\mu\nu}(x)\left[\frac{\partial}{\partial x_\mu}W_\nu(x) + \frac{\partial}{\partial x_\nu}W_\mu(x)\right] + S(x)\frac{\partial}{\partial x_\mu}W_\mu(x)\right\} + \text{H.c.}, \tag{2}$$

where $T_{\mu\nu}(x)$, $\theta_{\mu\nu}(x)$, and $S(x)$ are antisymmetric tensor, symmetric tensor, and scalar densities, respectively. Moreover, we assume that these new quantities are purely hadronic in origin without containing any leptonic field.

Up to the second order in g, the addition of the new Hamiltonian H_2 is effectively equivalent to re-