

Proton-Proton Bremsstrahlung Measurements at 64.4 MeV[†]

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The 30°-30° proton-proton bremsstrahlung cross section and its noncoplanar dependence were measured at 64.4 MeV. The measured noncoplanar dependence agrees with theoretical calculations, but the coplanar cross section, $2.32 \pm 0.20 \mu\text{b}/\text{sr}^2$, is significantly below the predicted values.

The interest in nucleon-nucleon bremsstrahlung in recent years was generated by the hope that since the description of the process involves the off-energy-shell elements of the two-nucleon transition matrix, experimental cross-section data might provide a basis for selecting among the various potentials which give reasonable fits to nucleon-nucleon elastic-scattering data. It is now apparent that the model dependence of the off-shell effects is small,^{1,2} and that extremely accurate bremsstrahlung cross-section data are needed before one can seriously consider selecting among various models of the nucleon-nucleon interaction. The available experimental data are scarce and sometimes contradictory. This is especially true for the 30°-30° coplanar symmetric geometry in the 30- to 70-MeV energy range, where individual measurements³⁻⁵ at nearly the same energy differ by as much as a factor of 4.

A practical data accumulation rate usually requires counters with large solid angles. A correction is then necessary for the effect of the finite azimuthal acceptance since the maximum angle of noncoplanarity allowed by kinematics is typically of the same order as the azimuthal acceptance, and the cross section is predicted⁶⁻⁸ to decrease rapidly as this maximum angle is approached. Gottschalk, Schlaer, and Wang⁹ measured this dependence at 157 MeV, but no precise measurements are available at lower energy. The aim of the present work was to measure the noncoplanar dependence at about 65 MeV and, at the same time, to provide an accurate measurement of the coplanar cross section.

A 64.4-MeV proton beam from the Oak Ridge isochronous cyclotron was focused at the center of a scattering chamber filled with hydrogen at a pressure of 1 atm. The final-state protons were detected in fourfold coincidence by a pair of ΔE , E scintillation telescopes which were placed at a polar angle of 30°, one on each side of the beam (we use a spherical polar coordinate system as described by Halbert, Mason, and Northcliffe¹⁰). The ΔE counters, which served as the

rear defining apertures, were used to distinguish low-energy protons from the much larger flux of elastically scattered protons illuminating the telescopes. This arrangement is similar to one used for earlier measurements⁵ except for two important additions. Events involving protons scattered from the edges of the tantalum front slits were rejected by a veto signal from strips of plastic scintillators placed just behind the slit edges. This reduced the number of low-energy protons accepted on each side by about a factor of 3 and allowed us to increase the beam intensity by an order of magnitude. The present data were taken at 600 nA.

The other innovation was the use of segmented E counters to provide azimuthal position sensitivity. Four segments were used on each side with a center-to-center azimuthal separation between adjacent segments of 2.5°. The entire system was thus equivalent to 16 left-right detector pairs centered at angles of noncoplanarity Φ of 0, 5, 10, and 15°. The maximum noncoplanarity

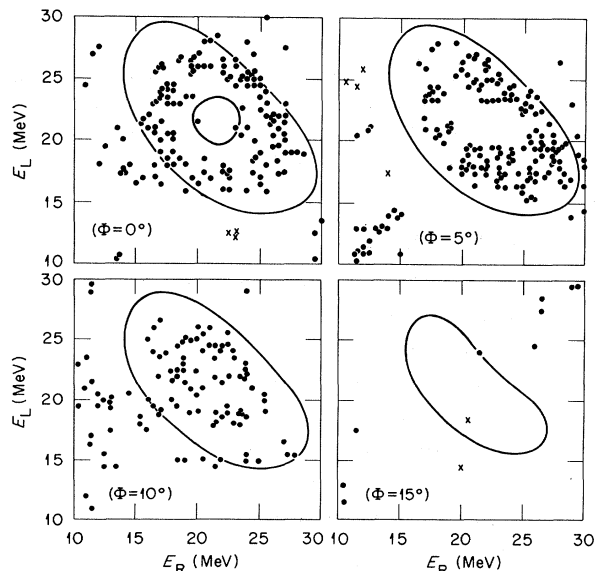


FIG. 1. Energy distribution of the data in the four noncoplanar groups with boundaries of kinematically allowed energies for each group.

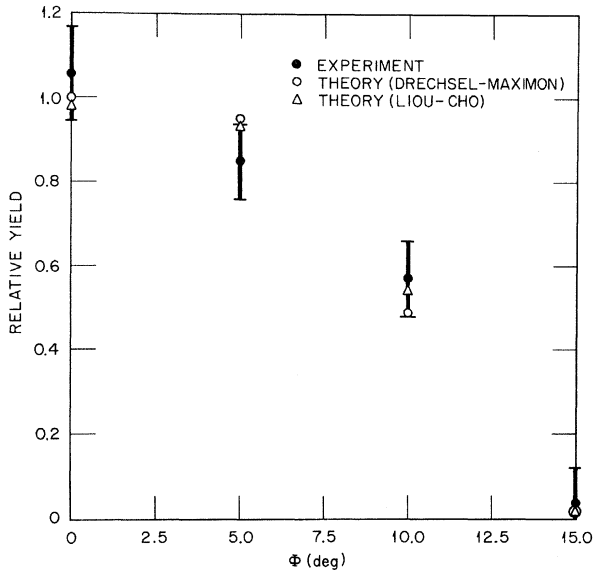


FIG. 2. Measured noncoplanar distribution compared with theoretical points based on calculations by Drechsel and Maximon (Ref. 6), and by Liou and Cho (Ref. 8).

allowed by kinematics is 12.2° . The overall noncoplanar resolution, including the effects of beam height and vertical divergence, was $\pm 2.5^\circ$. The polar acceptance was $\pm 2^\circ$.

As before,⁵ delayed coincidences were accumulated simultaneously with prompt coincidences as a means of evaluating random events. The prompt-to-delayed ratio in the proton-proton bremsstrahlung (PPB) region was 3.3. Events were sorted into four groups on the basis of noncoplanarity. Figure 1 shows the energy distribu-

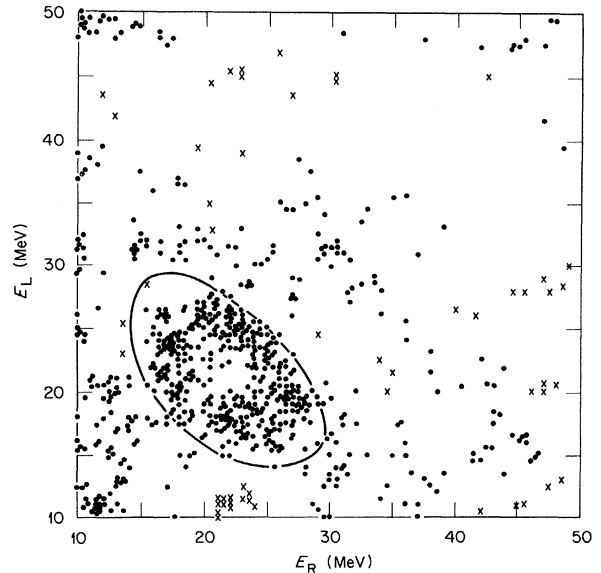


FIG. 3. Energy distribution of all events without regard to angle of noncoplanarity. The closed curve bounds the kinematically allowed energies as determined by the angular acceptance.

tion of events in each group after cancellation of neighboring prompt-delayed pairs. Each uncanceled delayed event is shown by an X. The closed curves bound the kinematically allowed energies as determined from the angular acceptance for each group. Finite energy resolution causes some of the bremsstrahlung events to appear outside these boundaries.

Figure 2 shows the measured noncoplanar distribution in comparison with theoretical predictions. The theoretical points were obtained by

Table I. Comparison of the present result ($2.32 \pm 0.20 \mu\text{b}/\text{sr}^2$) with theoretical coplanar proton-proton bremsstrahlung cross sections for $\theta_L = \theta_R = 30^\circ$, incident proton energy = 64.4 MeV. The theoretical cross sections were estimated from published graphs and numbers for 61.7 and 65 MeV.

Basis of Calculation	$d^2\sigma/d\Omega_L d\Omega_R$ ($\mu\text{b}/\text{sr}^2$)	$d^2\sigma_{\text{exp}}/d^2\sigma_{\text{th}}$	Ref.
Hamada-Johnston potential + Coulomb correction	2.79	$.83 \pm .07$	13
Hamada-Johnston potential + relativistic spin correction	2.86	$.81 \pm .07$	8
Hamada-Johnston potential	2.93	$.79 \pm .07$	14
Hamada-Johnston potential	3.00	$.77 \pm .07$	15
Hamada-Johnston potential + rescattering	3.15	$.74 \pm .06$	12
Bryan-Scott III potential	3.20	$.72 \pm .06$	15
Bryan-Scott III potential	3.40	$.68 \pm .06$	13
Bryan-Scott III potential	3.50	$.66 \pm .06$	12
Reid soft core, or Green-Ueda I potential	3.20	$.72 \pm .06$	15
Tabakin potential (HJ for $L > 2$)	2.99	$.78 \pm .07$	13
One-boson exchange model	2.76	$.84 \pm .07$	7

folding the theoretical distribution of Drechsel and Maximon⁶ and of Liou and Cho⁸ with the experimental noncoplanar resolution. The Drechsel-Maximon prediction⁶ was based on the Hamada-Johnston potential; an almost identical result was obtained using the Reid potential. The Liou-Cho calculation⁸ was also based on the Hamada-Johnston potential, but a relativistic spin correction was included. The agreement with either of these calculated distributions is good. This agreement validates the use of the Drechsel-Maximon distribution for correcting experimental data for noncoplanarity in this energy region.^{5,11}

Figure 3 shows the energy distribution of events plotted without regard to angle of noncoplanarity. A total of 370 ± 26 events is attributed to PPB. The Drechsel-Maximon noncoplanar distribution was used to calculate a multiplicative factor of 1.384 to correct for the effect of noncoplanar acceptance. This leads to a coplanar cross section

$$\frac{d^2\sigma_{\text{exp}}}{d\Omega_L d\Omega_R} = 2.32 \pm 0.20 \text{ } \mu\text{b/sr}^2.$$

If the correction factor is based on the Liou-Cho distribution, the coplanar cross section is 3.3% lower. Our result confirms earlier Oak Ridge measurements, which are generally below currently accepted predictions.

Table I shows our coplanar cross section in comparison with theoretical 30° - 30° cross sections estimated for 64.4 MeV from calculations at 61.7 and 65 MeV. The discrepancy between

this measurement and all these theoretical calculations is outside the experimental error.

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Borel Summability of the Ground-State Energy in Spatially Cutoff $(\varphi^4)_2$ †

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We show that the ground-state energy for a Hamiltonian $H_0 + \int g(x) : \varphi^4(x) : dx$ ($g \in L^1 \cap L^2$; $g \geq 0$; H_0 = free Hamiltonian for Bose particle of mass m in space-time of two dimensions) may be determined from the Feynman perturbation series by the method of Borel summability. This demonstrates that summability methods can be applicable to divergent series in systems with a continuous infinity of degrees of freedom.

We prove here that for a spatially cutoff $(\varphi^4)_2$ theory¹ the ground-state energy can be recovered from the (Feynman) perturbation series by the method of Borel summability. This represents a merging of two trends in mathematical physics. In the first place, considerable effort has gone into the study of the analytic properties of the levels of the anharmonic oscillator² and the proof that in various cases, one can recover the levels from the perturbation series by the Padé³ or Borel^{4,5} method despite the