Strain-Induced Electric Fields in Superconducting Aluminum

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Electromagnetic waves are excited in Al-Pb superconducting tunnel junctions by compressional 9.3-GHz acoustic waves when the acoustic wave front makes a small angle with the plane of the tunnel junction. The electromagnetic waves are induced by the electrostatic field in the superconducting Al film which is due to the compressional strain gradient in the Al. This result provides experimental confirmation of the theories of Dessler et al. and Herring developed for the case of gravitation-induced strains.

Recently considerable attention has been directed at the question of the existence of an electric field inside a metal due to a nonuniform strain. This interest arose in connection with the attempt of Witteborn and Fairbank[1] to measure the gravitational force acting on an electron. It was pointed out by Dessler et al.[2] and Herring[3] that the nonuniform strain induced by gravity in the copper cylinder used in the experiment to shield the electrons gives rise to an electric field within the metal. The force on the electron outside the metal due to this field is in the direction of the gravitational force and about four orders of magnitude larger. No such force was observed in the gravity experiment. To demonstrate directly the presence of an electric field outside a metal due to a nonuniform strain, Beams[4] measured the radial electrical potential gradient across a Duraluminum rotor spinning at high speed. He found reasonable agreement with the theoretical predictions, but "no clear-cut theoretical interpretation of the experiment was possible." More recently Craig[5] measured semiquantitatively the effect of uniform stress on the contact potential of several metals. The observed shifts in contact potentials were found to be in order-of-magnitude agreement with the theories that apply for the nonuniform gravitational strain,[2,3] provided one can assume that the gravitational stress is equivalent to a uniform hydraulic stress.

It was found some years ago[6,7] that the tunneling current in a superconducting Al-Pb tunnel junction is strongly affected by compressional strain waves (at microwave frequencies). In the case of a uniform strain, an extra tunneling current is induced through the relative motion of the Fermi levels with strain in the two metals. A simple modification of our previous experiment[7,8] enables us to produce a spatially varying compressional strain in the aluminum film and to determine the magnitude of the strain-induced electric field.

The aluminum films[9] used in the present experiment had a thickness $d \simeq 400$ Å, an electronic mean free path of $l \simeq 60$ Å, and a transition temperature $T_c \simeq 2.3$°K. The lead films were about 7000 Å thick with a bulk mean free path l of several microns. The thickness of the insulating aluminum oxide layer, w, was estimated to be about 30 Å. The junctions were 0.1 cm wide, had a length $L = 0.27$ cm, and were deposited on an X-cut quartz rod 2 cm long and 0.3 cm in diameter. The end faces of the rod were optically polished and parallel to 4 sec. The rod was inserted into the re-entrant microwave cavity shown schematically in Fig. 1(a) and was clamped at a position close to its entrance into the cavity. The assembly was immersed in liquid helium and the temperature could be varied by reducing the ambient pressure. The cavity was excited at its resonant frequency by 9.3-GHz microwave pulses of 1 μsec duration. A microwave pulse produced a compressional sound pulse of the same frequency which propagated along the length of the quartz rod. The sound pulse was reflected both at the tunnel-junction surface and at the cavity end of the rod and bounced back and forth many times thus producing a train of sound echoes in the rod. A detailed description of the experiment including the microwave setup, sample preparation, and measuring techniques is given in Ref. 8.

The main modification in the present experimental setup is the incorporation of the bending arrangement shown in Fig. 1(a). By means of three spring-loaded Teflon pins situated 120° apart, a bending moment was applied to the

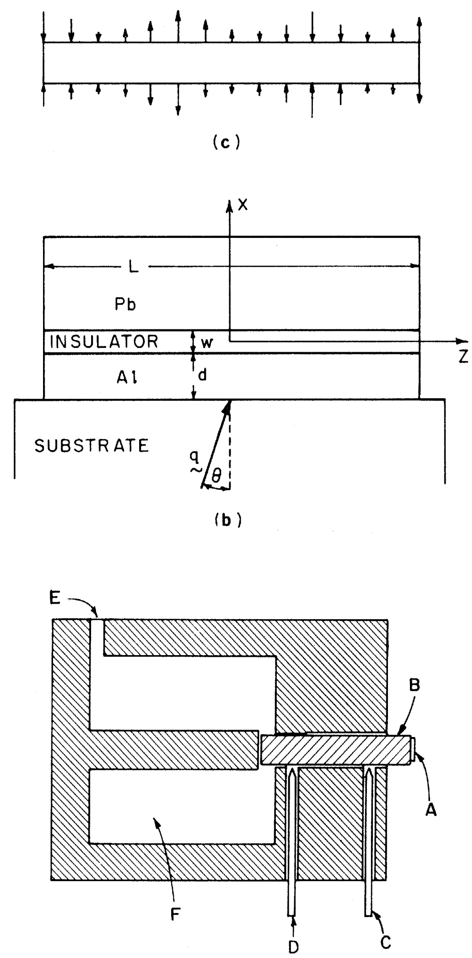

FIG. 1. (a) Schematic of microwave cavity and mechanism for bending the quartz rod: A, tunnel junction; B, quartz crystal; C, bending pin; D, clamping pin; E, coupling hole; and F, microwave cavity. (b) Schematic of tunnel junction. (c) Schematic of the strain distribution in the Al film.

quartz rod perpendicular to its axis. The purpose of this bending moment was to tilt the junction end of the rod by a small angle θ_1 and thus introduce an angle θ between the sound wave vector $\vec{q}$ and the normal to the junction surface. (Typically θ_1 was of the order of 15 sec.) The junction is shown schematically in Fig. 1(b). The non-normal incidence of the sound wave results in a strain wave in the z direction with a phase velocity $v_z = v/\sin\theta$, where v is the sound velocity. In the aluminum film the strain s is given by

$$s = s_0 \exp i(2\pi\nu t - q_z z), \tag{1}$$

where s_0 is the strain amplitude, ν is the sound frequency, t is the time, and $q_z = q \sin\theta \simeq q\theta$. The x variation of s in the aluminum film is neglected because $qd < 1$. (The value of q in Al was taken[10] as 9×10^4 cm^{-1}.) The z variation of the strain in the aluminum film is illustrated in Fig. 1(c). After each reflection of the sound beam the angle θ increases by $2\theta_1$ and thus for the nth echo q_z is given by

$$q_z = q\theta_1(2n-1). \tag{2}$$

This provides a built-in method of varying q_z.

The nonuniform strain gives rise to an electric field in the aluminum,[11] $-\partial\varphi/\partial z$, derivable from an electrostatic potential φ proportional to the strain gradient:

$$\partial\varphi/\partial z = (C/e)\partial s/\partial z, \tag{3}$$

where e is the absolute value of the electronic charge and C is defined as the deformation potential. For a free-electron gas, $C = -2E_F/3$ (-8 eV for Al), where E_F is the Fermi energy. The condition of locality $q\xi \ll 1$, where ξ is the Pippard coherence length,[12] required for Eq. (3) is satisfied in our aluminum films because of the short mean free path.

The electrostatic wave φ in the aluminum excites electromagnetic fields $\vec{E}$ and $\vec{H}$ in the strip line composed of the two superconductors (Al and Pb) and the insulator in between. When the phase velocity, $v/\sin\theta$, of the electrostatic wave φ is close to the characteristic electromagnetic strip-line wave velocity, resonant coupling occurs because of the traveling-wave interaction and the electromagnetic field builds up to a large value. The calculation, outlined in this Letter, shows that in that case the amplitude of the induced microwave voltage across the junction is approximately $QCs_0/2e$, where Q is the quality factor of the strip line. This voltage is rectified as a result of the nonlinear I-V characteristic of the tunnel junction and is determined from the extra dc tunneling current it produces.

The present calculation of the electromagnetic field in the junction departs from Swihart's[13] in that we include the field $-\partial\varphi/\partial z$ in the boundary condition at the aluminum-insulator interface. The general solution consists of the two characteristic waves in the z direction with wave numbers $\pm\gamma_p$, and a driven wave having the wave number q_z of the strain wave. We take into account the finite length L of the strip line by requiring the vanishing of the magnetic field at the boundaries $z = \pm L/2$, the usual boundary value for an open-ended strip line. From this latter condition we determine the amplitudes of the characteristic waves. The detailed calculation is straightforward but too lengthy to be presented here and will be published separately.[14] The solution for E_x in the insulator is given by

$$E_x = \frac{Cs_0 q_z^2}{we(q_z^2-\gamma_p^2)}\Big[\exp(-iq_z z) - \frac{\gamma_p}{q_z \sin\gamma_p L}\{\sin[(q_z-\gamma_p)L/2]\exp(i\gamma_p z) + \sin[(q_z+\gamma_p)L/2]\exp(-\gamma_p z)\}\Big], \quad (4)$$

where we have dropped the factor $\exp(2\pi i\nu t)$. It is convenient to express γ_p in the form

$$\gamma_p = \alpha_p[1-(i/Q)], \quad (5)$$

where α_p is the characteristic strip-line wave number given by[15]

$$\alpha_p = (\epsilon\nu/2\sigma_2 wd)^{1/2}, \quad (6a)$$

and

$$Q = 2\sigma_2/\sigma_1. \quad (6b)$$

Here σ_1 and $-\sigma_2$ are the real and the imaginary parts, respectively, of the complex conductivity, $\sigma = \sigma_1 - i\sigma_2$, of the aluminum and ϵ is the dielectric constant of the insulator.

Equation (4) exhibits a resonant peak when $q_z^2 = \mathrm{Re}(\gamma_p^2)$. When this condition is fulfilled the phase velocity of the electrostatic wave φ is close to the characteristic electromagnetic strip-line wave velocity.

A microwave field E_x across a superconducting tunnel junction induces an extra dc tunneling current δI_s. In the limit of small E_x [i.e., $(wE_x e/2h\nu)^2 \ll 1$] the extra tunneling current δI_s is given by

$$\delta I_s = (we/2h\nu)^2 f\langle|E_x^2\rangle, \quad (7)$$

where the coefficient f depends on the dc current-voltage (I-V) characteristic of the tunnel junction and on the microwave frequency ν:

$$f = I(V+h\nu) + I(V-h\nu) - 2I(V). \quad (8)$$

Equation (7) differs from the expression which applies for a uniform electric field[16] in that the uniform field is replaced here by $\langle|E_x^2|\rangle^{1/2}$, where the skew brackets indicate an average taken over the length of the strip line. We have neglected the contribution to δI_s due to the change in the electrochemical potential $\langle Cs\rangle$. This term should be added to E_x in Eq. (7) but is negligible for a rapidly varying strain ($q_z L \gg 1$), the case considered here. In the case of uniform strain ($q_z L \ll 1$), E_x is negligible and the major contribution comes from the term[6,8] $\langle Cs\rangle = Cs_0$.

The nonuniform strain in the lead also contributes an extra current. However, in the case of our lead films $q\xi > 1$, and thus the interaction of electrons with the strain wave is described by absorption and emission of phonons. This case has been treated theoretically by Cohen[17] who finds an extra current δI_q with a similar dependence on the bias voltage and on the strain amplitude as δI_s, but independent of the angle θ. Thus the total extra current δI is given by

$$\delta I = \delta I_s + \delta I_q. \quad (9)$$

The extra current δI was measured over the temperature range 1.39 to 2.03°K. In Fig. 2 we show typical experimental data for δI at two temperatures as a function of the echo number n. In agreement with theory the curves exhibit reso-

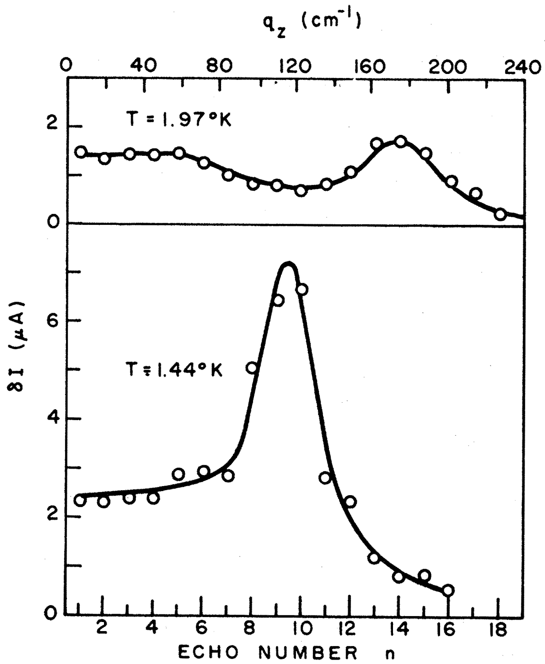

FIG. 2. The extra tunneling current δI in an Al-Pb junction as a function of echo number n (lower scale) and q_z (upper scale) taken at two different temperatures. The bending moment and microwave power were held constant. The normal junction resistance was 5 Ω, and T_c of the Al film was 2.25°K. The open circles are experimental and the curves were calculated from Eq. (9). The dc bias voltage V across the junction was set for maximum in δI.

Table I. Experimental and calculated values of parameters.

T (°K)	Experimental α_p (cm^{-1})	Experimental Q	Calculated α_p (cm^{-1})	Calculated Q	Experimental δI_q (μA)	Experimental $\lvert Cs_0 \rvert$ (μeV)
1.97	173	9	185	14	1.3	18
1.44	112	9	120	50	2.4	18

nant peaks due to the contribution of δI_s [see Eqs. (4) and (7)] and flat regions at low echo numbers due to the contribution of δI_q. The theoretical curves were calculated from Eq. (9) and are plotted against q_z.

In fitting the theoretical expression for δI to the experimental data, attenuation of the sound beam was taken into account. The attenuation is primarily due to the fact that if the sound beam is sufficiently deflected, part of it is not going to strike the junction area. It can be shown readily that for a bending moment along the y axis, the displacement along the z direction of the nth echo beam is given by $2n(n-1)d_0\theta_1$, where d_0 is the length of the quartz rod. As the center of the sound beam is displaced along the z direction one expects an attenuation in δI once the trailing edge of the beam starts to recede from the junction area. For the present geometry this occurs after the center of the beam is deflected by approximately 0.015 cm, and the echo at which this occurs is $n \approx 6$. After the beam is deflected by an additional 0.27 cm no sound energy is incident on the junction surface and we expect a cutoff in δI. To allow for this attenuation, δI in Eq. (9) was multiplied by an attenuation factor given by an exponential decrease, $\exp[b(6-n)]$, for $n > 6$, and by unity for smaller n. The coefficient b is a temperature-independent parameter.

The values of the parameters used in fitting the theoretical curves at the two temperatures are given in Table I. The calculated values of α_p and Q, also shown in the table, were derived from Eqs. (6) using for the dielectric constant the value[18] $\epsilon = 3$; for σ_2/σ_n and σ_2/σ_1 the theory of Mattis and Bardeen[19]; for σ_n the measured normal conductivity, 4×10^4 $(\Omega\ \mathrm{cm})^{-1}$; and for w and d the values given earlier. The small discrepancy between the calculated and experimental values of α_p is ascribed to the uncertainty in the thickness of the oxide layer, w, and the dielectric constant ϵ. The temperature dependence of the experimental α_p is in excellent agreement with theory over the whole temperature range measured. The experimental value of Q was found to be independent of temperature and smaller than the calculated one, indicating an additional contribution to the imaginary part of γ_p. The extra current δI_q due to the lead film was found to be in an order-of-magnitude agreement with that calculated for Pb-Pb junctions.[17] The value of the parameter b was found to be 0.21 and $|Cs_0| = 18$ μeV over the entire temperature range.

The possibility, in the present experiment, of adjusting the spatial variation of the strain in the films provided a means of separating the phonon-assisted tunneling current δI_q due to the nonlocal superconductor (Pb) from the strain-induced current δI_s due to the local superconductor (Al). The good agreement between the experimental and theoretical results for δI_s provide firm evidence for the existence of an electric field $-\partial\varphi/\partial z$ in the superconducting Al film proportional to the strain gradient $\partial s/\partial z$. This result provides an experimental confirmation of the theories of Dessler et al.,[2] Herring,[3] and Harrison[11] developed for the case of gravitation-induced strains. Using the experimentally determined values of $|Cs_0| = 18$ μeV given in Table I and the strain amplitude[20] $s_0 = (1-2)\times10^{-6}$ we obtain for the absolute value of the deformation potential in aluminum, $C = 9-18$ eV. This value is within experimental error of that deduced from Craig's data. It is noteworthy that the experiments of Craig and Beams were performed at room temperature and at strains orders of magnitude larger than ours.

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Electron Correlation in the Chemisorption of 5*d* Atoms on Tungsten

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The chemisorption energy of an atom in the 5*d* series on a tungsten substrate is calculated on the basis of a two-center Hubbard model, for which the ground-state energy may be obtained exactly. The experimental data can be fitted with a choice of parameters consistent with independent estimates.

Recently several attempts[1-3] at a theoretical understanding of the chemisorption energy ΔE of atoms in the 5*d* series on tungsten have been made; ΔE is here defined as the energy change on removing the atom from the surface to infinity. Whether the calculations are based on an explicit model Hamiltonian[3] or on the Friedel sum rule[1] the problem is treated as one of self-consistency within the Hartree-Fock (HF) approximation. The model is conceptually like that of Friedel[4] in that the broadened virtual 5*d* level of the adatom is found to drop through the Fermi level on moving to the right across the 5*d* series, with the effect of maximizing ΔE when half-filled, a situation apparently observed experimentally[5,6] at Re. These models predict a broad, parabolic maximum in ΔE as a function of the number N of electrons in the adatom d orbitals, in contrast to the triangular maximum observed.[5,6]

The effects of correlation on the total energy of the system should be greatest in the half-filled case owing to the greater number of configurations available, an effect to which this discrepancy might be attributed. To investigate this possibility within a mathematically tractable framework we here consider the following two-atom Hubbard[7] Hamiltonian:

$$H = \epsilon_a \sum_i n_{ai} + \epsilon_b \sum_i n_{bi} + \sum_i \beta_i [a_i^\dagger b_i + b_i^\dagger a_i] + \tfrac{1}{2} U_a \sum_{i,j} n_{ai} n_{aj} (1-\delta_{ij}) + \tfrac{1}{2} U_b \sum_{i,j} n_{bi} n_{bj} (1-\delta_{ij}). \quad (1)$$

Here $a_i^\dagger, b_i^\dagger$ are creation operators, $n_{ai} = a_i^\dagger a_i$, $n_{bi} = b_i^\dagger b_i$, and the two atoms a and b represent the substrate and the adsorbate atom, respectively. Neglecting s and p electrons, the index $i = 1 \cdots 10$ refers to atomic d-spin-orbitals $|ai\rangle$ and $|bi\rangle$. The d-orbital self-energies ϵ_a and ϵ_b are calculated, on the simplifying assumption[3] that the ionization potentials $I_a = \frac{1}{2}U_a$ and $I_b = \frac{1}{2}U_b$, as $\epsilon_a = -4.5U_a$, $\epsilon_b = -U_b(N-\frac{1}{2})$ taking $N=5$ for W. The third term in H describes hopping processes between the two atoms governed by the matrix element $\beta_i = \langle a_i | H | b_i \rangle$. The remaining terms describe a Hubbard-like intra-atomic electron-electron interaction in which only the largest Coulomb integrals F_0, in the notation of Condon and Shortley,[8] are retained. We note that the operators $N_i = n_{ai} + n_{bi}$, with eigenvalues ν_i, commute with H, as also do the set of permutation operators P_r (with eigenvalues p_r) operating on the spin-orbital indices i, if we make the additional assumption that all $\beta_i = \beta$. Evidently,

$$\Delta E = -\langle H \rangle - \tfrac{1}{2} U_b N^2 - 12.5 U_a, \quad (2)$$