right-hand side by 20%, indicating marginal instability.

In summary, a theory is proposed which is able to predict the wave parameters and the stabilization by shear of resistive drift waves. Resistive drift waves in a hydrogen plasma immersed in the sheared magnetic field of the spherator are experimentally observed to be stabilized in accordance with the derived stability criterion, and to possess the predicted frequencies and wavelengths in a marginally unstable condition.

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Anomalous Resistivity in a Steady-State, Current-Carrying Discharge-Tube Plasma

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The dc conductivity in a turbulent discharge-tube plasma has been measured as a function of dc electric field over the range 0.06 to 300 V/cm. Below approximately 60 V/cm the conductivity can be interpreted quantitatively in terms of the ion-acoustic wave instability. Above 60 V/cm the conductivity is approximately constant and about an order of magnitude lower than that calculated by Buneman for the two-stream instability.

Observations of anomalously low electrical conductivity in turbulent plasmas have been reported.[1-4] The conductivity observed has been explained by the presence of electrostatic instabilities such as ion-acoustic[5] or two-stream[6] instabilities. Collective electric fields associated with these instabilities provide an effective high collision rate for electrons which, experimentally, appears as anomalously low electrical conductivity.

We have measured the electrical conductivity of a plasma in a dc discharge tube (Fig. 1). The central part was constricted in order to increase the ratio of electron drift velocity to thermal velocity in this region. That this works is evidenced by the fact that both the low-frequency (ion-acoustic wave) and high-frequency (electron plasma oscillation) instabilities appeared only in the constricted region. The discharge was operated using either Ar or He at pressures typically less that 5×10^{-4} Torr. The maximum dc current used was approximately 1 A.

To generate the high electron drift velocities required to produce the instabilities of interest, we kept the current density $j=n_eev$ constant, while reducing n_e. We accomplished this by reducing the gas pressure in the system, thereby reducing the rate of plasma production. This re-

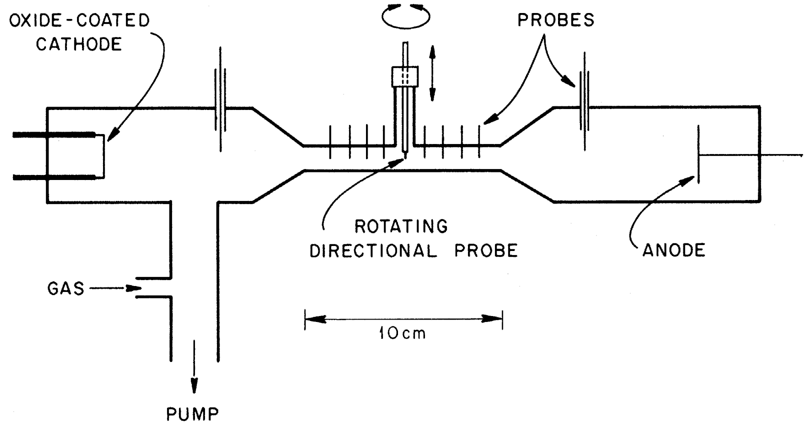

FIG. 1. Schematic of the discharge tube. The diameter of the constricted region is either 5 or 12 mm as compared with a diameter of 50 mm for the end sections. The rotating directional probe has a collecting surface of dimensions 1×1 mm^2.

gion of low gas pressure just before the discharge extinguishes has been known as a region of high voltage drop,[7] but has not been extensively investigated for high-frequency phenomena.

The electric field along the constricted plasma was observed by measuring the floating potential on each of several probes spaced 0.5 cm apart. The observed electric fields ranged from 0.06 to 300 V/cm. The electron density obtained from the Langmuir-probe method at low values of turbulence varied between 3×10^9 and 10^{11} cm^{-3}. The observed electron plasma frequency, as observed on a probe inserted into the plasma, was also used for density determination, especially at high values of turbulence and high electric fields ($E \geq 10$ V/cm). A directional rotating probe located at the center of the constricted region was used to obtain the ratio of electron drift velocity to thermal velocity.

An example of our conductivity measurements as a function of electric field is shown in Fig. 2, for an argon discharge at an electron density $n = 3 \times 10^{10}$ cm^{-3}. Also shown are the electron drift and thermal velocities obtained by means of the rotating probe. The electron temperature was also measured spectroscopically.[8]

Ion-acoustic noise has been measured as a function of electric field. The spectrum is rather continuous, and decreases monotonically in frequency. If we plot the peak intensity $|\varphi_k|^2$ of the

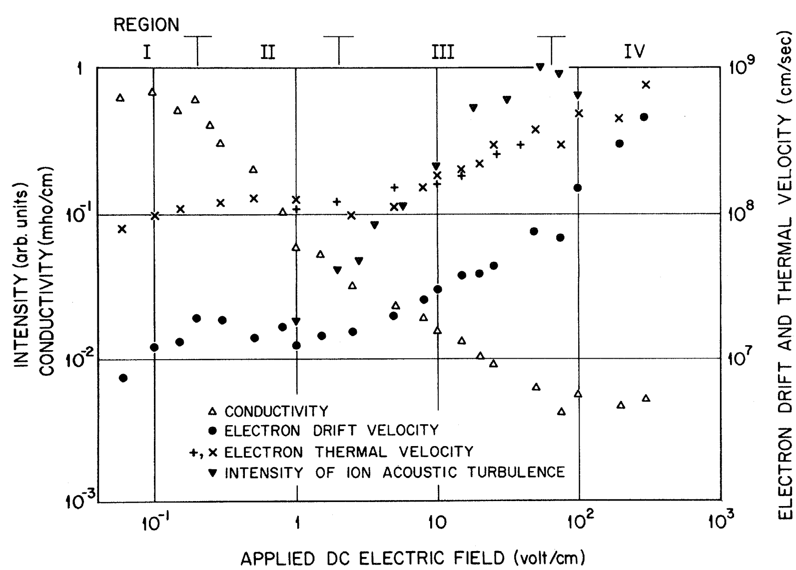

FIG. 2. Conductivity, intensity of ion-acoustic turbulence (amplitude squared), and electron drift and thermal velocities (plus signs, spectroscopic; crosses, probe) versus dc electric field. The gas used was argon. The plasma density was 3×10^{10} cm^{-3}.

noise spectrum versus the applied electric field E, as shown in Fig. 2, we find that it increases linearly in the range $1 \leq E \leq 50$ V/cm. Above 50 V/cm, the low-frequency noise decreases, while high-frequency noise (near ω_{pe}) increases. Below 1 V/cm, noise measurements are not complete. As seen from Fig. 2, the conductivity σ has four regions: Region I,

$$\sigma = \text{const}, \quad E \lesssim 0.2 \text{ V/cm};$$

Region II,

$$\sigma \propto 1/E, \quad 0.2 \lesssim E \lesssim 2 \text{ V/cm};$$

Region III,

$$\sigma \propto 1/E^{1/2}, \quad 2 \lesssim E \lesssim 60 \text{ V/cm};$$

Region IV,

$$\sigma = \text{const}, \quad 60 \text{ V/cm} \lesssim E.$$

It is also noted that the electron thermal velocity increases as $E^{1/2}$ in Region III and Region IV, indicating that electron heating occurs in these regions.

The highest conductivity in Fig. 2 (Region I) is already a factor of 20 lower than the value due to electron-neutral collisions. This region is characterized by the presence of low-frequency noise below 3 MHz. We conjecture that ion-acoustic waves are generated by the electron drift,[9,10] since the ratio of electron drift velocity v_d to thermal velocity β_e is about 0.07, which is large enough for generation of the instability if $T_e/T_i > 8$. The observed cutoff frequency is about $0.5\omega_{pi}$, giving a cutoff wave number of about 0.6 k_D, where k_D is the electron Debye wave number. Then we find $T_e/T_i \simeq 12$, or $T_i \simeq 0.08$ eV.

We have estimated the electrical conductivity limited by the ion-acoustic instability in low electric field where electron "runaway" may be neglected. Quasilinear theory has been employed in which a finite ion-ion collision frequency is assumed. We find an effective collision frequency which is lower than that given by Sagdeev and Galeev,[11] but is higher than that obtained by Sizonenko and Stepanov.[12] The electrical conductivity is given by

$$\sigma = \frac{ne^2}{m}\frac{1}{\nu_{\text{eff}}} \simeq \frac{\omega_{pe}}{4\pi}\left(\frac{MT_e}{mT_i}\right)^{1/2}$$

and the "critical" field, above which the conductivity begins to decrease,

$$E_c = \frac{m}{e}\left(\frac{kT_i}{M}\right)^{1/2}\omega_{pe}.$$

Using $\omega_{pe}/2\pi = 1.7$ GHz, $T_e/T_i \simeq 12$, and $M/m = 40 \times 1840$ (Ar), we expect $\sigma = 0.9$ mho/cm and $E_c = 0.15$ V/cm, compared with the experimental values of 0.6 mho/cm and 0.1 V/cm. We also note that the conductivity and critical field obtained above give the same order of magnitude as those reported by Demidov, Elagin, and Fan.[13]

For the second region, in which the conductivity decreases as E^{-1}, we may apply the theory[5,14] developed for the ion-acoustic instability for the case of the electric field higher than the critical field. The electron drift velocity is "frozen" at a value on the order of the critical drift velocity above which the ion-acoustic wave becomes unstable. The "frozen" constant drift velocity of electrons is about $0.1\beta_e$, which is on the right order of the critical velocity.

We notice that the results obtained in Region II are very similar to what was reported by Bovrovskii et al.,[15] on the anomalous resistance in TM-3 Tokamak. They observed the relationship

$$\sigma \propto 1/E$$

which, in our opinion, can be attributed to the onset of the ion-acoustic instability.

The third region ($\sigma \propto E^{-1/2}$) is accompanied by an increase in the electron temperature in direct proportion to the electric field. It is possible that the ion-acoustic instability is still responsible for the decreasing conductivity since the electron drift velocity stays relatively low ($0.1\beta_e$). We notice that the critical drift velocity v_c is proportional to[9]

$$T_e^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left[-\frac{1}{2}\left(\frac{T_e}{T_i}+3\right)\right].$$

Then, if we assume that the ion temperature also increases with the electric field so that T_e/T_i is independent of E, we obtain

$$\sigma \propto 1/E^{1/2},$$

as observed in the present experiment. The ion temperature has not been measured in detail. However, requiring that the energy gain of ions from the electric field equals the energy loss due to ambipolar radial diffusion, we have estimated the critical electric field at which the transition from Region II to Region III occurs. This gives $E \simeq 3$ V/cm, while we observe that the transition occurs at $E \simeq 1.5$ V/cm. Above this field, ions are expected to increase their temperature.

Region IV is accompanied by strong rf oscillation. The peak frequency observed by a probe inside the plasma is about 1.6 GHz, is roughly proportional to the square root of the plasma current, and is of the order of the electron plasma frequency calculated from the electron density obtained by Langmuir probes at low electric fields. The highest frequency observed was about 3 GHz, corresponding to an electron density of 10^{11} cm^{-3}.

The saturated conductivity is on the order of[6]

$$\sigma_B = (\omega_{pe}/4\pi)(M/m)^{1/3},$$

which is a factor of 2π lower than the value reported by Hamberger and Friedman.[3] Since the observed conductivity is rather insensitive to the electron thermal velocity and is not much affected by the variation of the plasma radius, we presume that it is mainly due to the turbulent energy absorption by the plasma, not by the wall.

The maximum, steady-state electron drift velocity in Region IV is observed to be on the order of the electron thermal velocity. Correlation studies of the electron plasma waves suggest that the resonance condition

$$\omega_{pe} \simeq kv_d$$

is satisfied, although the electron thermal velocity is already on the same order as the drift velocity. We have not observed, and do not use in our computations, the assumption that a double-layer sheath on the cathode side of the constriction accelerates an electron beam into a cold electron background in the constriction. Such an electron beam producing oscillations at ω_{pe} has been observed in many past experiments in constricted tubes. Verifying its presence, or eliminating the possibility of its presence (possibly by kinking the constricted tube, for example), is important for future experiments.

In earlier experiments, the rf signals observed external to the tube appeared at a frequency of $\omega \simeq \frac{1}{6}\omega_{pe}$. Correlator studies showed that this mode of oscillation corresponded to a half-wavelength standing wave extending along the length of the constricted region of the tube. This is a low-frequency form of surface wave, the general form of which was studied by Trivelpiece and Gould.[16]

An upper limit of the ion temperature has been established spectroscopically: $T_i \lesssim 1$ eV. This rather ineffective ion heating can be attributed to the fact that ions are lost to the wall rapidly.

We estimate the intensity of rf oscillation at the electron plasma frequency to be on the order of 100 V/cm or larger. That is, the turbulent high-frequency electric fields are on the order of the applied dc electric field. The density perturbation $|\Delta n/n|$ is calculated, using the value of the rf electric field observed, to be about 0.3 at our highest level of turbulence—presumably the maximum that could be produced.

A surprising feature of this experiment is that our conductivity calculations, made for an idealized, infinite plasma, agree reasonably well with the experiments made on an inhomogeneous, bounded gas-discharge column. The agreement may be fortuitous. However, the actual operating conditions may really approximate an ideal case. For example, the wavelengths involved are small compared with the plasma column size. More work needs to be done on this point.

In conclusion, we have measured the electrical conductivity of a steady-state plasma produced in a very simple discharge tube. The degree of turbulence can be varied over a wide range. The observed anomalously low conductivity has been interpreted quantitatively in terms of the ion-acoustic wave instability. For $E \gtrsim 60$ V/cm, however, the conductivity more nearly agrees with the two-stream instability result reported by Buneman.[6]

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Evolution and Large-Electric-Field Suppression of the Transverse Kelvin-Helmholtz Instability

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The interior of a plasma column is subjected to shear of arbitrary magnitude in the $\vec{E}\times\vec{B}$ rotation. The resultant Kelvin-Helmholtz instability undergoes an evolution in mode structure as the radial electric field is increased, and is suppressed when the Doppler-shifted wave frequency equals 0.5 to 0.65 times the ion-cyclotron frequency. Cyclotron waves appear concomitant with this suppression; these have properties in agreement with linear fluid theory.

A cylindrical plasma column in a uniform axial magnetic field and a nonuniform radial electric field is unstable to Kelvin-Helmholtz waves if the shear in the $\vec{E}\times\vec{B}$ rotation is sufficiently large. For example, the edge oscillation of Q machines is essentially a transverse Kelvin-Helmholtz instability.[1,2] In order to study this instability in detail, we have set up a velocity shear layer of externally controlled magnitude. In this Letter we describe how the observed instability evolves as the radial electric field is increased from zero to large values.

The significant experimental results, at large electric field, are the suppression of the low-frequency instability when the Doppler-shifted wave frequency is slightly greater than one-half the ion-cyclotron frequency ω_c and, concomitant with this suppression, the appearance of coherent, higher-frequency oscillations near ω_c. A radial wave equation valid for instabilities of arbitrary frequency in a nonuniformly rotating plasma cylinder is presented. This equation has two sets of solutions: One set corresponds to the observed high-frequency modes near ω_c; the other set consists of the low-frequency Kelvin-Helmholtz modes. The suppression of the low-frequency waves occurs at the resonant condition for nonlinear Landau damping.[3]

The experiment was performed in a single-ended Q-machine plasma with a specially designed segmented hot end plate. This consisted essentially of two concentric tantalum sections, insulated from each other and separated by an annular nonemitting region. The entire end-plate assembly was heated by electron bombardment. Controlled $E\hat{r}\times B\hat{z}$ velocities in the annular region were established by applying a static voltage between the concentric plate sections. Typical radial profiles of the equilibrium floating potential are shown in Fig. 1(a). These potential distributions can be represented by $\varphi=\varphi_0\tanh[A\times(r-r_0)]$; here, $A=7.0\ \text{cm}^{-1}$ and $r_0=1.1$ cm. The corresponding radial variation of $\vec{E}\times\vec{B}$ rotation frequency is shown in Fig. 1(b), where

$$\omega_E(r)=-mcE(r)/rB, \tag{1}$$

and m is the mode number. We define L as the width of the velocity shear layer.

When the applied voltage is sufficiently large (the value depends on B, $k_\parallel$, n, and dn/dr), oscillations localized in the gap region and traveling in the $\vec{E}\times\vec{B}$ direction are observed. In these experiments, using a potassium plasma, there are as many as seven ion gyroradii in L,