Effect of ϵ ($J^P = 0^+, I = 0$) Resonance on $e^+e^- \to \pi^+\pi^-\gamma$†

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The product of the coupling constants for $\epsilon \to \pi\pi$ and $\epsilon \to 2\gamma$ is estimated from a superconvergent sum rule obtained from the helicity-flip amplitude of pion-Compton scattering. It is then used to show that the contribution of ϵ (even C) to the process $e^+e^- \to \pi^+\pi^-\gamma$ (to order α^3) is about two orders of magnitude larger than that when the pions are in odd-C states.

Recently it has been suggested[1] that the reaction (to order α^3)

$$e^+ + e^- \to \pi^+ + \pi^- + \gamma \tag{1}$$

can be used to study the di-pion system in states of even charge conjugation. In particular, it would provide reliable information about the existence of the ϵ ($J^P = 0^+, I = 0$) meson resonance at 760 MeV. The diagram contributing to this process is shown in Fig. 1(a). However, to the same order α^3, the diagrams of Fig. 1(b) would also contribute where the two pions have C odd. The differential cross section $d\sigma_{+-}$ for $e^+e^- \to \pi^+\pi^-\gamma$ is proportional to $|A+B|^2$. Under the exchange of two pions B changes sign while A does not; so $d\sigma_{+-} + d\sigma_{-+}$ is $|A|^2 + |B|^2$. The magnitude of B is known from the knowledge of the pion form factors. Thus from the measured cross sections $d\sigma_{+-} + d\sigma_{-+}$ the amplitude $|A|^2$ can be isolated, and if ϵ exists it would appear as a resonance peak in A. Until such an experiment is performed, it is worthwhile to give a theoretical estimate of the contribution of ϵ to (1). This requires a knowledge of the coupling $\epsilon \to 2\gamma$, a reliable estimate of which has not been given previously.

In this Letter we first obtain a superconvergent sum rule from the fixed-s dispersion relation for the helicity-flip amplitude of pion Compton scattering. Then saturating the sum rule by appropriate meson resonances we estimate the product of the coupling constants for $\epsilon \to \pi\pi$ and $\epsilon \to 2\gamma$. This, along with the assumption of vector-meson dominance of the electromagnetic currents, is then used to calculate the contribution of ϵ to (1) when the colliding-beam energy is at the φ-meson resonance, for comparison with the contribution of the diagrams in Fig. 1(b). It is found that the former is about 10^2 times larger than the latter (in cross section) and is also peaked [depending on the width $\Gamma(\epsilon \to \pi\pi)$]. Thus a measurement of $d\sigma_{+-} + d\sigma_{-+}$ of (1) would unambiguously settle the question of the existence of ϵ and determine the width of $\epsilon \to 2\pi$.

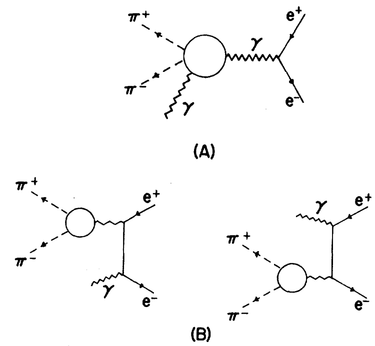

FIG. 1. Contributing diagrams to the process $e^+e^- \to \pi^+\pi^-\gamma$ (to order e^3).

We consider the pion Compton scattering $\pi^+(q) + \gamma(k) \to \pi^+(q') + \gamma(k')$, the helicities of the initial and final photons being denoted by λ and λ'. The Lorentz-invariant helicity amplitudes are denoted by $M_{\lambda,\lambda'}(s,t,u)$.[2] It has been shown[3] that the amplitude $M_{1,-1}/t$ is free of both s and t kinematic singularities and, further, sufficiently well behaved at large t so that one can write a fixed-s unsubtracted dispersion relation:

$$\frac{M_{1,-1}(s,t)}{t} = -\frac{2e^2 m_\pi^2}{(m_\pi^2 - s)(m_\pi^2 - u)} + \frac{1}{\pi}\int \frac{dt'}{t'-t}\,\frac{\mathrm{Im}M_{1,-1}(s,t')}{t'} + \frac{1}{\pi}\int \frac{du'}{u'-u}\,\frac{\mathrm{Im}M_{1,-1}(s,u')}{2m_\pi^2 - s - u'}, \tag{2}$$

where the first term on the right-hand side of (2) is the one-pion Born contribution. In a purely Reggeized world, the large-t behavior of $M_{1,-1}(s,t)/t$ is given by $t^{\alpha(s)-1}$, where the trajectory $\alpha(s)$ is one for which $\alpha(m_\pi^2) = 0$. Since the slope of α is positive, $\alpha(s) < 0$ for $s < m_\pi^2$, and $M_{1,-1}/t$ would decrease faster than t^{-1} for large t. Thus, from (2) we obtain a superconvergent sum rule[4]

$$-2e^2 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt'\,\frac{\mathrm{Im}M_{1,-1}(s=0,t')}{t'} + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} du'\,\frac{\mathrm{Im}M_{1,-1}(s=0,u')}{2m_\pi^2 - u'} = 0, \tag{3}$$

where we have put $s=0$ in (3).

We saturate the sum rules (3) with ϵ and f meson resonances in the t channel and A_1 and A_2 mesons in the u channel. We define

$$(2\pi)^3(4q_0q_0')^{1/2}\langle\pi(q),\,\pi(-q')\,|\,\epsilon(p)\rangle = g_{\epsilon\pi\pi}, \tag{4}$$

$$(2\pi)^3(4k_0k_0')^{1/2}\langle\gamma(-k),\,\gamma(k')\,|\,\epsilon(p)\rangle = e^2 g_{\epsilon 2\gamma}[k\cdot k'\delta_{\nu\nu'} - k_{\nu'}k_{\nu}]\epsilon_{\nu}(-k)\epsilon_{\nu'}(k), \tag{5}$$

$$(2\pi)^3(4q_0q_0)^{1/2}\langle\pi(q),\,\pi(-q)\,|f(p)\rangle = g_{f\pi\pi}Q_{\tau}Q_{\tau'}\epsilon_{\tau\tau'}(p), \tag{6}$$

$$\begin{aligned}(2\pi)^3(4k_0k_0')^{1/2}\langle\gamma(-k),\,\gamma(k)\,|f(p)\rangle = e^2\{&g_{f2\gamma}[k\cdot k'\delta_{\nu\nu'} - k_{\nu'}k_{\nu}']K_{\tau}K_{\tau'} + g_{f2\gamma}'[(k\cdot k')^2\delta_{\nu\tau}\delta_{\nu'\tau'} \\ &- k\cdot k'(\delta_{\nu\tau}k_{\tau'}'k_{\nu'} - \delta_{\nu'\tau'}k_{\nu}'k_{\tau}) + k_{\nu}'k_{\tau}k_{\tau'}'k_{\nu'}] \\ &\times\epsilon_{\nu}(-k)\epsilon_{\nu'}(k)\epsilon_{\tau\tau'}(p),\end{aligned} \tag{7}$$

$$(2\pi)^3(4q_0k_0')^{1/2}\langle\pi(q),\,\gamma(-k')\,|A_1(p)\rangle = eg_{A_1\pi\gamma}[q\cdot k'\delta_{\nu'\alpha} - q_{\nu'}k_{\alpha}']\epsilon_{\nu'}(-k)\epsilon_{\alpha}^{A_1}(p), \tag{8}$$

$$(2\pi)^3(4q_0k_0')^{1/2}\langle\pi(q),\,\gamma(-k')\,|A_2(p)\rangle = eg_{A_2\pi\gamma}\tfrac{1}{2}\epsilon_{\nu'\alpha\beta\mu}q_{\alpha}k_{\beta}'\epsilon_{\pi\tau}(p)(q+k')_{\tau}, \tag{9}$$

where $\epsilon_{\alpha}^{A_1}$ and $\epsilon_{\tau\tau'}^{A_2}$ are the polarization vector and tensor for A_1 and A_2.

The contributions of ϵ, f, A_1, and A_2 meson resonances to $\mathrm{Im}M_{1,-1}(s,t)$ are then calculated to be

$$\mathrm{Im}M_{1,-1}^{(\epsilon)}(s,t) = \tfrac{1}{2}\pi e^2 t g_{\epsilon\pi\pi}g_{\epsilon 2\gamma}\delta(t-m_{\epsilon}^2), \tag{10}$$

$$\mathrm{Im}M_{1,-1}^{(f)}(s,t) = \tfrac{1}{8}\pi e^2 t g_{f\pi\pi}\tilde{g}_{f2\gamma}[(s-m_{\pi}^2)^2 + t(s-m_{\pi}^2) + \tfrac{1}{3}t(\tfrac{1}{2}t+m_{\pi}^2)]\delta(t-m_f^2), \tag{11}$$

$$\mathrm{Im}M_{1,-1}^{(A_1)}(s,u) = -\pi\tfrac{1}{4}e^2 t g_{A_1\pi\gamma}^2 u\delta(u-m_{A_1}^2), \tag{12}$$

$$\mathrm{Im}M_{1,-1}^{(A_2)}(s,u) = -\pi\tfrac{1}{32}e^2 t g_{A_2\pi\gamma}^2[(2m_{\pi}^2-2s-u)u + m_{\pi}^2(s+u-2m_{\pi}^2)]\delta(u-m_{A_2}^2), \tag{13}$$

where $\tilde{g}_{f2\gamma} = g_{f2\gamma} - g_{f2\gamma}'$.

Using (10)-(13) in (3), we obtain

$$-2 - \tfrac{1}{2}g_{\epsilon\pi\pi}g_{\epsilon 2\gamma} - \tfrac{1}{48}g_{f\pi\pi}\tilde{g}_{f2\gamma}m_f^4 - \tfrac{1}{4}g_{A_1\pi\gamma}^2 m_{A_1}^2 + \tfrac{1}{32}g_{A_2\pi\gamma}^2 m_{A_2}^4 = 0, \tag{14}$$

where we have neglected terms of the order of m_{π}^2 compared with m_f^2, etc. From the Pagels-Harari[5] and Singh[6] sum rules, we obtain

$$\tfrac{1}{8}g_{A_2\pi\gamma}^2 = (0.462\pm 0.069)/\mathrm{GeV}^4,$$
$$g_{A_1\pi\gamma}^2 = (0.04\pm 1.14)/\mathrm{GeV}^2, \tag{15}$$

where we have used the experimental information $\Gamma(\omega\to\pi\gamma) = 1.19\pm 0.33$ MeV.[7] From $\Gamma(f\to 2\pi) = 151\pm 25$ MeV and $\Gamma(f\to 2\pi^+2\pi^-) \lesssim 6$ MeV,[7] along with vector dominance, we get

$$|g_{f\pi\pi}| = (1.82\pm 0.14)\times 10^1/\mathrm{GeV}, \tag{16}$$

$$|\tilde{g}_{f2\gamma}| \lesssim 0.845/\mathrm{GeV}^3. \tag{17}$$

Now using (16), (17), and (15) in (14), we obtain

$$g_{\epsilon\pi\pi}g_{\epsilon 2\gamma} \lesssim -2.056\pm 0.60 \text{ (dimensionless)}. \tag{18}$$

We should point out that the superconvergent relation predicts $g_{\epsilon\pi\pi}g_{\epsilon 2\gamma}$ to be a constant, while from perturbation theory one obtains $g_{\epsilon 2\gamma}$ to be proportional[8] to $g_{\epsilon\pi\pi}$.

We are now in a position to calculate the contribution of ϵ to the process (1). As already pointed out by Creutz and Einhorn,[1] the contribution of Fig. 2(a) can be enhanced compared with that of the diagrams in Fig. 1(b) by setting the colliding-beam energy to the φ-meson resonance (1020 MeV). To minimize the contributions of external bremsstrahlung from pions one has to observe a hard photon at right angles to the beam direction (in the e^+e^- rest system) and the pions coming out symmetrically about the photon direction. In this configuration we write

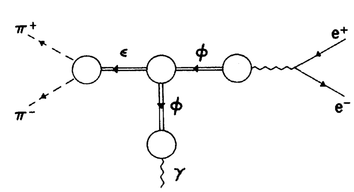

FIG. 2. ϵ- and φ-meson dominance of Fig. 1(a).

the contribution of the diagram of Fig. 2 as

$$|A|^2=\frac{e^6}{8}\frac{g_{\epsilon\pi\pi}{}^2 g_{\epsilon\varphi\varphi}{}^2}{(t-m_\epsilon{}^2)^2+m_\epsilon{}^2\Gamma_\epsilon{}^2}\left(\frac{1}{g_\varphi{}^2}\right)^2\frac{m_\varphi{}^2}{\Gamma_\varphi{}^2}\frac{(m_\varphi{}^2-t)^2}{m_\varphi{}^2}, \tag{19}$$

where we have set the beam energy to m_φ^2.

From (18), and the assumption of vector dominance, we have

$$\frac{g_{\epsilon\pi\pi}g_{\epsilon\varphi\varphi}}{g_\varphi{}^2}\gtrsim\frac{2.056}{3}\cos^2\theta_v, \tag{20}$$

where θ_v is the $\omega-\varphi$ mixing angle and is given by $\cos\theta_v=0.768$ from the mass formula. Using (20) in (19) and taking $t=m_\epsilon{}^2$, we obtain

$$|A|^2\gtrsim\frac{0.28}{8}\left(\frac{m_\varphi}{\Gamma_\varphi}\right)^2\left(\frac{m_\varphi}{m_\epsilon}\right)^2\left(1-\frac{m_\epsilon{}^2}{m_\varphi{}^2}\right)^2\frac{1}{\Gamma_\epsilon{}^2}, \tag{21}$$

which is an order of magnitude larger than the corresponding expression given by Creutz and Einhorn.[1]

The spin-average contribution from the diagrams of Fig. 1(b) is given by Creutz and Einhorn and is equal to (approximating the pion form factor by ρ meson resonance)

$$|B|^2\simeq\frac{e^6 m_\rho{}^4}{(m_\rho{}^2-m_\epsilon{}^2)^2+m_\rho{}^2\Gamma_\rho{}^2}\left(\frac{2}{m_\epsilon{}^2}\right) \tag{22}$$

$$=2e^6/\Gamma_\rho{}^2, \tag{23}$$

where we have used $m_\epsilon=m_\rho$ in (22). Using $\Gamma_\varphi=3.9$ MeV, $\Gamma_\rho=125$ MeV,[7] and $\Gamma_\epsilon=300$ MeV in (21) and (23), we get

$$|A|^2/|B|^2\gtrsim 6.96\times10^1. \tag{24}$$

Thus, $|A|^2$ is about two orders larger than $|B|^2$. The reason for the relative enhancement of the term $|A|^2$ in the present paper over that of Creutz and Einhorn[1] is our estimate $g_{\epsilon\pi\pi}{}^2 g_{\epsilon\varphi\varphi}{}^2/g_\varphi{}^4 \geq 0.163$ [Eq. (20)], which is about $2\frac{1}{2}$ times the value assumed by these authors. We also observe that the expression (21) is inversely proportional to the width $\Gamma(\epsilon\to2\pi)$, while the corresponding expression given by Creutz and Einhorn is independent of $\Gamma(\epsilon\to2\pi)$. Moreover, expression (24) is peaked between $0.6m^2$ and $0.9m^2$ in t depending upon the width $\Gamma(\epsilon\to2\pi)$, 500-300 MeV. Thus, a measurement of $d\sigma_{+-}+d\sigma_{-+}$ of (1) would determine the width $\Gamma(\epsilon\to2\pi)$ unambiguously.

Details of this work and also that of kaon Compton scattering to find the effect of δ meson resonance on $e^+e^-\to K\bar{K}\gamma$ and $e^+e^-\to\pi^0\eta^0\gamma$ will be published elsewhere.

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