Formation-Zone Effect in Transition Radiation Due to Ultrarelativistic Particles*

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The formation-zone effect in transition radiation due to charged particles with large Lorentz factor γ, both in air and in aluminum, has been investigated. A stack of aluminum foils spaced uniformly apart was used as the transition radiator. Positrons of 1- to 4-GeV energy ($\gamma = 2000$ to 8000 corresponding to protons of energy 2000-8000 GeV) were employed. The formation-zone effect sets in at about 5 mils in air and at around 0.5 mil in aluminum.

In a previous publication,[1] we reported the linear dependence of the intensity of x-ray transition radiation on the energy of the incident charged particle. In this paper, we present some measurements of an effect which is of fundamental importance in the process of generating transition radiation.

According to theoretical arguments, when a charged particle traverses the boundary surface between two different media, transition radiation is emitted if the so-called "formation zone" condition is satisfied. According to Frank,[2] at ultrarelativistic velocities, the equilibrium field entrained by the particle in a vacuum is formed along a considerable path length, namely the formation zone. A general formula for such a formation zone has been derived by Garibian for both vacuum and material.[3] For a medium with dielectric constant ϵ, the formation zone is given by

$$Z = (c/\omega)\beta/|1-\beta(\epsilon-\epsilon\sin^2\theta)^{1/2}|, \tag{1}$$

where c is the velocity of light, β is the velocity of the charged particle, ω is the frequency of the emitted transition radiation, $\epsilon = 1-(\omega_p/\omega)^2$ is the dielectric constant of the medium, θ is the angle of emission of the transition radiation with respect to the particle path, $\omega_p = (4\pi Ne^2/m_e)^{1/2}$ is the plasma frequency of the dielectric under consideration, N is the density of electrons in the dielectric medium, and m_e is the mass of the electron.

If we consider only the x-ray region of the transition radiation from ultrarelativistic charged particles, expression (1) can be reduced to a more easily applicable form. In the x-ray region, the dielectric constant ϵ is close to 1, because $\hbar\omega_p$ for aluminum is ~32 eV and $\hbar\omega$ is of the order of keV and higher. For particles in the relativistic region, $\theta \approx 1/\gamma$ where γ is the Lorentz factor $(1-\beta^2)^{-1/2}$. Therefore, expession (1) becomes

$$Z = c/\omega[1/\gamma^2 + \tfrac{1}{2}(\omega_p/\omega)^2]. \tag{2}$$

For vacuum where $\omega_p = 0$, the formation zone Z is proportional to γ^2. For material where $\omega_p \neq 0$, the formation zone is much smaller than that for vacuum. (The formation zone for air is the same as that for vacuum in the x-ray region.)

We have made some investigations on the formation-zone effect in the x-ray region of the transition radiation. These investigations were carried out at the Cambridge Electron Accelerator, employing the positron beam from the 6-GeV electron synchrotron. The experimental setup has been described in previous publications.[1,4]

We first studied the formation-zone effect in air. The basic transition radiator consists of 231 foils of 1-mil-thick aluminum uniformly spaced apart. For a fixed positron energy, the air spacing between the aluminum foils was varied from 50 down to 2 mils. The measurements on the peak transition x-ray intensity were made for four different positron energies. These results are shown in Fig. 1 where the relative transition x-ray intensity is plotted as a function of air spacing between foils for the four positron energies, namely, 1, 2, 3, and 4 GeV, respectively. The range of x-rays of the transition radiation detected extends from 3 to 270 keV. Here, the background, due to bremsstrahlung and other area background, was measured in each case by replacing the 231 aluminum foils with a solid aluminum block of the same thickness but without the multitude of interfaces. It is seen that the transition x-ray intensity remains constant when

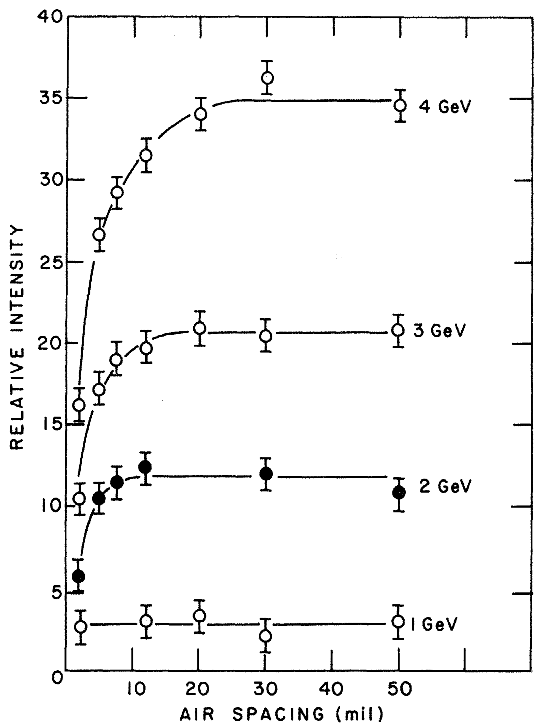

FIG. 1. Relative intensities of transition radiation for different air spacings. Each radiator is made of 231 aluminum foils 1 mil thick.

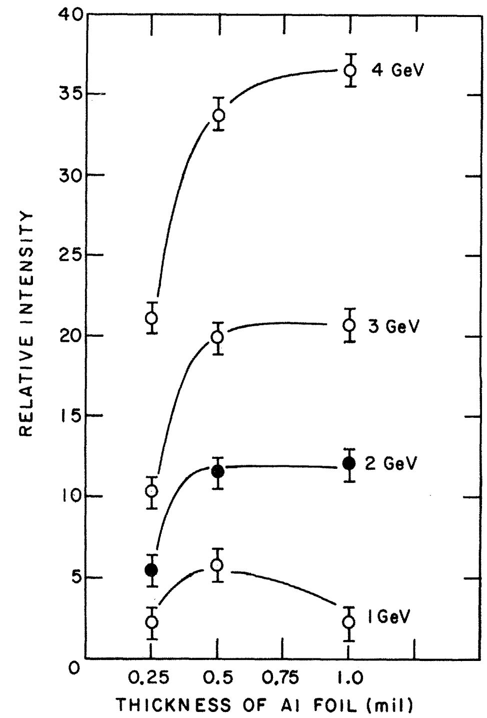

FIG. 2. Relative intensities of transition radiation for different thickness of aluminum foils. Each radiator is made of 231 aluminum foils with an air spacing of 30 mils.

the air spacing is varied from 50 down to about 10 mils, but it drops sharply when the air spacing is reduced to 2 mils. The constant intensity region between 20 and 50 mils spacing seems to hold true for energies of the positrons ranging from 1 to 4 GeV (i.e., γ from 2000 to 8000). It appears that the formation-zone effect takes place somewhere between 2 and 20 mils for γ from 2000 to 8000 under the present experimental conditions.

We also have some preliminary findings on the formation-zone effect in aluminum. Three different thickness of aluminum foils were investigated, namely, 1, 0.5, and 0.25 mil thick, respectively. The air spacing was kept at 30 mils when the thickness of aluminum foils was varied. The measurements were repeated for four different positron energies. These results are shown in Fig. 2, where the relative intensity of the transition x-ray radiation is plotted as a function of the foil thickness. The curves are drawn to show the general trend of this effect. It is seen that the transition x-ray intensity remains approximately unchanged when the aluminum foil thickness varies from 1 to 0.5 mil for all measured positron energies except at 1 GeV and drops off sharply when the foil thickness is reduced further down to 0.25 mil. At 1 GeV the intensity shows an increase from 1 to 0.5 mil thickness. This may be explained by the fact that a reduced thickness in the foils would reduce the absorption of the low-energy x rays which are predominant at this energy thus causing an increase in the detected x-ray intensity. However, when the foil thickness is further reduced to 0.25 mil, the transition x-ray intensity also drops quite sharply instead of increasing as it does in the case when the foil thickness is reduced from 1 to 0.5 mil, and this is true for all positron energies. This would seem to indicate that the formation-zone effect sets in at about 0.5-mil thickness within the range of γ considered, i.e., from 2000 to 8000.

Figure 3 shows the comparison of the experimental with the theoretical formation zone values for air and aluminum as a function of γ of the charged particle as calculated from expression (2) using the average energies of the observed x-ray spectra at the respective γ's. It is seen that the formation zone increases with increasing γ, and that the value of the formation zone in air is much larger than in aluminum. The experimental formation-zone values shown in Fig. 3

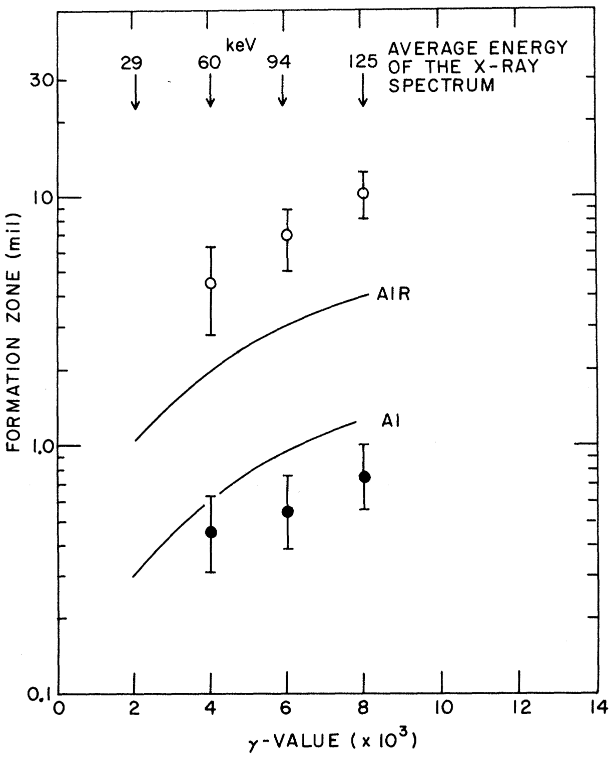

FIG. 3. Comparison of the experimental and theoretical formation-zone values. The curves represent the calculated values for air and aluminum as a function of γ, the circles represent the measured values for air spacing, and the dots represent the measured values for aluminum.

are taken as those at 90% dropoff from the constant plateaus. The circles represent the measured values for air spacing; the dots represent the measured values for aluminum. It is seen that the calculated values of the formation zone are in qualitative agreement with the experimental values. The data were also analyzed with the formula for multilayers derived by Ter-Mikaelyan.[5] The experimental results are not inconsistent with this theory under the present experimental conditions, namely, the air spacing of 2 to 50 mils, the thickness of aluminum of 1 to 0.25 mils, and the beam energies of 1 to 4 GeV. A more detailed investigation of the formation-zone effect is being undertaken at higher γ values of the charged particle where this effect is expected to become more pronounced.

We wish to express our thanks to the staff and crew of the Cambridge Electron Accelerator for their generous assistance and cooperation in carrying out this experiment. We wish to thank Dr. Fritz Dell in particular for his valuable help in various ways. We wish also to express our deep appreciation to the Grumman Aerospace Corporation for the partial financial assistance which helped us carry out this work. The Li-drifted germanium detector was supplied through the courtesy of the Instrumentation Division of Brookhaven National Laboratory.

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