Commission.
†Work performed under the auspieces of the U. S. Atomic Energy Commission under Contract No. AT(11-1) 1764.
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Effect of the Earth's Revolution Around the Sun on the Proposed Gyroscope Test of the Lense-Thirring Effect

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(Received 6 April 1970)

We analyze the proposed Stanford experiment (precession of the spin of a gyroscope in an Earth satellite) to test the Lense-Thirring effect. We show that the sun also makes a contribution to the precession which must be included, particularly if one wishes to distinguish between the Einstein and Brans-Dicke theories.

Modern technology is making possible new tests of Einstein's general theory of relativity. One of these is Schiff's[1,2] proposed gyroscope experiment. Everitt and Fairbank[3] and Fairbank[4] expect to carry out this experiment in the near future by launching a satellite containing two pairs of superconducting gyroscopes into a polar orbit around the Earth; the spin of one pair (gyro No. 1) will be parallel to the Earth's axis and the spin of the other pair (gyro No. 2) will be perpendicular to the plane of the orbit.[2-4] Not only is this test capable of distinguishing[5] between the gravitational theories of Einstein and of Brans and Dicke[6] (BD), but it is <u>the only experiment which is sensitive to the off-diagonal terms in the metric tensor</u>. The latter terms result from the Earth's rotation and were calculated by Lense and Thirring[7] soon after Einstein's work.

The angular velocity of precession of the spin axis $\vec{S}$ of a gyroscope in Einstein theory, $\vec{\Omega}_E$ say, may be written as[1,2,5]

$$\vec{\Omega}_E = \vec{\Omega}_T + \vec{\Omega}_{DS} + \vec{\Omega}_{LT} + \vec{\Omega}_Q, \tag{1}$$

where $\vec{\Omega}_T$, $\vec{\Omega}_{DS}$, $\vec{\Omega}_{LT}$, and $\vec{\Omega}_Q$ are the so-called Thomas, de Sitter, Lense-Thirring, and quadrupole-moment[8,9] contributions, respectively. From henceforth, we will regard the $\vec{\Omega}$'s as being averaged over a period of the motion. It is possible to have $\vec{\Omega}_T$ essentially zero[1] by putting the gyroscope in a satellite. The importance of selecting a <u>polar</u> orbit results from the fact that $\vec{\Omega}_{DS}$ and Ω_{LT} are at right angles[1-4] for such an orbit. For definiteness, consider the Earth's angular velocity to be in the z direction and the polar orbit to be in the xz plane so that the orbital angular momentum of the satellite points in the y direction. Then $\vec{\Omega}_{DS}$ lies along y and $\vec{\Omega}_{LT}$

along z. Thus gyro No. 1 (with spin along z) will be affected only by $\vec{\Omega}_{DS}$, and gyro No. 2 (with spin along y) only by $\vec{\Omega}_{LT}$. Since we are interested here in a test of the Lense-Thirring effect, henceforth we consider only gyro No. 2.

The magnitude[10] of $\vec{\Omega}_{LT}$ for a satellite in a circular polar orbit 300 miles above the Earth is 43.8×10^{-3} sec/yr (at this altitude the magnitude of $\vec{\Omega}_{DS}$ is the oft-quoted value of 7.0 sec/yr). Using BD theory, this value is reduced[5] by a factor of $\frac{1}{16}$, i.e., to 2.7×10^{-3} sec/yr. As before,[5] we take $\omega=6$, where ω is the dimensionless coupling constant which appears in BD theory.[6] This is the usual value taken for ω but of course it is a trivial matter to calculate the various quantities for different ω values. Thus to distinguish between the Einstein and BD theories the experiment should be capable of measuring such small precession angles. In fact, measurement accurate to 10^{-3} sec/yr will be possible[4] by use of the London moment readout technique. The question we wish to consider here is whether there are any perturbations of magnitude greater than 10^{-3} sec/yr along the z axis, in addition to $\vec{\Omega}_{LT}$. With regard to $\vec{\Omega}_{Q}$, this contribution turns out to be in the same direction as $\vec{\Omega}_{DS}$ for a polar orbit[9] (though this is not true in general[9]) and thus it has no component along z.

Consider now the precession of the gyroscope due to its journey in space around the sun. The only contribution of significance is the de Sitter contribution due to the sun, which we call $\vec{\Omega}_{DS}{}^{S}$. The magnitude of this term is easily shown to be 19.2×10^{-3} sec/yr! Since the Earth's equator is inclined at an angle θ of 23.44° to the ecliptic, the z component is $0.917\ \vec{\Omega}_{DS}{}^{S}$, i.e., 17.6×10^{-3} sec/yr. It is more than 6 times as large as the difference between the Lense-Thirring contributions arising from the Einstein and BD theories and 17.6 times larger than what can be measured! It is important to note that this de Sitter contribution due to the sun is different[5] in Einstein and BD theories. Thus the z components of the total $\vec{\Omega}$, in both Einstein[1] ($\Omega_{E,z}$) and BD[5] ($\Omega_{BD,z}$) theories, are

$$\Omega_{E,z}=\Omega_{LT}+\cos\theta\Omega_{DS}{}^{S}=61.4\times10^{-3}\text{ sec/yr} \quad (2)$$

and

$$\Omega_{BD,z}=\tfrac{15}{16}\Omega_{LT}+\tfrac{11}{12}(\cos\theta\Omega_{DS}{}^{S}) =57.2\times10^{-3}\text{ sec/yr.} \quad (3)$$

It should be noted that $\vec{\Omega}_{DS}{}^{S}$ also has a component in the x direction, to which gyro No. 2 is responsive. The magnitude of this component is $\sin\theta\cos u\ \Omega_{DS}{}^{S}$, i.e., $7.64\times10^{-3}\cos u$ sec/yr, where u is the angle between the autumnal equinox and the y direction. In conclusion, we note that it is important that the satellite orbit should not deviate from the polar axis by more than 30″ since, for deviations greater than this, $\vec{\Omega}_{DS}$ (which we saw above has the relatively large magnitude of about 7″/yr) will contribute more than 10^{-3} sec/yr to the precession of gyro No. 2.

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